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THE
ACCOMPLISHED TUTOR;
OR,
COMPLETE SYSTEM
OF
LIBERAL EDUCATION.

VOL. II.

S. GOODELL, Printer,
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THE
ACCOMPLISHED TUTOR;

OR,
Complete System



OF
LIBERAL EDUCATION:

CONTAINING THE
MOST IMPROVED THEORY AND PRACTICE

OF THE
FOLLOWING SUBJECTS:

- | | | |
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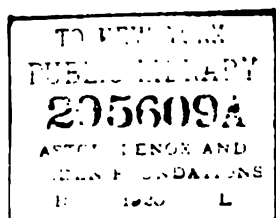
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THE
ACCOMPLISHED TUTOR.

CHAP. VII.
OF ALGEBRA.

SECT. I.
OF NOTATION.

Definitions.

1. ALGEBRA is the art of resolving difficult questions more readily than by the rules of common arithmetic.

In algebra, the value of quantities is expressed by some letters of the alphabet, which have sometimes figures, and certain characters added to them, whereby their value is increased, or diminished; and each letter may represent any quantity at pleasure. But, generally, the first letters in the alphabet, *a, b, c, d, &c.* are used to signify quantities, the value whereof is known; and the latter letters, as *u, x, y, z, &c.* are used for quantities which are unknown: the letters are then managed according to the rules of the art.

2. The sign $+$ signifies *addition*, and in algebra it is called *plus*; it denotes that the characters or letters placed on each side of it are to be added together, thus, $a + b$ signifies that

to it; as $6d$; here 6 is the coefficient, and signifies that the quantity d is multiplied thereby.

21. A *binomial quantity* consists of two terms; as $b+c$. A *trinomial quantity*, of three terms; as $a+b+c$. A *quadrinomial quantity*, of four terms; as $a+b+c+d$.

22. A *residual quantity* is a binomial, where one of the terms is a negative one; as $a-b$.

23. A *rational quantity* has no radical sign.

24. A *surd quantity* is that which has not a proper root; as the square root of b (\sqrt{b}), the biquadrate root of bb ($\sqrt[4]{bb}$).

25. The sign $:::$ signifies *proportion*; as $5:10::40:80$, that is, as 5 to 10, so is 40 to 80.

26. An *equation* is the comparison of two quantities which are equal to one another, having the sign of equality between them, as $5, 9=6, 8$, which signifies that 5 and 9 is equal to 6 and 8, or 14. Of equations there are several sorts:

1. A *dependent equation* is that which may be deduced from some others.—2. An *independent equation*, that which cannot be deduced from another.—3. A *pure equation*, that which contains but one power of the unknown quantity.—4. An *affected equation*, that which has several powers of the unknown quantity.

Axioms.

1. If equal quantities be added to equal quantities, the sums will be equal. And if equal quantities be taken from equal quantities, the remainders will be equal.

2. If equal quantities be multiplied by equal quantities, the products will be equal. And if equal quantities be divided by equal quantities, the quotients will be equal.

3. If equal quantities be raised to equal powers, the products will be equal.

4. Quantities equal to any other quantity are equal to one another.

5. The whole is equal to all its parts taken together.

SECT.

S E C T. II.

OF THE FOUR SINGLE RULES OF ALGEBRA.

OF ADDITION.

RULE 1. When like quantities having like signs are to be added together, add together the coefficients, (if there be any,) and to the sum prefix the sign, and subjoin the common quantity.

2. When the quantities are like, but have unlike signs, take the difference between the sum of the affirmative coefficients, and the sum of the negative ones; to which difference prefix the sign of the greater sum, and annex the common quantity.

3. But when the quantities are all unlike, they cannot be brought into one sum, but must be written down one after another, prefixing to each its proper sign; as in the following examples :

$7ab$	$12a^2c + 2ab$	$-10x^2y^2 - 2a^2$
$12ab$	$9x^2c + 5ab$	$= 7x^2y^2 - 7a^2$
$9ab$	$10a^2c + 2ab$	$= 4x^2y^2 - 2a^2$
$2ab$	$4a^2c + 3ab$	$= 31x^2y^2 - 1a^2$
<hr/>	<hr/>	<hr/>
$30ab$ Sum.	$35a^2c + 12ab$ Sum.	$-52x^2y^2 - 12a^2$ Sum.
<hr/>	<hr/>	<hr/>

When there is no coefficient prefixed to a quantity, the coefficient is 1. And when there is no sign prefixed, the quantity is affirmative; as in the quantities of the first and second of the foregoing examples.

Examples

Examples of like Quantities, and unlike Signs.

$-3ac-7cd+de$	$+12a^2b^2-de-12x^2$
$+7ac+4cd-de$	$+4a^2b^2+de+6x^2$
$-6ac-8cd+de$	$+5a^2b^2-de-10x^2$
<hr/>	<hr/>
$-2ac-5cd+de$	$+21a^2b^2-de-16x^2$
<hr/>	<hr/>

Unlike Quantities, and unlike Signs.

$$\begin{array}{r}
 +15a-6a^2-2b^2+5cd \\
 -2de+4xy-x^2+xy \\
 +3b^2-8x+y^2 \\
 \hline
 +15a-6a^2+b^2+5cd-2de+5xy-x^2-8x+y^2
 \end{array}$$

SUBTRACTION.

RULE. Place the quantities one under the other, and change all the signs of the subtrahend; that is, where there is an affirmative sign, place a negative one; and *vice versa*. Then add the quantities together, as in addition.

Examples.

From $+2a^2b^2-cd$	From $-ab+6a^2-9cd$
Take $+a^2b^2+cd$	Take $+4ab-2cd+x^2$
<hr/>	<hr/>
$+a^2b^2-2cd$	$5-ab+6a^2-7cd-x^2$
<hr/>	<hr/>

Subtraction, as well as each of the other four rules in algebra, is proved in the same manner as in common arithmetic *.

* The reason of this rule is evident from hence, that if a decrement or a negative quantity be taken away from an affirmative quantity, the remainder will be the same as if an increment or an affirmative quantity of equal quantity be added to the original quantity. For every negative quantity always decreases the value of any quantity with which it is joined. Thus, if $-b$ be taken from $a-b$ there will remain a , and if $+b$ be added to $a-b$, the sum is likewise a .

MULTIPLICATION.

RULE. Multiply each term of the multiplier into every term of the multiplicand: that is, the coefficients into the coefficients, and the letters into the letters; and to each prefix its sign; viz. + to like signs, and — to unlike signs,

Examples,

$$\begin{array}{r} \text{Multiply } 7a \\ \text{By } 3b \\ \hline \text{Product } 21ab \end{array}$$

$$\begin{array}{r} \text{Multiply } 3d^2 \\ \text{By } 5c^2 \\ \hline \text{Product } 15d^2c^2 \end{array}$$

$$\begin{array}{r} \text{Multiply } -5c^2d^2 \\ \text{By } +7c^2d^2 \\ \hline \text{Product } -35c^4d^4 \end{array}$$

$$\begin{array}{r} \text{Mult. } 7a-3c \\ \text{By } 12a+4c \\ \hline 84aa-36ac \\ +28ac-12cc \end{array}$$

$$\begin{array}{r} \text{Multiply } a^2-ab+c^2 \\ \text{By } -a+2b^2 \\ \hline -a^3+a^2b-ac^2 \\ +2a^2b^2-2ab^3+2c^2b^2 \end{array}$$

$$\begin{array}{r} \text{Pro. } +84aa-8ac-12cc \\ \hline \hline \text{P. } -a^3+a^2b+2a^2b^2-ac^2-2ab^3+2c^2b^2 \end{array}$$

The foregoing examples may be proved by division, as in common arithmetic *.

* This rule depends upon the same principle as multiplication in common arithmetic. And that two quantities having like signs, should give a product with the sign +, and two quantities of unlike signs the sign —, may be proved from hence: viz. 1. If an affirmative quantity be multiplied by an affirmative quantity, the product must of course be an affirmative quantity.—2. If a negative quantity be multiplied by an affirmative one, the negative quantity must be taken as often as there are units in the affirmative one, and the sum of any number of negative quantities will be negative. And if an affirmative quantity be multiplied by a negative one, the affirmative quantity must be subtracted as often as there are units in the negative one; and the sum of any number of negatives will be negative.—3. Again, if a negative quantity be multiplied by a negative quantity, the multiplicand is to be subtracted, as often as there are units in the multiplier; but, to subtract a negative quantity is the same thing as to add an equal affirmative one: therefore, the product will be affirmative. From hence the general rule, that like signs produce +, and unlike signs —.

DIVISION.

RULE. If the quantities be simple, divide the coefficient of the dividend, by the coefficient of the divisor, and place the answer in the quotient; annexing thereto those letters in the dividend, which are not found in the divisor: observing that like signs produce +, and unlike signs —.

2. But if the quantities be compound, divide the first term of the dividend by the first term of the divisor, and place the result in the quotient. Then multiply the whole divisor thereby, and subtract the product from the dividend, and to the remainder bring down the next term in the dividend. And repeat the operation as in common arithmetic. But the terms in the dividend should be ranged in a proper order, that is, according to the dimensions of some letter; the quantities represented by a being generally placed first; those by b or $a b$ next, as follows:

Examples.

$$\begin{array}{r} 9ac)36abcd(4bd \\ \underline{36abcd} \\ 0 \end{array} \qquad \begin{array}{r} 4a^2b^2)24a^2b^2d(-6bd \\ \underline{24a^2b^2d} \\ 0 \end{array}$$

$$\begin{array}{r} 4a-6c)16abbb-24cbb(4bb \\ \underline{16abbb-24cbb} \\ 0 \end{array}$$

$$\begin{array}{r} 3a-b)3a^3-12a^2-ba^2+10ab-ab^2(a^2-4a+2b) \\ \underline{3a^3} \qquad \qquad \underline{ba^2} \\ -12a^2 \qquad \qquad +10ab \\ -12a^2 \qquad \qquad +4ab \\ \hline \qquad \qquad \qquad +6ab-2b^2 \\ \qquad \qquad \qquad +6ab-2b^2 \\ \hline \qquad \qquad \qquad \qquad \qquad \qquad 0 \end{array}$$

When

When the divisor will not exactly divide the dividend, as is often the case, the dividend is to be placed over the divisor, and a line drawn between them, like a fraction, throwing out such letters as are found in both the divisor and dividend. Thus, If $ab + c^3$ was to be divided by $a b - c^2$, it would stand thus, $\frac{ab + c^3}{ab - c^2} = \frac{+c^3}{-c^2}$. When the power of a quantity is to be divided by any other power of the same quantity, it is done by subtracting the exponent of the divisor, from that of the dividend :

$$\text{Thus, } \frac{b^8}{b^4} = b^{4*}.$$

SECT. III.

OF FRACTIONAL QUANTITIES.

BEFORE the student proceed to equations, it is necessary that he know how to manage fractional quantities; and to raise a quantity to any given power; and, on the contrary, to extract the root of any quantity; to manage surd quantities, &c.

The rules for managing Algebraic Fractions are exactly the same as those for Vulgar Fractions in arithmetic, and therefore need not be repeated; as few persons would attempt Algebra, till they were sufficiently skilled in common arithmetic. An example or two may, however, be of service.

* To prove the reason of this rule, that like signs give +, and unlike signs —, it is only necessary that the divisor be multiplied by the quotient, and the product will be equal to the dividend.

EXAMPLE 1. Reduce the mixed quantity $a - \frac{c}{d}$, to a fraction. Here $\frac{ad-c}{d}$, is the fraction required.

EXAMPLE 2. Let the fraction $\frac{bd-b^2}{d}$, be reduced to a mixed quantity. Here dividing $bd-b^2$ by d , the quantity will be b , and the remainder $-b^2$, and therefore the mixed quantity will be $b - \frac{b^2}{d}$.

EXAMPLE 3. Reduce the fractions $\frac{a}{b} \frac{c}{d}$ and $\frac{e}{f}$ to fractions of the same value, having a common denominator. Here $\frac{adf}{bdf} \frac{cbf}{bdf} \frac{ebd}{bdf}$ are the fractions required.

Involution; or, to find any given Power of any given Quantity.

RULE. Multiply the quantity into itself as often as the index contains units, except one, and the last product will be the required power; or, which is more convenient, multiply the index of the quantity by the index of the power.

Thus, let it be required to raise the quantities $b+c$ and $b-c$ to the third power, or the cube.

$$\begin{array}{rcl}
 b+c & \text{Root} & \\
 \hline
 b+c & & \\
 b^2+bc & & \\
 +bc+c^2 & & \\
 \hline
 b^2+2bc+c^2 & = \text{Square} & \\
 b+c & & \\
 \hline
 b^3+2b^2c+bc^2 & & \\
 +b^2c+2bc^2+c^3 & & \\
 \hline
 b^3+3b^2c+3bc^2+c^3 & = \text{Cube} &
 \end{array}$$

$b-c$

$$\begin{array}{r}
 b - c \quad \text{Root} \\
 \hline
 b - c \\
 \hline
 b^2 - bc \\
 \quad - bc + c^2 \\
 \hline
 b^2 - 2bc + c^2 = \quad \text{Square} \\
 \hline
 b - c \\
 \hline
 b^3 - 2b^2c + bc^2 \\
 \quad - b^2c + 2bc^2 - c^3 \\
 \hline
 b^3 - 3b^2c + 3bc^2 - c^3 \quad \text{Cube.}
 \end{array}$$

These quantities are raised to the third power only; but by the same method of proceeding, quantities may be raised to any higher power.

If a trinomial, or quadrinomial, &c. be required to be raised to any power, it will be best done by taking the first or the last term of the quantity, and for all the other terms substitute any single term. These two terms being raised to the required power, the answer will be obtained by replacing, instead of the substituted term, the proper value.

Thus, if it be required to raise $b - c + d - ef + g$ to the fourth power, for the terms $b - c + d - ef$, substitute the term a , then the quantity will be $a + g$, which being raised to the fourth power, the quantity a may be taken away, and the proper value placed instead thereof in the product, which the learner may prove at his leisure.

In involving a fractional quantity, both the numerator and denominator must be raised to the required power, by which a new fraction will be obtained, being the answer of the question. Thus, the third power $\frac{a^2}{b} = \frac{a^6}{b^3}$.

The *Binomial Theorem*, invented by *Sir Isaac Newton*, is the most elegant and concise method of raising a quantity to any power, and is as follows: let x denote any number at

pleasure, and let $a+b$ be a binomial, then the n^{th} power thereof will be as follows :

$$\begin{aligned} & \text{by } a + na^{\frac{n-1}{1}}b + \frac{n \cdot n-1}{1 \cdot 2}a^{\frac{n-2}{2}}b^2 + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3}a^{\frac{n-3}{3}}b^3 \\ & + \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4}a^{\frac{n-4}{4}}b^4 + \frac{n \cdot n-1 \cdot n-2 \cdot n-3 \cdot n-4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}a^{\frac{n-5}{5}}b^5, \&c. \end{aligned}$$

And the n^{th} power of $a-b$, is expressed in the same manner, except that the signs of every other term will be negative.

To illustrate this theorem, let $a+b$ be involved to the third power. In this case, the index is 3, which must be placed in the theorem instead of n , then the first term will be a^3 , the second term $3a^{3-1}b=3a^2b$; the third term, $\frac{3 \times 2}{2}a^{3-2}b^2=3ab^2$, the fourth term $\frac{3 \times 2 \times 1}{2 \times 3}a^{3-3}b^3=b^3$; the fifth and following terms are equal to nothing. Therefore these four terms together, or the third power of $a+b$, is $a^3 + 3ab^2 + 3ab^2 + b^3$.

It may here be observed that the coefficients increase till the indices of the two letters a and b become equal or change values; then they return or decrease again in the same order: thus, having the coefficients of half the terms, the rest are known.

Evolution; or, to extract the Root of a given Power.

RULE 1. If the quantities be simple, extract the root of the coefficient for the new coefficient, and divide the index of the letters by the index of the power, and the quotient will be the root required.

Thus, the square root of $25b^2$ will be $5b^{\frac{2}{2}}=5b$, and the cube root of $27x^3=3x^{\frac{3}{3}}=3x$.

RULE

RULE 2. If the quantities be compound, after ranging the terms according to the dimensions of some letter, so that the highest power of that letter may stand first in order, and the lower powers of the same letter follow, according to the dimensions of their power: take the root of the first term, and place it in the quotient, and if it be the square, or cube root, subtract the square, or cube thereof, from the first term, bring down two, or three of the next terms for a dividend, according as the case shall be the square, or cube root; then proceed to find the divisor as in extracting the square, or cube root; is common arithmetic. And if the root of a higher power be to be extracted, it is performed in the same manner as in common arithmetic.

Examples.

EXAMPLE 1. What is the square root of

$$\begin{array}{r} 36x^4 + 108x^2 + 81(6x^2 + 9) \\ 36x^4 \\ \hline 108x^2 + 81 \\ + 108x^2 + 81 \\ \hline \end{array}$$

EXAMPLE 2. What is the square root of $9x^4 + 16a^4 + 4b^4 - 24a^2x^2 + 12b^2x^2 - 16a^2b^2$. Answer $3x^2 - 4a^2 + 2b^2$.

EXAMPLE 3. What is the cube root of

$$\begin{array}{r} x^3 - 6x^2y + 12xy^2 - 8y^3(x - 2y) \quad \text{Root} \\ x^3 \\ \hline 3x^26xy + 4y^2) - 6x^2y + 12xy^2 - 8y^3 \\ \hline - 6x^2y + 12xy^2 - 8y^3 \\ \hline \end{array}$$

Surd Quantities.

When a quantity has not a perfect root, it is called a surd quantity; and the root cannot be expressed any other way, than by either inserting the quantity with its proper radical sign, or throwing it into an infinite series. Thus, the square

root

root of a , can be expressed in no other way than by $\sqrt[n]{a}$, or $a^{\frac{1}{n}}$; the cube root of b^2 by $\sqrt[3]{b^2}$ or $b^{\frac{2}{3}}$; the cube root of $\frac{a^2b}{c^2}$ by $\sqrt[3]{\frac{a^2b}{c^2}}$.

To reduce a rational Quantity to the Form of a Surd.

RULE. Multiply the index of the quantity by the index of the surd, and over the product place the radical sign, and it will be the form required. Thus, let 3 be reduced to the form of $\sqrt{5}$. Here $3^1 \times 2$, or $3^2 = 9$, therefore, $\sqrt{9}$ is the surd required.

Again, let a^2 be reduced to the form of a cube surd of the form of $\sqrt[3]{b}$. Here $a^2 \times 3 = a^6$ and $\sqrt[3]{a^6}$ is the surd quantity.

To reduce Quantities of different Indices to other Quantities equal in Value, and having one given Index.

RULE. Divide the indices of the quantities by the given index, and the quotients will be the new indices of those quantities. Then over the said quantities with their new indices place the given index, and they will be the equivalent values required.

EXAMPLE. Let $12\frac{1}{2}$, and $9\frac{1}{3}$, be reduced to equal quantities, having the common index $\frac{1}{5}$. Here, $\frac{1}{2} \div \frac{1}{5} = \frac{5}{2} = 2\frac{1}{2}$, the index of the first quantity, and $\frac{1}{3} \div \frac{1}{5} = \frac{5}{3} = 1\frac{2}{3}$, the index of the second quantity, therefore, $12\frac{1}{2}\frac{1}{5}$ and $9\frac{1}{3}\frac{1}{5}$, are the quantities required.

To reduce a Surd Quantity to its most simple Terms.

RULE. Divide the surd by the greatest power which it contains, and place the root of such power before the quotient with the radical sign between them.

Thus,

Thus, let $\sqrt{32}$ be reduced to its most simple terms. Here 16 is the greatest square number that will divide 32, which being divided by 16, the quotient is 2, therefore, $4\sqrt{2}$ is the surd required.

Again the most simple term of the surd $\sqrt{61a^2b}$ is $9a\sqrt{b}$.

To find whether Surds are commensurable, or not.

RULE. Reduce the surds to the least common index; and the quantities, if fractions to a common denominator, except when like terms are commensurable; then divide them by the greatest common divisor, or by such a divisor as will give one rational quotient, and if both the quotients are rational, the surds are commensurable; otherwise, not.

EXAMPLE. Let $\sqrt{27}$ and $\sqrt{12}$ be given to find whether they are commensurable, these two surds have already one common index, and are equal to $\sqrt{3 \times 9}$, and $\sqrt{3 \times 4}$, respectively. Therefore, divide 27 and 12 by 3, and the quotients are $\sqrt{9}$ and $\sqrt{4}$, that is, 3 and 2; therefore, they are commensurable.

To add, or subtract Surd Quantities.

RULE. If the quantities have unlike indices, reduce them to quantities with like indices; and fractional quantities must be reduced to a common denominator, or to other fractions that have rational denominators or numerators; then reduce the quantities to their simplest terms, and if the surd part be the same, in all, annex it to the sum or difference with the sign \times ; but if the surd part is not the same in all, the quantities must be added or subtracted by joining them together with the sign $+$ or $-$.

EXAMPLE

EXAMPLE 1. Let $\sqrt{32}$ $\sqrt{72}$ be added together. Here $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$; and $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$, and the sum $= 4 + 6 \times \sqrt{2} = 10\sqrt{2}$.

EXAMPLE 2. Let $\sqrt{4a}$, and $\sqrt[4]{a^6}$ be added together. Here $\sqrt{4a} = \sqrt{16a^2} = 2\sqrt[4]{a^2}$. And $\sqrt[4]{a^6} = \sqrt[4]{a^4 \times a^2} = a\sqrt[4]{a^2}$. Therefore, their sum $= a + 2 \times \sqrt[4]{a^2} = a + 2 \times \sqrt{a}$. If it were required to have subtracted $\sqrt{4a}$ from $\sqrt[4]{a^6}$, the remainder would have been $a - 2 \times \sqrt{a}$. Also if $\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7}$ be added to $2\sqrt[3]{a^2} + \sqrt{a} - \sqrt{3}$, the sum will be $3\sqrt[3]{a^2} - \sqrt{a^3} + \sqrt{7} + \sqrt{a} - \sqrt{3}$.

To multiply and divide Surds.

RULE. Reduce the surds to the same index; and the product or quotient of the rational quantities being annexed to the product or quotient of the surds, will give the product or quotient required.

EXAMPLE 1. Multiply $2\sqrt{2}$ by $3\sqrt{3}$, these surds have the same index already, therefore, $2 \times 3 \times \sqrt{2 \times 3} = 6\sqrt{6}$, thus, $6\sqrt{6}$ is the product required.

EXAMPLE 2. Multiply $a^{\frac{1}{3}}$ by $b^{\frac{1}{2}}$. Here $a^{\frac{1}{3}} = a^{\frac{2}{6}}$, and $b^{\frac{1}{2}} = b^{\frac{3}{6}}$, therefore, their product is $a^{\frac{2}{6}} b^{\frac{3}{6}}$.

EXAMPLE 3. Let $x^{\frac{1}{2}}$ be divided by $x^{\frac{1}{3}} + y^{\frac{1}{2}}$, this is the same as if the dividend $x^{\frac{1}{2}}$ was multiplied by $\frac{1}{x^{\frac{1}{3}} + y^{\frac{1}{2}}}$; therefore, the quotient $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{2}}} \bigg| \frac{1}{6}$

Involution or Evolution of Surd Quantities.

RULE. If the surd be a simple quantity, multiply the index of the quantity by the index of the power, to which the surd is to be involved; or by the fraction, expressing the root to which it is to be evolved; and if there is a rational part, its proper power or root is to be prefixed thereto.

Compound surds are involved and extracted as integers or rational quantities, having regard to the operation of simple surds.

EXAMPLE 1. What is the square of $a\sqrt{x}$? Here the square of $a=aa=a^2$ and $\sqrt{x}=x$; therefore the square of $a\sqrt{x}=a^2x$.

EXAMPLE 2. What is the cube of $a\sqrt{x\frac{1}{2}}$? Here the cube of $a=a^3$, and the cube of $\sqrt{x\frac{1}{2}}=\sqrt{x^{\frac{1}{2}}}$, therefore the cube of $a\sqrt{x\frac{1}{2}}=a^3\sqrt{x^{\frac{1}{2}}}$.

EXAMPLE 3. What is the cube root of $\overline{a+x}\frac{1}{2}$? Here $\overline{a+x}\frac{1}{2} \times \frac{1}{2} = \overline{a+x}\frac{1}{4}$.

SECT. IV.

OF EQUATIONS.

AN Equation is the mutual comparing of two equal quantities, having the sign $=$ between them. Thus, if a be equal to 3, and b to 6, and c to 4, and d to 13, then, a added to b , will be equal to d made less by c ; and is thus expressed in algebra $a+b=d-c$.

To reduce an Equation.

When a question is brought to an Equation, in order to understand the value thereof, the quantity or quantities sought must be placed on one side of the equation, and the known quantities on the other side. For this purpose, the following rules must be attended to:—

First. When any quantity is expressed on both sides of the equation, it may be entirely rejected, or thrown out of both. Thus, if $3d + 7x = 4c - 2b + 7x$. Here $7x$ should be rejected from both sides of the equation, then it will stand thus, $3d = 4c - 2b$.

Second. When known and unknown quantities are both on the same side of the equation, the known quantities must be brought to one side of the equation, and the unknown quantities to the other side, and those quantities so transposed must have their signs changed. That is, those which have the sign +, must, after they are transposed to the other side, have the sign —. And those which have the —, must, after transposition, have the +. Thus, if $10 + 5 = x - 5$: here if x be the quantity sought, -5 must be transposed to the other side of the equation with the sign +; it will then stand thus, $+5 + 10 + 5 = x$; therefore $x = 20$. Again, what is the value of x in this equation? $24 - 4x + 10 = 60 - 12x$. Here $+24$ and $+10$ on the first side of the equation must be transferred to the other side, and $+12x$ in the second side of the equation being transferred to the first side, it will stand thus, $-4x + 12x = 60 - 24 - 10$. And by subtracting $4x$ from $12x$; and 24 and 10 from 60 , the equation will be $8x = 26$: therefore $x = 3\frac{1}{4}$.

Third.

Third. If there be fractions in the equation, multiply both sides of the equation by the denominators of the fractions; and the product will be the true integral quantities.

EXAMPLE 1. Reduce the fractional equation $a + \frac{b^2}{x} = c$ to integral quantities. Here, by multiplying the whole by x , we shall have $ax + b^2 = cx$.

Again, If there be given $\frac{a^2}{a+x} + \frac{b^2}{x} = c$, then will $a^2x + b^2(a+x) = ca + x \times x$.

Fourth. If in the unknown quantity there be a surd, all the other terms must be transposed to the contrary side, and each side of the equation involved according to the index of the surd; and if there be more surds than one, the operation must be as often repeated as there are surd quantities. Thus, if $\sqrt{x^2 + ax} + d = c$, by transposing d , the equation will be $\sqrt{x^2 + ax} = c - d$, and by squaring both sides, the equation is $x^2 + ax = c^2 - 2cd + d^2$, thus the equator is freed from the surd.

Fifth. When any quantity is multiplied into both sides of the equation, or into the highest term of the unknown quantity, divide the whole equation thereby. Thus, the equation $5bx^2 = 3bc$ is divided by b , and it becomes $5x^2 = 3c$; again if it were divided by 5 it would be $x^2 = \frac{3c}{5}$.

Sixth. When the side of the equation containing the unknown quantity is a pure power, or when being affected it has a rational root, extract the root from both sides of the equation. Thus in the equation $a^2 = b^2 + ax$, the equation will be $a = \sqrt{b^2 + ax}$. Again, if $x^2 + 4x + 9 = 25c$ be given by taking the square root we have $x + 2x + 3 = 5c$.

Each or all of the foregoing rules are to be used as may be necessary, till the equation be brought to a proper form.

Examples, wherein the foregoing Rules appear.

EXAMPLE 1. What is the value of x , in the equation
 $10 + \frac{36}{12-x} = 16.$

By subtracting 10 from each side of the equation, we have
 $\frac{36}{12-x} = 6$, both sides of which divided by 6, the quotient
 is $\frac{6}{12-x} = 1$, this multiplied by $12-x$, gives $6 = 12-x$,
 whence by transposing x and 6, we have $x = 12-6$, or
 $x=6$.

EXAMPLE 2. What is the value of x in the equation
 $\frac{ax^2+ac^2}{a+x} = ax+b^2$. Here multiplying by $a+x$, there comes
 out $ax^2+ac^2 = \overline{ax+b^2} \times \overline{a+x}$, or $ax^2+ac^2 = a^2x+ab^2+ax^2+$
 b^2x , which transposed and ordered according to the foregoing
 rules, is $a^2x+b^2x = -ab^2+ac^2$, wherefore, $x = \frac{-ab^2+ac^2}{a^2+b^2}$,
 so that if $a=1$, $b=2$, $c=3$, then will $x = \frac{-4+9}{1+4} = 1$.

EXAMPLE 3. What is the value of x in the equation
 $\sqrt{a^2+x^2} = \sqrt{b^2+x^2}$. Both sides of this equation being
 raised to the fourth power, we have $a^4+2a^2x^2+x^4 = b^4+x^4$,
 which by transposition, &c. becomes $2a^2x^2 = b^4-a^4$, which
 divided by $2a^2$, becomes $x^2 = \frac{b^4-a^4}{2a^2}$, therefore $x = \sqrt{\frac{b^4-a^4}{2a^2}}$.

EXAMPLE 4. What is the value of x in the following
 equation: $x = \sqrt{c^2+x\sqrt{b^2+x^2}}-c$. In this equation $x+c =$
 $\sqrt{c^2+x\sqrt{b^2+x^2}}$, which squared gives $x^2+2cx+c^2 = c^2+$
 $x\sqrt{b^2+x^2}$

$x\sqrt{b^2+x^2}$, or $x^2+2cx=x\sqrt{b^2+x^2}$; dividing this by x , it quotes $x+2c=\sqrt{b^2+x^2}$; this squared gives $x^2+4cx+4c^2=b^2+x^2$, and by transposition it becomes $4cx=b^2-4c^2$; and dividing by $4c$ we have $x=\frac{b^2-4c^2}{4c}$.

To exterminate an unknown Quantity out of several Equations; or, to reduce two or more Equations to a single one.

RULE. If the quantity to be exterminated, has but one dimension in the equation, find the value of it in two equations, and put those values equal to each other; or having found the value in one equation, substitute it in the room of the quantity in the other equations. Proceed in the same manner with every unknown quantity. But if the quantity to be exterminated be of several dimensions, find the value of its highest power in two equations. Then if the coefficients are not the same, multiply the less quantity, so that it may become equal to the greater. Put these values equal to each other, and there will arise a new equation, with a less power of the unknown quantities: and the operation must be repeated till the quantity be exterminated.

Examples.

EXAMPLE 1. What is the value of x and y in these two equations, $7x-5y=28$ and $3x+4y=55$, by transposing 28 in the first equation, and 5y, we have $7x-28=5y$, therefore the value of y is $\frac{7x-28}{5}$.

In

In the second equation by proceeding in the same manner, viz.:—By transposing $3x$, the value of y is found to be $\frac{55-3x}{4}$; therefore, these two quantities being put equal

to each other, we have the equation $\frac{7x-28}{5} = \frac{55-3x}{4}$. In

this equation only x is concerned. Multiply this equation by 20, which is the production of 4 and 5; or, which is the same thing, multiply the numerators and the denominators cross ways, and there will arise the equation $28x-112=275-15x$, which by transposition becomes $43x=387$, or $x=\frac{387}{43}$;

therefore, $x=9$; therefore, 9 being substituted in either of the given equations instead of x , the value of y will be found. Thus, in the first equation if 9 be substituted for x , it will be $63-5y=28$, which transposed is $\frac{63-28}{5}=y$, or $7=y$.

EXAMPLE 2. Required the value of x , y , and z , in the three following questions:

$x+100=y+z$; $y+100=2x+2z$; $z+100=3x+3y$, by transposing 100 in the first equation, $x=y+z-100$ arises, which value substituted in the other two equations, instead of x , we have the two following:—

$$y+100 (=3y+2z-200+2z)=2y+4z-200$$

$z+100 (=3y+3z-300+3y)=6y+3z-300$, then by transposing y and $4z-200$, in the first of these two equations we have $300-4z=y$, which substituted for the y in the last equation, is $z+100=1800-24z+3z-300$, that is $z+100=1500-21z$; wherefore $22z=1400$, or $z=\frac{1400}{22}=63\frac{1}{11}$; therefore, $y=300-4z=45\frac{1}{11}$, and $x=y+z-100=9\frac{1}{11}$.

Of the Nature and Composition of Equations, containing different Dimensions of the same unknown Quantity.

It often happens that the unknown quantity will be of several different dimensions, then such equation is called a quadratic, a cubic, a biquadratic equation, &c. according as the dimension of the highest power is a square, cube, or biquadrate: in such equations we must discover the root or value of the unknown quantities.

All equations are derived, (or may be considered so,) from those of a more simple form. Thus, if $x-b=0$, which is a simple equation, be raised to the second power, there arises $x^2-2bx+b^2=0$, which is called a quadratic equation; if the former equation be raised to the third power, we have $x^3-3bx^2+3b^2x-b^3=0$, which is called a cubic equation, and so on. It is but seldom that equations occur in this regular form, for the coefficients of the terms will generally be more or less than those produced by the involution of one quantity, as $x-b$, and therefore, a quadratic equation is a compound one, generally derived from $\overline{x-b} \times \overline{x-c}$. A cubic equation is derived from $\overline{x-b} \times \overline{x-c} \times \overline{x-d}$. A biquadratic equation from $\overline{x-b} \times \overline{x-c} \times \overline{x-d} \times \overline{x-e}$, or from a quadratic squared, &c. But, the letters b, c, d , &c. may have either affirmative or negative signs.

In equations of this nature, as the whole is equal to nothing, it is obvious that some or other of the factors must be equal to nothing. It is also evident that any such equation may be divided by its factors, till there remain only one factor; and as each of the inferior equations obtained by such division, must still be equal to nothing, it must follow that each of these factors themselves are equal to nothing; therefore, b, c, d, e , &c. exhibit so many different values of x with contrary signs; therefore every equation has as many roots

roots as there are dimensions of the unknown quantity in its highest power. And where b, c, d, e , &c. are found negative, x is affirmative; and where any of these are affirmative, x is negative. By multiplying the factors or roots together, under different signs, it is observable that when b, c, d , &c. are all negative, or, which is the same thing, when all the values of x are affirmative, the signs in the equation are +, and — alternately. But when there is a negative root, one affirmative quantity will follow another; therefore, there will be as many affirmative roots in the equation as there are changes of the signs from + to —, and from — to +, and all the rest will be negative.

What is here delivered, concerns only possible roots. An impossible root is when b, c, d , &c. denote the square or any other even root of a negative quantity: an equation derived from such roots is an impossible or imaginary one: if there be one possible root, the equation will admit of one possible answer.

In the multiplication of the roots of such equations, the coefficient of the second term is the sum of all the roots with contrary signs; the coefficient in the third term is equal to the sum of the rectangles of those roots; or, of all the products that can possibly arise by combining them two and two; the coefficient of the fourth term is equal to the sum of all the products that can possibly arise by the combination of them three and three, &c. and the last term is always equal to the product of all the roots with contrary signs.

The Resolution of Quadratic Equations.

If it be a pure quadratic, as $x^2 = b^2$, or $x^2 - b^2 = a$, it is produced from the rectangle of $x - b$ and $x + b$, and therefore has one affirmative, and one negative root, and the affirmative root is equal in number to the negative. The root in this case is found by extracting the square root of the number denoted by b^2 . Thus, if $x^2 + = 576$, then $x = +\sqrt{576} = +24$.
All

All other quadratics are comprehended under some of the following forms: viz. $x^2 + 2bx = +d = 0$; or, $x^2 - 2bx + d = 0$; or, $2bx - x^2 + d = 0$; and this last form, by transposition, becomes the same as the second form, only the negative roots are changed into affirmative roots, and the affirmative into negative; therefore we may consider the two other forms as applicable to all cases, and in the solution of them it will be more commodious to transpose d , and then they will stand thus: $x^2 + 2bx = +d$ and $x^2 - 2bx + d = 0$. Now if to each of these equations be added b^2 (the square of half the coefficient of the second term) we shall have in the former case $x^2 + 2bx + b^2 = +d + b^2$, and in the latter case $x^2 - 2bx + b^2 = +d + b^2$, and by extracting the square roots, the equations become $x + b = \sqrt{+d + b^2}$ and $x - b = \sqrt{+d + b^2}$ respectively; and x in the former case $= \sqrt{+d + b^2} - b$, and in the latter $= \sqrt{+d + b^2} + b$ which expressions give the affirmative values of x : but the square roots of the above equations may also be $-x - b = -\sqrt{+d + b^2}$ and $-x + b = -\sqrt{+d + b^2}$ respectively; and therefore in the former case $x = -\sqrt{+d + b^2} + b$, and in the latter $= -\sqrt{+d + b^2} - b$. That is, if $B = \sqrt{+d + b^2}$, where d is $+$ or $-$, according as it is $+$ or $-$ in the second side of the given equation; then in the first case, where $x^2 + 2bx = +d$ the values of $+x$ are $+B + b$, where b is $-$ or $+$ according as x is affirmative or negative; and in the second case, where $x^2 - 2bx = +d$, the values of $+x$ are $+B + b$, where b is $+$ or $-$, as x is $+$ or $-$.

Examples.

EXAMPLE 1. What are the two values of x in this equation, $x^2 + 6x = 2295$? Here $b = 3$, and $2295 = +d$, and $\sqrt{+d + b^2} = \sqrt{2295 + 9} = 48 = B$, and $+B + b = +48 + 3 = +51$, and -51 for the two values of x .

EXAMPLE 2. What are the two values of x in $x^2 - 12x = -28$? Here $b = 6$, and $-28 = +d$, and $\sqrt{+d + b^2} = \sqrt{-28 + 36} = 4 = B$, and $+B + b = +4 + 6 = +10$, and -10 for the two values of x .

$\sqrt{-d+b^2} = \sqrt{-28+30 \cdot 25} = \sqrt{2 \cdot 25} = 1 \cdot 5 = B$; and
 $+B+b = +1 \cdot 5 + 5 \cdot 5 = +7$, and $+4$ the two values of x ,
 which are here both affirmative.

To increase or diminish the Roots of Equations.

RULE. Substitute a new letter for the unknown quantity;
 the given increment, or $+$ the given decrement; and sub-
 stitute the powers thereof in the equation, instead of the un-
 known letter.

EXAMPLE. Increase the roots of the following equation by
 $2 \cdot x^3 + x^2 - 10x + 8 = 0$. Let $x+2=z$, or $z-2=x$, then
 $x^3 = \overline{z-2}^3$, $x^2 = \overline{z-2}^2$ and $-10x = -10 \times z - 2 = -10z + 20$
 then the powers of x being actually involved, and the se-
 veral terms collected, we have $x^3 + x^2 - 10x + 8 = z^3 - 5z^2 -$
 $2z + 24 = 0$, where the roots of z are greater than those of x
 by 2.

Thus all the negative roots of an equation may be made
 affirmative, by increasing them with a proper quantity.

To complete a deficient Equation.

RULE. Increase or diminish the roots of the equation, by
 some given quantity, as shewn in the last example.

*To multiply or divide the Roots of any Equation, by a
 given Quantity.*

RULE. Multiply or divide any new letter by the given
 number; and substitute its powers in the equation for the
 unknown quantity.

EXAMPLE. Divide the roots of the equation $x^3 - 2x + \sqrt{3} = 0$
 by $\sqrt{3}$. Here by putting $x = y\sqrt{3}$, and substituting it for x ,
 we have $3y^3\sqrt{3} - 2y\sqrt{3} + \sqrt{3} = 0$, which, by dividing by
 $\sqrt{3}$, is $3y^3 - 2y + 1 = 0$ for the equation required.

By this rule, fractions and surds may be taken out of an
 equation, viz. by dividing the new letter by the common
 deno-

denominator; or by multiplying the new letter by the surd quantity.

To take away any Term out of an Equation.

RULE. Add an unknown quantity to a new letter, and substitute this sum and the powers thereof, for the root in the given equation; then any term, or any of those quantities wherein the new letter is of the same power, being put into an equation, and made equal to nothing, will give the value of the unknown quantity, which being put into the equation with the new letter, the power of the new letter, which was equated, will vanish.

Example.

$$\text{Suppose } x^4 - 3x^3 + 3x^2 - 5x - 2 = 0$$

$$\text{Put } y + e = x \text{ then } x^4 = y^4 + 4y^3e + 6y^2e^2 + 4ye^3 + e^4$$

$$- 3x^3 = - 3y^3 - 9y^2e - 9ye^2 - 3e^3$$

$$+ 3x^2 = + 3y^2 + 6ye + 3e^2$$

$$- 5x = - 5y - 5e$$

$$+ 2 = + 2$$

If the second term is to be taken away, we have $4y^3e - 3y^3 = 0$; and dividing by y^3 , $4e - 3 = 0$, or, $e = \frac{3}{4}$, which substituted for e , the second term will vanish. If the third term is to be taken away, we have $6y^2e^2 - 9y^2e + 3y^2 = 0$, and dividing by the y^2 , we have $6e^2 - 9e + 3 = 0$, from which quadratic equation e may be determined. In like manner the fourth term may be taken away by solving the cubic equation: and the fifth term by solving a biquadratic equation, &c.

To resolve or extract the Root of a cubic Equation.

RULE. Take the second term of the equation away, as taught in the last example; then the equation will be in this form, $x^3 + ax = b$, and the following general expression will give the value of x .

$$\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}^{\frac{1}{3}} \quad \frac{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}^{\frac{1}{3}}}{\frac{1}{3}}$$

There are several other particular rules for the solution of biquadratic and other higher equations; but the method of approximating the roots of equations, has superseded all the other methods, on account of its dispatch.

To approximate the Roots of Equations, in general.

RULE. By several trials, choose some number to represent the unknown quantity, and such a number that approaches pretty near to the true value. Then assume some letter, as v , to denote the defect or excess of the number so found, and put that number + or $-v$, instead of the unknown quantity in the equation; by which a new equation will arise affected with v only, and known quantities; wherein all the terms that contain two or more dimensions of v may be rejected as inconsiderable in respect to the rest. This being done, the value of v will be found by a simple equation, which, added to or subtracted from the said number, according as it was taken, too little or too big, will give a number still nearer the truth. Then with this number and the letter v , proceed as before to find another value of v , which must be applied as above: repeat the operation till the unknown quantity be discovered to a sufficient degree of exactness.

EXAMPLE. Let it be required to find the value of x , in this equation, $x^3 + 24x = 587814$. Here by a few trials it will be found that x is something above 80; wherefore, let $80 + v = x$; then, $x^3 = 512000 + 192000v + 16800v^2 + v^3$, and $24x = 1920 + 24v$. Therefore (rejecting those terms affected with

with v^2 and v^3) we have $513920 + 19224v = 587914$, and

$$v = \frac{587914 - 513920}{19224} = \frac{73994}{19224} = 3.8$$
: this, added to 80,

gives 83.8 for the approximate value of x ; this being substituted in the equation, it will be found too great; therefore, for another operation take $83.8 - v = x$, and $x^3 = 588480.472 - 21067.32v$, and $24x = 2011.2 - 24v$, and consequently

$590491.672 - 21091.32v = 587914$, and $v = \frac{2577.672}{21091.32} =$

0.1222 nearly. Hence $x = 83.8 - 0.1222 = 83.6778$. true to five figures, and the sixth being too much by only two.

If the operation be repeated, it will give the answer true to eleven figures. But when five or six figures of the root have been obtained, and more exactness is still required, it will shorten the work to seek a correction for v , instead of one for the whole root, which may be had by substituting the last value of $v +$ or $-$ (new) v , instead of the last v in the equation, including all the powers of the last v , but rejecting those of the new v , (in the equation, thence arising,) as before.

Thus, in the last example, the whole equation, including the terms affected with v^2 and v^3 , is $513920 + 19224v + 1680v^2 + v^3 = 587914$, or $19224v + 1680v^2 + v^3 = 73994$, and in this equation putting $3.8 - v$, instead of v , and rejecting the terms of v^2 and v^3 as before; a correction will be obtained for the last found v , which will give the answer as above. In like manner, a second or third correction may be found, and the operation carried on to any degree of exactness. This rule doubles the number of figures true in the root at each operation. If the term wherein v^3 is found to be retained, v will be had by solving a quadratic, and then treble the number of figures will be had each time; therefore, if the first figure only be taken true, nine or ten figures will be had at two operations. This rule affords various theorems for solving particular equations as well as general

fore, assign the limits of such equations in the following cases :

CASE I.

*When several unknown Quantities are in one Equation ;
to find the Limits.*

RULE. Transpose all the negative quantities to the contrary side, that all the terms may be affirmative ; then to find the limits of any one quantity, suppose all the rest to vanish in the equation, then the value of that one will become determinate, and will be one limit thereof. And to know which limit it is, suppose the other quantities to increase, and become of some certain value ; then if the value of the unknown quantity under consideration, increase, it is the least limit ; but if it decrease, it is the greatest.

When fractional quantities are to be excluded, instead of supposing the other quantities to vanish, put each of them = 1, and an equation will arise, from which the limits of the remaining quantity will be found as before. Proceed in the same manner to find the limits of the other unknown quantities.

EXAMPLE. What are the limits of x and y , in the equation $4x + 5y = 67$. Let $y = 0$ or be supposed to vanish, and then $4x = 67$, and $x = 16\frac{1}{4}$. Now let y be supposed to be equal to some quantity ; then it is evident, that as y increases, x decreases, therefore $16\frac{1}{4}$ is the greater limit ; wherefore x is less than $16\frac{1}{4}$.

If $x = 0$ then $5y = 67$ and $y = 13\frac{2}{5}$. Now if x be supposed to increase, y will decrease ; and therefore $13\frac{2}{5}$ is the greater limit of y ; whence y is less than $13\frac{2}{5}$; and the less limit of both x and y is 0.

CASE II.

To determine the Limits of three or more unknown Quantities, when they are in two Equations.

RULE. Fix upon a quantity to be limited, and expunge one of the other quantities; then there will be had one limiting equation. Do the same with another unknown quantity, and there will be had another limiting equation; from each of which equations find a limit for the quantity fixed on.

EXAMPLE. What is the limit of x in the two following equations? $x+y+z=56$, and $32x+20y+16z=1232$. To exterminate y , multiply the first equation by 20, and there arises $20x+20y+20z=1120$, subtract this from the second equation, and we have $12x-4z=112$, then excluding the fractions, the less limit of x in this equation is $9\frac{1}{2}$.

Again, to exterminate z , multiply the first equation by 16, and subtract the product from the second equation, and there remains $16x+4y=336$. And the greater limit of x in this equation is $20\frac{1}{4}$. Hence x is greater than $9\frac{1}{2}$, and less than $20\frac{1}{4}$. In the same manner may y and z be limited; and so also in any other equation.

Of indeterminate Problems.

EXAMPLE. What is the least integer for the value of x , that will also cause the value of the following fraction to be an integer: $\frac{ax+b}{c}$

RULE. Divide the denominator (c) by the coefficient (a) of the indeterminate quantity; then divide the divisor by the remainder, and the last divisor again by the last remainder, and continue this operation till a unit only remains. Write down all the quotients in a line, as they rise, under the first quotient,

quotient write an unit, and under the second quotient write the first quotient; then multiply these two together: to the product, add the first term of the lower line, or an unit, and place the sum under the third term of the upper line; multiply in like manner the next two corresponding terms of the two lines together, and add the second term of the lower line to the product; put down the sum under the fourth term of the upper line; proceed in the same manner till you have multiplied by every number in the upper line. Then multiply the last number, thus found by the absolute number (b), in the numerator of the fraction, and divide the product by the denominator; then the remainder will be the true value of x required, provided the number of terms in the upper line be even, and the sign of b be negative; or that the number be odd, and the sign of b affirmative: but if the number of terms be even, and the sign of b affirmative, or *vice versa*, then the difference between the said remainder and the denominator of the fraction will be the true answer.

Operation.

$$\begin{array}{r} \text{The given fraction } \frac{ax+b}{c} = \frac{71x+10}{89} \\ \begin{array}{r} 71)89(1 \\ 18)71(3 \\ 17)18(1 \\ \hline 1 \end{array} \quad \begin{array}{r} 1.3.1 \text{ Total quotients} \\ 1.1.4.5 \\ 10=b \\ \hline 50 \text{ Product.} \end{array} \end{array}$$

Here, if the product 50 be divided by 89, the remainder is 50 = the least value of x .

In this rule it is always supposed, that a is less than c ; and that they are prime to each other; for if they were to admit of a common measure, whereby b is not divisible, no integer could be assigned for x , so as to give the value of the fraction $\frac{ax+b}{c}$ an integer.

series with unknown coefficients to represent it; which being multiplied or involved as the question requires, and the quantities of the same dimension being put equal to each other, new equations will be had, wherein the coefficients may be discovered.

EXAMPLE 2. Suppose $\frac{1}{a-x}$ be the given quantity, and the assumed series be $A + Bx + Cx^2 + Dx^3 + Ex^4$, &c. $= \frac{1}{a-x}$. Multiply both by $a-x$, and there arises $1 = aA + aBx + aCx^2 + aDx^3 + aEx^4$, &c. And $-Ax - Bx^2 - Cx^3 - Dx^4$, &c. and by equating the coefficients of the same powers of x , $aA = 1$, $aB - A = 0$, $aC - B = 0$, $aD - C = 0$, $aE - D = 0$, &c. Thus, in the first equation $A = \frac{1}{a}$; in the second equation $B = \frac{A}{a} = \frac{1}{a^2}$; in the third $C = \frac{B}{a} = \frac{1}{a^3}$; in the fourth $D = \frac{C}{a} = \frac{1}{a^4}$, and in the like manner $E = \frac{1}{a^5}$; therefore, $\frac{1}{a-x}$ brought to a series, is $\frac{1}{a} + \frac{x}{a^2} + \frac{x^2}{a^3} + \frac{x^3}{a^4} + \frac{x^4}{a^5}$, &c.

Some of the Properties of Square Numbers.

1. All even square numbers are divisible by 4; therefore, if a number consists of two even square numbers, it will be divisible by 4.

2. Any odd square number divided by 4 leaves a remainder of 1; therefore, if a number consisting of two odd square numbers be divided by 4, there will be a remainder of 2.

3. Therefore, if a number consisting of an odd and an even square number, be divided by 4, there will be a remainder of 1.

F 2

4. From

4. From hence it follows, that if any number composed of two square numbers be divided by 4, it cannot leave a remainder of 3; therefore, a number composed of two square numbers, cannot fall within this progression 3, 7, 11, 15, 19, 23, &c.

5. Any number ending in 2, 3, 7, or 8, is not a square number.

6. The sum of any number of terms of the series 1, 3, 5, 7, 9, 11, &c. beginning with the first, is a square number, whose root is equal to the number of terms.

7. The difference between any two square numbers is equal to the product of the sum and difference of their roots. Thus, if a and b be the roots, then $a + b \times a - b = a^2 - b^2$, and the same is also equal to the sum of the two roots, together with twice the sum of the roots of all the intermediate square numbers. Thus, the difference between 36 and 9 = $6 + 3 + 2 \times \sqrt{16 + 25} = 9 + 18 = 27$.

To resolve questions of this nature, the chief point is, to make such assumptions for the root of the required square, or cube, as shall, when involved, cause either the given number, or the highest power of the unknown quantity, to vanish from the equation; whereby at length there will be only one dimension of the unknown quantity, and so the question will be solved by reducing the equation.

SECT. V.

THE RESOLUTION OF SEVERAL ALGEBRAIC PROBLEMS.

Qv. 1. What are those two numbers, the sum whereof is 108, and the proportion of the less to the greater is as 5 to 7?

Let x represent the greater number, then $108-x$ is equal to the less number; and the proportion of the numbers will be as follows: $108-x:x::5:7$, these four quantities being in a direct proportion, the product of the two means $5x$ is equal to the product of the two extremes $756-7x$, therefore, we have this equation $5x=756-7x$; and by transposing $7x$, we have $12x=756$. Hence, by dividing 756 by 12, x is found equal to 63, which is the greater number, therefore, $108-63=45$, the less number.

Qv. 2. Bought apples at six for a penny, and pears at five for twopence. The number of apples and pears together was 100; the money given for the whole was 2s. 2d.; how many were there of each sort?

Let a represent the number of apples, then $100-a$ will be the number of pears: and as $6:1d.::a:\frac{a}{6}=$, price of the apples. Also, as $5:2d.::100-a:\frac{200-2a}{5}=$, price of the pears; then $\frac{a}{6}+\frac{200-2a}{5}=26$. This equation multiplied by 30, gives $5a+1200-12a=780$. By transposing and dividing $7a=1200-780=420$, and by division $a=60$, the number of apples; and $100-60=40$, the number of pears.

Qv. 3. It is required to divide the number 128 into four such parts, that if the first part be added to 7, the second made

made less by 7, the third multiplied by 7, and the fourth divided by 7, the results may be equal among themselves.

Let the four parts into which the number is divided be represented by the letters v, x, y, z . Then $v+7=x-7=7y=\frac{z}{7}$ will represent the four quantities; and from the equality of the two first equations we have $x=v+7+7$; from the equality of the first and third equations $y=\frac{v+7}{7}$, and from the equality of the first and fourth $z=\overline{v+7} \times 7 = 7v+49$; therefore, by collecting these together, there arises $\overline{v+14} + \frac{v+7}{7} + 7v+49 = 128$. By collecting the terms and transposition $9v + \frac{v+7}{7} = 65$; and multiplying this by 7, collecting the terms, transposing and dividing, we have $v=7$. And hence $x=7+14=21$; $y=\frac{7+7}{7}=2$, and $z=\overline{7+7} \times 7 = 98$, the several parts required.

Qv. 4. There are two cubical pieces of marble, the side of one exceeding the side of the other by three inches, the solid inches of both are 2457 inches; what is the length of the side of each piece?

Let the side of the less piece be represented by x , then the side of the greater will be $x+3$, and $x^3 + \overline{x+3}^3 = 2457$ inches; therefore, $2x^3 + 9x^2 + 27x = 2430$; this equation solved, gives $x=9$, and consequently $y=12$.

Qv. 5. A gentleman left a sum of money to be divided among three servants, in such proportion, that one half of the share of the first, one third of the second share, and one quarter of the share of the third, should be equal to 62*l*.; and one third of the first, one fourth of the second, and one fifth of the third, equal to 47*l*.; and one fourth of the first, one fifth of the second, and one sixth of the third, equal to 38*l*.;—what is each servant's share?

Put $a=62$, $b=47$, and $c=38$, and let the three shares required, be denoted by x, y , and z ; then the conditions

.of

of the question will stand thus: $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = a$, $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = b$, $\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = c$. These equations brought out of fractions, give $6x + 4y + 3z = 12a$, $20x + 15y + 12z = 60b$, $15x + 12y + 10z = 60c$. Here by subtracting the second equation from four times the first, in order to exterminate z , there arises $4x + y = 48a - 60b$; then, by taking three times the third equation from ten times the first, we have $15x + 4y = 120a - 180c$, which, subtracted from four times the last equation, leaves $x = 72a - 240b + 180c = 24$; wherefore $y = 48a - 60b - 4x = 60$ and $z = \frac{12a - 6b - 4y}{3} = 120$. demonstrated thus:

$$\frac{24}{2} + \frac{60}{3} + \frac{120}{4} = 12 + 20 + 30 = 62$$

$$\frac{24}{3} + \frac{60}{4} + \frac{120}{5} = 8 + 15 + 24 = 47$$

$$\frac{24}{4} + \frac{60}{5} + \frac{120}{6} = 6 + 12 + 20 = 38$$

Qu. 6. A grocer bought 120 pounds of tea, and as many pounds of coffee; he had one pound of coffee more for 20 shillings than of tea, and the whole price of the tea exceeded that of the coffee by 6*l*.; I demand how many pounds of tea he had for 20 shillings, and how many pounds of coffee?

Let the number of pounds of tea bought for 20 shillings be represented by x , then the number of pounds of coffee, for 20 shillings, will be $x + 1$, and the whole price of the tea will be $\frac{120}{x}$ pounds, and that of the coffee $\frac{120}{x+1}$ pounds; therefore, $\frac{120}{x} - \frac{120}{x+1} = 6$; wherefore, $120x + 120 - 120x = 6x^2 + 6x$, therefore $x^2 + x = 20$; which resolved gives $x = 4$ the pounds of tea for 20 shillings, and $x + 1 = 5$ the pounds of coffee for 20 shillings.

Qv. 7. Two travellers set out on a journey at the same time; the one sets out from C to go to B, the other from B to go to C; they both travel uniformly, and in such proportion that he that set out from B, four hours after meeting the other, arrives at C, and the other arrives at B, nine hours after meeting. How many hours did each person take to perform his journey?



In this figure, D is the place of their meeting; put $a=4$, $b=9$, and x for the number of hours which they travel before they meet. Then the distances they have travelled, with the same uniform pace, will be to each other as the times in which they are described; therefore, $BD:DC::a$, (the time in which the traveller who set out from B goes the distance BD,) $a=$ the time in which he travels from D to C; and by the same manner as $BD:DC::b$, (the time the other traveller goes from D to B,) $x=$ the time he goes from C to D; now as x is to a in the ratio of BD to DC, and b to a in the same ratio, it will follow, that as $x:a::b:x$, whence $a^2=ab$ and $x=\sqrt{ab}=6$, therefore, $a+\sqrt{ab}=10$, and $b+\sqrt{ab}=15$, are the two numbers required.

Qv. 8. The sum of three numbers in geometrical proportion, and the sum of the squares of three numbers, being given; to find the numbers themselves.

Put s for the sum of the three numbers, and b for their squares, and x , y , and z for the numbers themselves, then we shall have $x+y+z=s$, and $x^2+y^2+z^2=b$ and $xyz=y^3$, whence, by transposing y in the first equation, and involving both sides to the second power, there arises $x^2+2xz+z^2=s^2-2xy+y^2$, from which subtracting the second equation, we have $2xz-y^2=s^2-2xy+y^2-b$; but $2xz$ by the third equation is $=2y^3$, therefore, $2y^3-y^2=s^2-2xy+y^2-b$, or $s^2-2xy-b=0$, whence, $y=$

$$\frac{s}{2}$$

$\frac{a}{2} - \frac{b}{2x}$. Now to find x and z , we must look upon y as a known quantity, and then by the second equation we shall have $x^2 + z^2 = b - y^2$ from which subtracting $2xz = 2y^2$, we have $x^2 - 2xz + z^2 = b - 3y^2$, and by taking the root we have $x - z = \sqrt{b - 3y^2}$, but by the first equation $x + z = a - y$, therefore, $x = \frac{a - y + \sqrt{b - 3y^2}}{2}$ and $z = \frac{a - y - \sqrt{b - 3y^2}}{2}$

Q^y. 9. A farmer fold as many sheep and oxen as brought him 100*l*.; for the sheep he received 17 shillings each, and for the oxen 7*l*. each. It is required to know how many he fold of each?

Let the number of sheep be x , and that of the oxen y ; then we have this equation $17x + 140y = 2000$, and consequently $x = \frac{2000 - 140y}{17} = 117 - 8y + \frac{71 - 4y}{17}$ which being a whole number $\frac{71 - 4y}{17}$ or $\frac{4y - 11}{17}$ must therefore be a whole number likewise; whence by proceeding as above, we have $y = 7$, and $x = 60$, and this is the only answer the question will admit of.

Q^y. 10. What are the dimensions of a cubical block of marble, whose side in inches is expressed by two digits; the superficies of the block is equal to 864 times the sum of the said digits; and its solidity is equal to 576 times the square of the sum of the said digits?

Put x for the digit in the place of tens, and y for the digit in the place of units, then $10x + y$ is equal to the side of the cube and $(10x + y)^2 \times 6 = 864 \times x + y$; or $(10x + y)^2 = 144 \times x + y$. Also, $(10x + y)^3 = 576 \times x + y$ per question; and multiplying these equations cross ways, we have $(10x + y)^3 \times 576 \times x + y^3 = 144 \times x + y \times (10x + y)^3$. Then dividing both sides by $144 \times$

$\overline{x+y \times 10x+y}^2$, we have $x+y \times 4 = 10x+y$, or $4x+4y=10x+y$, and by transposition $3y=6x$, therefore, $y=2x$. This being substituted for y in the former equation, we have $\overline{10x+2x}^2 = 144 \times x+2x$ or $144x^2 = 144 \times 3x$, and dividing by $144x$ we have $x=3$ and $y=2x=6$; therefore, the side of the cube is 36.

CHAP. X.

OF THE VALUE OF LIVES;

OR,

DOCTRINE OF ANNUITIES.

SECT. I.

THE VALUE OF AN ANNUITY FOR A SINGLE LIFE.

AN ANNUITY is a sum of money payable yearly, half-yearly, or quarterly; to continue either for life, for a certain number of years, or for ever.

When an annuity remains unpaid after it is due, it is said to be in *arrear*. When the purchaser of an annuity does not immediately enter upon possession, the annuity is said to be in *reversion*.

The

The interest upon annuities in arrear may be computed either in the way of simple or compound interest. But compound interest being found most equitable, both for buyer and seller, is in most general use.

Annuities may be divided into certain and uncertain.

A certain annuity is that which continues for a certain time, or for ever. An uncertain annuity depends upon one or more lives.

Before I proceed to give the doctrine of contingent annuities, it will be necessary to deliver the rules for calculating of annuities certain.

PROBLEM I.

To find the Amount of an Annuity for a given Term of Years, at a given Rate of Interest.

EXAMPLE. What will an annuity of 50*l.* amount to, at the end of 8 years, at the rate of 5 per cent. per annum, simple interest?

In this example, the interest being at 5 per cent. multiply the rate of interest of *l.* for 1 year, or .05 by 50 the annuity, and the product by 8, the number of years, and the product hence arising is 20; the half whereof (10) multiplied by the number of years, made less by one, (7,) produces 70, the simple interest; which added to the product of 50, and 8, (400,) give 470, the amount required.

PROBLEM II,

To find the Amount of an Annuity, at compound Interest.

RULE. Multiply the amount of *l.* for 1 year, as often into itself as there are years, except one; or, which is the same, raise it to the power whose index is equal to the number of years, and from the result, subtract 1; then divide

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the

the remainder by the interest of 1*l.* for 1 year, and multiply the quotient by the annuity, and the product will be the amount required.

EXAMPLE. What is the amount of an annuity of 50*l.* for 3 years, at 5 *per cent. per annum, compound interest*? Here the amount of 1*l.* for 1 year is 1.05, which multiplied twice into itself, produces 1.157625, and 1 subtracted from this, the remainder is .157625, which divided by .05, the quotient is 3.1525; this multiplied by 50, produces 157.625, or 157*l.* 12*s.* 6*d.* the answer required.

NOTE. If the payments are half-yearly or quarterly, the amount, and interest of 1*l.* must be taken for a half, or a quarter of a year. And then the double or quadruple of the time must be taken. And the amount of 1*l.* for half a year at compound interest is equal to the square root of the amount for a year; and the amount for a quarter of a year is equal to the square root of that for half a year.

PROBLEM III.

To find the present Value of an Annuity, having the Time and Rate.

RULE. Multiply the amount of one year as often into itself as there are years, less 1; or involve it to the power, denoted by the time: by this result, divide 1, and subtract the quotient from 1, divide the remainder by the interest of 1*l.* for a year; then multiply this last quotient by the annuity, and the product will be the present value.

EXAMPLE. What is the present value of an annuity of 40*l.* for 5 years, discounting 5 *per cent. per annum, compound interest*? Here 1.05 involved to the fifth power is 1.27628. By which dividing 1, the quotient is .78353, which subtracted from 1, leaves .21647; this divided by .05 gives 4.3294, multiplied by 40 is 173.176, or 173*l.* 9*s.* 6½*d.* the worth.

PROBLEM

PROBLEM IV.

Having the present Worth, Rate, and Time, to find the Annuity.

RULE. Find the present value of 1*l.* annuity at the given rate and time; and then by the rule of three, say, as the present worth, thus found, is to 1*l.* annuity, so is the present worth given to its annuity; that is, divide the given present worth by that of 1*l.* annuity.

EXAMPLE. What annuity will 173*l.* 3*s.* 7*d.* purchase to continue 5 years, allowing *compounded interest at 5 per cent. per annum?*

$$.05:1::1:201.$$

$$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$$

$$1.2762815625)20.00000000(15.6705$$

20

$$\underline{15.6705}$$

$$\underline{4.3295} \text{ present worth of 1} \text{ l. annuity.}$$

$$4.329)173.179(40 \text{ l. annuity, Answer.}$$

Annuities for ever, or Freehold Estates.

In calculating the value of an annuity for ever, commonly called an *Annuity in fee simple*, three things are to be considered: 1. The annuity, or yearly rent. 2. The price, or present worth. 3. The rate of interest.

PROBLEM I.

Having the Rent and Rate of Interest, to find the Price or Value.

RULE. As the interest of 1*l.* is to 1*l.* so is the rent to the price or value.

EXAMPLE.

the remainder by the interest of $1l.$ for 1 year, and multiply the quotient by the annuity, and the product will be the amount required.

EXAMPLE. What is the amount of an annuity of $50l.$ for 3 years, at 5 per cent. per annum, compound interest? Here the amount of $1l.$ for 1 year is 1.05, which multiplied twice into itself, produces 1.157625, and 1 subtracted from this, the remainder is .157625, which divided by .05, the quotient is 3.1525; this multiplied by 50, produces 157.625, or 157*l.* 12*s.* 6*d.* the answer required.

NOTE. If the payments are half-yearly or quarterly, the amount, and interest of $1l.$ must be taken for a half, or a quarter of a year. And then the double or quadruple of the time must be taken. And the amount of $1l.$ for half a year at compound interest is equal to the square root of the amount for a year; and the amount for a quarter of a year is equal to the square root of that for half a year.

PROBLEM III.

To find the present Value of an Annuity, having the Time and Rate.

RULE. Multiply the amount of one year as often into itself as there are years, less 1; or involve it to the power, denoted by the time: by this result, divide 1, and subtract the quotient from 1, divide the remainder by the interest of $1l.$ for a year; then multiply this last quotient by the annuity, and the product will be the present value.

EXAMPLE. What is the present value of an annuity of $40l.$ for 5 years, discounting 5 per cent. per annum, compound interest? Here 1.05 involved to the fifth power is 1.27628. By which dividing 1, the quotient is .78353, which subtracted from 1, leaves .21647; this divided by .05 gives 4.3294, which multiplied by 40 is 173.176, or 173*l.* 3*s.* 6*d.* the present worth.

PROBLEM

PROBLEM VII.

Having the Value of an Annuity in Reversion, the Time prior to the Commencement of the Reversion, and Rate of Interest, to find the Annuity.

RULE. Find the amount of the price of the reversion, for the years prior to the commencement, by the tables: then find the annuity, which that amount will purchase.

EXAMPLE. If the reversion of an annuity to commence 20 years hence, be bought for 675.577*l.* what is the annuity, compound interest, at 6 per cent. per ann.?

Here by the tables, the amount of 675.577*l.* for 20 years, at 6 per cent. is 2166.6*l.* which will purchase an annuity of 130*l.*

Life Annuities.

Annuities for lives, are calculated from observations made on the bills of mortality, which, however imperfect they may seem to be in themselves, when individually applied, they are nevertheless become in very general use, and are found to answer pretty accurately, for such persons, or public offices, as buy and sell annuities.

It must, however, be observed, that calculations of this nature are at best but bare probabilities, or mere chance-work; yet every one must be sensible, that a person of 60 years of age has not so great a chance of living as a person of 30, provided both be in good health; therefore, an annuity to continue for the life of the former, cannot be worth near so much as an equal annuity for the life of the latter.

The principal writers on calculations of this kind, are Dr. Halley, Mr. Simpson, Monsieur de Moivre, and Dr. Price, each of whom has formed tables on the probability of human life, deduced from the bills of mortality.

Dr. Halley, from the bills of mortality at Breslaw, constructed a table, in which he shews how many persons died in every year out of 1000 promiscuously chosen, and all born in the same year, till the death of the last. He chose the city of Breslaw, the capital of Silesia, for this purpose, which he thought might serve as a standard to the rest of Europe, being a central town, at a distance from the sea, and not much crowded with foreigners. His table is, however, found not to be so well adapted to the probability of human life in England, as those of some others since his time. I shall, however, give his table, as well as that of Mr. Simpson, and Dr. Price, that the reader may see the comparative merit of each.

DR. HALLEY'S TABLE,

CALCULATED

From the Bills of Mortality at Breslaw.

TABLE I.

Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.
1	1000	145	32	515	8	63	212	10
2	855	57	33	507	8	64	202	10
3	798	38	34	499	9	65	192	10
4	760	28	35	490	9	66	182	10
5	732	22	36	481	9	67	172	10
6	710	18	37	472	9	68	162	10
7	692	12	38	463	9	69	152	10
8	680	10	39	454	9	70	142	11
9	670	9	40	445	9	71	131	11
10	661	8	41	436	9	72	120	11
11	653	7	42	427	10	73	109	11
12	646	6	43	417	10	74	98	10
13	640	6	44	407	10	75	88	10
14	634	6	45	397	10	76	78	10
15	628	6	46	387	10	77	68	10
16	622	6	47	377	10	78	58	9
17	616	6	48	367	10	79	49	8
18	610	6	49	357	11	80	41	7
19	604	6	50	346	11	81	34	6
20	598	6	51	335	11	82	28	5
21	592	6	52	324	11	83	23	3
22	586	7	53	313	11	84	20	5
23	579	6	54	302	10	85	15	4
24	573	6	55	292	10	86	11	3
25	567	7	56	282	10	87	8	3
26	560	7	57	272	10	88	5	2
27	553	7	58	262	10	89	3	2
28	546	7	59	252	10	90	1	1
29	539	8	60	242	10	91	0	0
30	531	8	61	232	10			
31	523	8	62	222	10			

TABLE II.

SHAWING

*The Probabilities from Ten Years Observation on the
Bills of Mortality in London,*

BY MR. SIMPSON.

Age.	Persons living.	Decre- ment of Life.	Age.	Persons living.	Decre- ment of Life.	Age.	Persons living.	Decre- ment of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	2
26	327	6	53	141	6	80	23	2

TABLE III.

The Probabilities of Life, calculated from Forty-six Years Observation on the Bills of Mortality at Northampton: viz. from 1735 to 1780.

Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.	Age.	Persons living.	Decrement of Life.
0	11650	1894	34	4085	75	69	1312	80
1	9756	1106	35	4010	75	70	1232	80
2	8650	1367	36	3935	75	71	1152	80
3	7283	502	37	3860	75	72	1072	80
4	6781	335	38	3785	75	73	992	80
5	6446	197	39	3710	75	74	912	80
6	6249	184	40	3635	76	75	832	80
7	6065	140	41	3559	77	76	752	77
8	5925	110	42	3482	78	77	675	73
9	5815	80	43	3404	78	78	602	68
10	5735	60	44	3326	78	79	534	65
11	5675	52	45	3248	78	80	469	63
12	5623	50	46	3170	78	81	406	60
13	5573	50	47	3092	78	82	346	57
14	5523	50	48	3014	78	83	289	55
15	5473	50	49	2936	79	84	234	48
16	5423	50	50	2857	81	85	186	41
17	5373	53	51	2776	82	86	145	34
18	5320	58	52	2694	82	87	111	28
19	5262	63	53	2612	82	88	83	21
20	5199	67	54	2530	82	89	62	16
21	5132	72	55	2448	82	90	46	12
22	5060	75	56	2366	82	91	34	10
23	4985	75	57	2284	82	92	24	8
24	4910	75	58	2202	82	93	16	7
25	4835	75	59	2120	82	94	9	5
26	4760	75	60	2038	82	95	4	3
27	4685	75	61	1956	82	96	1	1
28	4610	75	62	1874	81			
29	4535	75	63	1793	81			
30	4460	75	64	1712	80			
31	4385	75	65	1631	80			
32	4310	75	66	1552	80			
33	4235	75	67	1472	80			
34	4160	75	68	1392	80			

TABLE IV.

*The Expectation of Life at every Age, according to the Bills
of Mortality for both London and Northampton.*

Age.	Expectation.		Age.	Expectation.		Age.	Expectation.	
	London.	North.		London.	North.		London.	North.
0	19.5	25.18	34	22.4	26.20	68	9.9	9.50
1	27.5	32.74	35	22.0	25.68	69	9.6	9.05
2	32.5	37.79	36	21.6	25.16	70	9.3	8.60
3	34.5	39.55	37	21.2	24.64	71	8.9	8.17
4	36.1	40.58	38	20.8	24.12	72	8.6	7.74
5	36.5	40.84	39	20.4	23.60	73	8.3	7.33
6	36.5	41.07	40	20.1	23.08	74	8.0	6.92
7	36.3	41.03	41	19.7	22.56	75	7.7	6.54
8	36.1	40.79	42	19.3	22.04	76	7.3	6.18
9	35.7	40.36	43	19.0	21.54	77	6.9	5.83
10	35.3	39.78	44	18.6	21.03	78	6.5	5.48
11	34.8	39.14	45	18.3	20.52	79	6.0	5.11
12	34.2	38.49	46	17.9	20.02	80	5.5	4.75
13	33.6	37.83	47	17.5	19.51	81		4.41
14	33.0	37.17	48	17.2	19.00	82		4.09
15	32.4	36.51	49	16.8	18.49	83		3.80
16	31.8	35.85	50	16.5	17.99	84		3.58
17	31.2	35.20	51	16.1	17.50	85		3.37
18	30.6	34.58	52	15.7	17.02	86		3.19
19	30.0	33.99	53	15.4	16.54	87		3.01
20	29.4	33.43	54	15.0	16.06	88		2.86
21	28.8	32.90	55	14.7	15.58	89		2.66
22	28.2	32.39	56	14.3	15.10	90		2.41
23	27.7	31.88	57	13.9	14.63	91		2.09
24	27.1	31.36	58	13.6	14.15	92		1.75
25	26.6	30.85	59	13.2	13.68	93		1.37
26	26.1	30.33	60	12.9	13.21	94		1.05
27	25.6	29.82	61	12.5	12.75	95		0.75
28	25.1	29.30	62	12.1	12.28	96		0.50
29	24.6	28.79	63	11.7	11.81			
30	24.1	28.27	64	11.3	11.35			
31	23.6	27.76	65	11.0	10.88			
32	23.2	27.24	66	10.6	10.42			
33	22.8	26.72	67	10.3	9.96			

TABLE V.

Shewing the Number of Years Purchase an Annuity on a single Life is worth according to the Probabilities of Life, at London and Northampton, from the Age of 6 to 75 Years; at 3, 4, and 5 per Cent.

Age.	3 per Cent.		4 per Cent.		5 per Cent.		Age.	3 per Cent.		4 per Cent.		5 per Cent.	
	Lon.	North	Lon.	North	Lon.	North		Lon.	North	Lon.	North	Lon.	North
6	18.8	20.73	16.2	17.48	14.1	15.04	41	13.0	14.62	11.4	13.02	10.2	11.70
7	18.9	20.85	16.3	17.61	14.2	15.17	42	12.8	14.39	11.2	12.84	10.1	11.55
8	19.0	20.89	16.4	17.66	14.3	15.23	43	12.6	14.16	11.1	12.66	10.0	11.41
9	19.0	20.81	16.4	17.63	14.3	15.21	44	12.5	13.93	11.0	12.47	9.9	11.26
10	19.0	20.66	16.4	17.52	14.3	15.14	45	12.3	13.69	10.8	12.28	9.8	11.11
11	19.0	20.48	16.4	17.39	14.3	15.04	46	12.1	13.45	10.7	12.09	9.7	10.95
12	18.9	20.28	16.3	17.25	14.2	14.94	47	11.9	13.20	10.5	11.89	9.5	10.78
13	18.7	20.08	16.2	17.10	14.1	14.83	48	11.8	12.95	10.4	11.68	9.4	10.62
14	18.5	19.87	16.0	16.95	14.0	14.71	49	11.6	12.69	10.2	11.47	9.3	10.44
15	18.3	19.66	15.8	16.79	13.9	14.59	50	11.4	12.44	10.1	11.26	9.2	10.27
16	18.1	19.44	15.6	16.62	13.7	14.46	51	11.2	12.18	9.9	11.06	9.0	10.10
17	17.9	19.22	15.4	16.46	13.5	14.33	52	11.0	11.91	9.8	10.85	8.9	9.93
18	17.6	19.01	15.2	16.31	13.4	14.22	53	10.7	11.67	9.6	10.64	8.8	9.75
19	17.4	18.82	15.0	16.17	13.2	14.11	54	10.5	11.41	9.4	10.42	8.6	9.57
20	17.2	18.64	14.8	16.03	13.0	14.01	55	10.3	11.15	9.3	10.20	8.5	9.38
21	17.0	18.47	14.7	15.91	12.9	13.92	56	10.1	10.83	9.1	9.98	8.4	9.19
22	16.8	18.31	14.5	15.80	12.7	13.83	57	9.9	10.61	8.9	9.75	8.2	9.00
23	16.5	18.15	14.3	15.68	12.6	13.75	58	9.6	10.34	8.7	9.52	8.1	8.80
24	16.3	17.98	14.1	15.56	12.4	13.66	59	9.4	10.06	8.6	9.28	8.0	8.60
25	16.1	17.81	14.0	15.44	12.3	13.57	60	9.2	9.78	8.4	9.04	7.9	8.39
26	15.9	17.64	13.8	15.31	12.1	13.47	61	8.9	9.49	8.2	8.80	7.7	8.18
27	15.6	17.47	13.6	15.18	12.0	13.38	62	8.7	9.21	8.1	8.55	7.6	7.97
28	15.4	17.29	13.4	15.05	11.8	13.28	63	8.5	8.91	7.9	8.29	7.4	7.74
29	15.2	17.11	13.2	14.92	11.7	13.18	64	8.3	8.61	7.7	8.03	7.3	7.51
30	15.0	16.92	13.1	14.78	11.6	13.07	65	8.0	8.30	7.5	7.76	7.1	7.28
31	14.8	16.73	12.9	14.64	11.4	12.97	66	7.8	7.99	7.3	7.49	6.9	7.03
32	14.6	16.54	12.7	14.50	11.3	12.87	67	7.6	7.68	7.1	7.21	6.7	6.79
33	14.4	16.34	12.6	14.35	11.2	12.74	68	7.4	7.37	6.9	6.93	6.6	6.54
34	14.2	16.14	12.4	14.20	11.0	12.62	69	7.1	7.05	6.7	6.65	6.4	6.28
35	14.1	15.94	12.3	14.04	10.9	12.50	70	6.9	6.73	6.5	6.36	6.2	6.02
36	13.9	15.73	12.1	13.88	10.8	12.38	71	6.7	6.42	6.3	6.08	6.0	5.76
37	13.7	15.52	11.9	13.72	10.6	12.25	72	6.5	6.10	6.1	5.79	5.8	5.50
38	13.5	15.30	11.8	13.55	10.5	12.12	73	6.2	5.79	5.9	5.51	5.6	5.24
39	13.3	15.08	11.6	13.38	10.4	11.98	74	5.9	5.49	5.6	5.23	5.4	4.99
40	13.1	14.85	11.5	13.20	10.3	11.84	75	5.6	5.20	5.4	4.96	5.2	4.74

THE USE OF THE TABLES.

Table I.

Shews the probability of life, according to Dr. Halley's computation: the first column shews the ages; the second column, the number of persons living at those ages; and the third column, the decrement of life, or the number of persons that died each year: thus, opposite age 1, is 1000 in the second column, and 145 in the third column; which shews, that of 1000 persons born in the same year, 145 died before the expiration of the year; and of 855, the remainder of the persons living, 57 died the second year, and so on.

But the calculations according to this table differ from those made from the tables of the probabilities of life in London, partly owing to the different situations of these two places. Breffaw being an inland town not much frequented by strangers or foreigners, and London being a mercantile port, and crowded with traffickers and travellers from all parts of the world; and partly owing to the difference of climate, difference of food, and different manners of life, between the inhabitants of these two places, which is always found to occasion a different proportion in the deaths at the same ages. These considerations induced Mr. Simpson to compose a table of the probability of life, calculated from the bills of mortality of London, and which table will consequently much better answer the purpose of calculating the value of an annuity for a life at London, than the other table of Dr. Halley. This table may be seen page 51.

It must, however, be observed, that Dr. Halley's table is better adapted for the use of all Europe in general than any other particular table.

Table III.

Shews the probability of life at all ages, from 46 years observation on the bills of mortality at Northampton: viz.
from

from the year 1735 to 1780 inclusive. This table is justly reckoned to be the most correct of any extant, as it is taken from a large manufacturing town, and which consists generally of the same persons: and also that the table is constructed from a larger number of persons born than any other table, being 11650, which affords an opportunity of observing the decrement of life to a greater exactness.

Table IV.

Shews the expectation of life, or the number of years which any person may be supposed to have a fair chance of living, according to an equality of chance at every age, according to the bills of mortality for both London and Northampton. Thus, against age 2 stands 32.5 in the second column, under London; and 37.79 in the third column, under Northampton: which shews, that a child of the age of two years has an equal chance of living 32.5 years, according to the London tables, or 32 years 6 months; and according to the Northampton tables 37.79 years, or 37 years and upwards of 9 months.

Table V.

Shews the value of an annuity for a single life, according to the probabilities of life at London and Northampton, from the age of 6 to 75 years inclusive, at 3, 4, and 5 *per cent.* Thus, suppose it were required to find the number of years purchase which an annuity is worth to a person of the age of 30: here in the second column, and opposite the age 30, stands 15.0, which shews that an annuity for a person of 30 years of age in London, and at 3 *per cent.* is worth 15 years purchase; and an annuity for a Northampton life, at the same rate of interest, is worth 16.92 years purchase: and for a London life, at 4 *per cent.* an annuity is worth 13.1 years purchase; and for a Northampton life, at the same rate and interest,

is worth 14.78 years purchase: and an annuity at 5 *per cent.* for a London life, is worth 11.6 years purchase; and for a Northampton life, at the same rate of interest, 13.07 years purchase. This table also shews the value of an annuity of 1*l.* for a single life, at all the above-mentioned rates of interest: thus, an annuity of 1*l. per ann.* for a single life, at 39 years of age, according to the London tables, at 3 *per cent.* is worth 15*l.*; and the same, according to the Northampton tables, is worth 16*l.* and upwards of 18*l.* &c.

This table is esteemed the best of any extant, and preferable to any other of a different form. But those who sell annuities have generally a table of 2 years more value than the lives in this table, for purchasers who are upwards of twenty years of age.

Definitions.

1. The *probability of life* is the chance that any person or persons have of living to any certain time, and is denoted by a fraction, whose numerator is the chance of living, and denominator that of living and dying. Thus, suppose it were required to find the probability of a person of the age of 20 attaining to the age of 37, according to Mr. Simpson's table. Here it must be observed, that of 360 persons living at the age of 20, only 252 survive to the age 37; therefore, 108 persons have died between the two ages. Thus, 252 is the chance of the said person's living to the age of 37, and 360 the chance of the said person's dying before he attains the age of 37, and the probability of life of that person is expressed by the fraction $\frac{252}{360}$ or $\frac{7}{10}$; therefore, the odds in that person's favour, or the chance that he shall live to that age, is 7 to 3.

2. The *probability of dying* is expressed by a fraction, which is the difference between the former fraction and unity. Thus, the probability that the aforesaid person shall die before the age of 37 is expressed by $\frac{108}{360}$ or $\frac{3}{10}$, which

shews that the chance of that person's dying before the said age is as 3 to 7.

3. The *extremity of life* is the period beyond which there is no probability of surviving. In the Northampton tables this is 96 years.

4. The *complement of life* is the number of years which any person's age wants of the full extremity of life; this in the Northampton table, for a life aged 80, is 17 years.

5. The *expectation of life* is the number of years due to the life of a person of a certain age, upon an equality of chance. And it is the number of years purchase, which an annuity for life is worth in ready money, without allowing any interest. And in single lives it is always equal to the sum of all the probabilities of surviving to the extremity of life.

6. The *number of years purchase* of annuities, at any rate of interest, is that number which, if multiplied by the annuity, is equal to the present value thereof, according to such rate of interest; therefore, it is the present value of an annuity of 1*l.* according to a given rate of interest, as seen in Table V.

7. The *reversion* of a life annuity is where two or more lives are in joint possession, and the expectation depends upon the probability of one particular life surviving the rest.

PROBLEM I.

To find the Value of an Annuity for the Life of any Person at a given Rate of Interest.

RULE. Seek the age of the person in the first column of Table V. and against it, under the proper interest, is the number of years purchase for either the London or Northampton life; or, which is the same thing, the present value of an annuity of 1*l.* during such life. Multiply this by the annuity, and the product is the answer.

EXAMPLE. Suppose the given age be 49, the rate of interest 3 *per cent.* and the annuity 20*l.* Here again 49, and under 3 *per cent.* stands 11.6 according to the London bills, which, multiplied by 20*l.* gives 232*l.* for the value of a London life. And in the next column stands 12.69, the value according to the Northampton bills, which multiplied by 20*l.* as before, produces 253*l.* 16*s.* for the value of a Northampton life.

S E C T. II.

OF THE VALUE OF AN ANNUITY DURING JOINT LIVES.

PROBLEM I.

To find the Value of an Annuity for the joint Continuance of two Lives; that is, one Life failing, the Annuity to cease.

CASE I.

When both Persons are of the same Age.

RULE. Find the value of any one of the lives from Table V. Multiply this value by the interest of 1*l.* for a year at the given rate; subtract the product from 2, divide the aforesaid value by this remainder, and the quotient will be the value of 1*l.* annuity, or the number of years purchase.

EXAMPLE. What is the value of 100*l.* annuity, for the joint lives of two persons, aged 40 years each, according to the London tables, reckoning interest at 5 *per cent.*? Here, by the table, one life for 40 years is,

	10.3
Multiply by	<u>.05</u>
Subtract this product	.515
From	<u>2.000</u>
Remains	<u>1.485</u>

And 1.485×10.3 (6.9 value of *1l.* annuity, which multiplied by 100 is 690 $\frac{1}{2}$ the value of the annuity sought.

CASE II.

When the two Persons are of different Ages.

RULE. Find the values of the two lives in Table V. Multiply them one into the other, and call the result the first product; then multiply the said first product by the interest of *1l.* for a year, at the given rate, calling the result the second product; add the values of the two lives together, and from the sum subtract the second product; divide the first product by the remainder, and the quotient will be the value of *1l.* annuity, or the number of years purchase.

EXAMPLE. What is the value of 50 $\frac{1}{2}$ annuity, for the joint lives of two persons, whereof one is 20, and the other 30 years of age, according to the Northampton tables, interest at 4 per cent. ?

The value of 20 years is	16.03
And the value of 30 years is	<u>14.78</u>
First product	236.9234
	<u>.04</u>
Second product	9.476936
Sum of the two lives	<u>30.810000</u>
Remainder	<u>21.333064</u>

And $21.333064 \times 236.9234$ (11.1 value of *1l.* annuity.

50
555.0 Value required.

PROBLEM

PROBLEM III.

To find the Value of an Annuity during the Life of the longest Survivor of two Lives; that is, as long as either of the two Parties live.

RULE. From the sum of the values of the single lives, subtract the value of the joint lives, and the remainder will be the value sought.

EXAMPLE. What is the value of an annuity of 1*l.* to continue during the longest of two lives: the one person being 30, and the other 40 years of age; interest at 4 per cent. the life of 30 years of age valued according to the London bills, and that of 40 years of age according to the Northampton bills?

By the table, the value of 30 years is	13.1
The value of 40 years is	13.20
	<u>26.30</u>
The value of their joint lives, by Problem II.	
Cafe 2, is	8.9
The value sought	<u>17.4</u>

If the annuity be any other than 1*l.* multiply the above found value, by the given annuity; and if the two persons be of equal ages, the value of their joint lives must be found by Cafe 1, of Problem II.

PROBLEM IV.

To find the Value of an Annuity for the joint Continuance of three Lives; that is, one Life failing, the Annuity to cease.

RULE. Multiply the value of the three single lives continually into each other, calling the result the product of the three lives; multiply that product by the interest of 1*l.* and that product again by 2, calling the result the double product; then from the sum of the several products of the said lives,

taken

taken two and two, subtract the double product; divide the product of the three lives by the remainder, and the quotient will be the value of the three joint lives.

EXAMPLE. What is the value of an annuity of 1*l.* during the joint lives of three persons, whereof A is 10 years of age, B 20, and C 30, at 4 *per cent.* according to the London tables?

Here, by the table, the value of A's life is 16.4, that of B's 14.8, and C's 13.1, which three multiplied together is 3179.63; this multiplied by .04, the interest of 1*l.* gives 127.18528, which, multiplied again by 2, gives 254.370 for the double product. Then

The product of A and B is	342.72
And the product of A and C is	214.84
The product of B and C is	193.88
The sum of all taken two and two	651.44
Double product to subtract	254.37
Remainder	397.07

And $397.07 \div 3179.63 = 8.007$ Value sought.

PROBLEM V.

To find the Value of the Annuity for the longest Life of three or more Persons.

RULE. Find an age answerable to the value of the longest life of any two lives, which substitute in lieu of the two, and then find the value of the longest life of that life and the other, and that will be the value of the longest life among three lives. If there are four or more lives, substitute the age corresponding to this value in lieu of the three lives, and find the value of the longest life of this age, and the other remaining age, and it will be the value of the longest of four lives; proceed in the same manner for five or more lives, having regard to the rate of interest.

The examples in the foregoing problems will be found sufficient to instruct the learner how to perform all the following

following problems. I shall, therefore, give a few problems with their rules, leaving their operation for the exercise of the learner.

SECT. III.

THE VALUE OF CONTINGENT REMAINDERS AND REVERSIONS.

PROBLEM I.

To find the Value of the Reversion of an assigned Life after a given Term.

RULE. Subtract the value of the annuity for the given term of years, from the value of the proposed life, on the contingency of its ceasing, upon the extinction of the aforesaid life, and the remainder will be the answer.

PROBLEM II.

To find the Value of the Reversion of an Annuity for the Remainder of a given Term of Years after an assigned Life.

RULE. From the value of an annuity, certain for the given term of years, subtract the value of the annuity for the said term, on the contingency of its ceasing, upon the failing of the proposed life; and the remainder will be the value of the reversion.

PROBLEM

PROBLEM III.

To find the Value of the Reversion of one Life after another.

RULE. Subtract the value of the two joint lives from the value of the life in expectation, and the remainder will be the value of the reversion.

PROBLEM IV.

To find the Value of the Reversion of two Lives after one,

RULE. Subtract the value of the life in possession from the value of the longest of the three lives, and the remainder will be the value of the reversion.

PROBLEM V.

To find the Value of the Reversion of one Life after two Lives,

RULE. If the two lives are joint lives, subtract the value of the three joint lives from the value of the life in expectation, and the remainder will be the answer. But if the reversion takes place after the extinction of either life, subtract the value of the two lives in possession from the value of the three lives, and the remainder will be the value of the reversion.

PROBLEM VI.

To find the present Value of any Number of Lives in Succession.

RULE. Multiply the value of each life by the interest of $1l.$ for one year, and subtract each product from unity or 1; multiply all the remainders continually together, and subtract this product from unity; then the remainder multiplied

plied by the perpetuity*, will be the value of all the successive lives.

PROBLEM VII.

A given sum of money is to be received as a legacy on the decease of D, who is at a given age; what is the value thereof in present money?

RULE. Subtract the value of the life of D from the perpetuity; then say, as the perpetuity is to the remainder, so is the proposed sum to the present value.

PROBLEM VIII.

To find the Value of a Sum of Money to be received at the Decease of B, in case A is then deceased also.

RULE. Subtract the value of the oldest life from the value of an annuity for as many years as are expressed by the complement of B's age; then say, as the complement of the younger life is to the remainder, so is the proposed sum to its present value.

ANNUITIES UPON TONTINES.

PROBLEM IX.

To find the Value of an Annuity for either Person of two, who have a joint Annuity, which at the Decease of either one, is to become the sole Property of the Survivor.

* The perpetuity, or value of an annuity to continue for ever, is found by dividing 100*l.* by the rate of interest per cent. or by dividing 1*l.* by the interest of 1*l.* for a year; and the quotient is the perpetuity, or value of an annuity of 1*l.* to continue for ever. The perpetuity is also equal to the number of years purchase, which a perpetual annuity is worth, without allowing any interest.

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RULE.

RULE. From the value of the life of either of the two persons, subtract half the value of the two joint lives, and the remainder will be the value of the other person's life.

PROBLEM X.

What is the present value of an annuity to be possessed by D and his heirs, as soon as any two of the three lives, A, B, and C, become extinct; D and his heirs holding the same during the life of the survivor of the three lives, A, B, C?

RULE. Add thrice the value of the three joint lives, A, B, and C, to the sum of the value of the three single lives, deducting therefrom twice the sum of the value of each two joint lives: viz.—of A and B, A and C, and B, C; and the remainder will be the answer.

PROBLEM XI.

What is the value of the right of any one of the three following persons: viz.—A, B, and C, who enjoy an annuity equally among them, which, upon the decease of any one, is to become the property of the two survivors, during their joint lives, and on the decease of the next person to become the property of the last survivor during his life?

RULE. Subtract half the sum of the values of the joint lives, A and B, and the joint lives of A and C, from the value of the life of A; then to the remainder add one third of the value of the three joint lives, and the sum will be the value of the right of A. The value of the right of either of the other parties may be found by a similar method.

PROBLEM XII.

What is the value of the two successive lives, A and B, A having an annuity for life, and to have the nomination of a successor, who is to hold the annuity for his own life, after the decease of A?

R

RULE.

RULE. Multiply the value of the life of A, by the value of the life put in at his decease; divide the product by the perpetuity, and subtract the quotient from the sum of the said values, and the remainder will be the answer.

SECT. IV.

OF ASSURING LIVES.

By assuring a life, is meant, obtaining security for receiving a certain sum of money, should the assured life fail in a certain given time: in consideration of which, a premium is given to the assurer, which is a sufficient compensation for the loss he is likely to sustain, in case the life should drop. This compensation, called the *premium*, is varied according to the two following causes:—First, The rate of interest at which the money is supposed to be improved; and, secondly, the probability of the duration of the life to be assured. If the interest be high, and the probability of life high also, the value of the assurance will be low in proportion: on the contrary, if the interest be low, and the probability of life also low, the value of the assurance will be proportionably high. For example:—Let 100*l.* be supposed to be assured on a life, for 1 year; that is, let 100*l.* be payable a year hence, provided a person of a given age dies in that time.

K 2

Now,

Now, if the interest of the money be *5 per cent.* and the life *sure** of failing, the value of the assurance would be the same as the present value of 100*l.* payable at one year's end, reckoning interest at *5 per cent.* and would be that sum, which being put out to interest now, at *5 per cent.* would produce the 100*l.* at the end of the year, which sum is 95*l.* 4*s.* 9*d.*

If it be an even chance, or the odds are equal, whether the life does or does not fail in the year, which is the case when one half of a given number of lives fail in a given time, the value of the assurance will be half as much as the former value, or 47*l.* 12*s.* 4½*d.*

If the odds against the person's life failing are two to one, which is the case when one third of a given number of lives fail in the time, the value of the assurance will also be one third of the first value, (if the interest be the same), or 31*l.* 14*s.* 11*d.*

If the odds are nineteen to one against the life failing, which is the case when the twentieth part of the lives fail in the given time, the value of the assurance will be a twentieth part of the first value, or 4*l.* 15*s.* 2½*d.*

If the odds are forty-nine to one against the life failing, or when only one out of fifty of such lives fails in the given time, the assurance will be only a fiftieth part of the first value, or 1*l.* 18*s.* 1*d.*

Now the odds of two to one, according to Dr. Halley's table, are, that a life aged 85 years will not drop in a year. The odds of nineteen to one are, that a life aged 64 will not drop in a year. And the odds of forty-nine to one are, that a life aged 39 will not drop in a year. Therefore, the value of the assurance of 100*l.* for one year, on a life aged 85, is

* By this word is to be understood a certainty, according to the calculations from the tables.

31*l.* 14*s.* 11*d.*;—on a life aged 64, 4*l.* 15*s.* 2½*d.*;—on a life aged 39, 1*l.* 18*s.* 1*d.* at 5 *per cent.* interest. But if interest be reckoned at 3 *per cent.* these three values will be 32*l.* 7*s.*—4*l.* 17*s.*—1*l.* 18*s.* 10*d.* respectively.

This calculation supposes the value of the assurance to be paid in one single present payment. . But the value may be paid in *annual payments*, and be continued till the failure of the life, should that happen within the given term; or, if not, till the determination of the time.

The value of an assurance upon a life cannot be discovered by any one ignorant of the method of calculating the value of life annuities, delivered in the former part of this chapter. But those who understand what has been delivered, may form any calculations upon this subject, from the following examples :

EXAMPLE. What money in hand, and also in annual payments during life, ought a person of a given age to pay for a given sum of money, payable at his death to his heir ?

RULE. Subtract the value of the given life at the rate *per cent.* as given in Table V. from the *perpetuity*; multiply the remainder by the product of the sum to be assured, and the rate of interest for a year; divide this last product by 100*l.* increased by its interest for a year, and the quotient will be the answer, or the money which ought to be given in a single present payment: and this payment divided by the value of the life, will quote the sum that ought to be paid in annual payments during the whole continuance of life.

CASE I.

Where the Premium is to be paid immediately, in a single Payment.

QUESTION 1. What premium should be given to secure 100*l.* at the decease of a person aged 45 years, interest 3 *per cent.* ?

Operation.

Operation.

The perpetuity	33.333
The value of the life by	
Table V. at 3 per cent.	12.3
	<hr/>
Remainder	21.033
The given sum multiplied	
by its interest	300
	103)6309.9(61.26116 Answer.

Thus it appears that 61.26116, or 61*l.* 5*s.* 2½*d.* is the premium which ought to be immediately paid to secure 100*l.* on the decease of a person aged 45 years, at 3 per cent. per annum, according to the probability of life for London.

CASE II.

When the Premium is to be paid in fixed annual Payments, during the whole Continuance of Life.

QUESTION 2. What money should be given in equal annual payments, during the life of a person aged 45 years, to secure 100*l.* on the decease of the said person; interest at 3 per cent. per annum?

In this case, the value of the assurance in one present payment is to be found as in the foregoing case, which value divided by the value of the life, quotes the sum to be paid annually during the life of the person:—Thus, 61.26116 divided by 12.3, quotes 4.98, or 4*l.* 19*s.* 7*d.* which is the sum to be paid annually during life, in order to secure the sum of 100*l.* at the extinction of the said life.

If

If the foregoing questions be repeated, reckoning the interest at $3\frac{1}{2}$ *per cent. per annum*, the premium will be less, viz. in one present payment at $3\frac{1}{2}$ *per cent.* it will be 57*l.* 11*s.* and the annual payments 4*l.* 19*s.* 7*d.*

Thus it appears upon what very easy terms a large sum of money might be secured at the decease of any person, if the premium be paid by annual payments. Hence the great advantage of institutions for the assurance of lives, provided they be properly conducted, and managed by persons sufficiently skilled in numbers to avoid errors in making their calculations, which are most detrimental to societies of this nature, and from which there are hardly any of these institutions exempt.

Assurances of this nature might be extended considerably more than they are at present; and rendered not only subservient to the parochial poor, but also of infinite advantage to the nation at large, particularly to the revenue in a financial respect, were the subject to meet the approbation of the legislature; and perhaps more pecuniary assistance might be derived from establishments of this nature, under proper modifications, than from any other mode of funding, and creating permanent debts, as I shall prove in another treatise.

When an estate, or a perpetual annuity, is to be assured for the duration of another life, after the failure of the assured life, instead of assuring a gross sum, the value of a single payment will be the value of the life subtracted from the perpetuity, and the remainder multiplied by the annuity, or by the rent of the estate. And the value in annual payments to begin immediately, will be the single payment divided by the value of the life, increased by unity. Therefore, an assurance of an estate or annuity, after any given life or lives, is worth as much more than the assurance of a corresponding sum, as 100*l.* increased by its interest for a year, is
greater

greater than 100*l.* Thus the present values, in single and annual payments, of the assurance of an estate of 5*l.* *per annum* for ever, and of 100*l.* in money, are to one another as 105*l.* is to 100*l.* The reason of the difference is, that the algebraical calculations, by which these values are determined, suppose the gross sum and the first yearly payment of the annuity are to be received at the same time, after the expiration of the life or lives.

The examples here given will be found sufficient to instruct any person in the method of finding the value of annuities, in all cases of reversion: as also in the principles of assurances upon lives.

CHAP. XI.

OF LOGARITHMS.

SECT. I.

OF THE ORIGIN AND NATURE OF LOGARITHMS.

LOGARITHMS are certain artificial numbers, which are the ratios of other natural numbers; and are the indices of the ratio of numbers to one another; or, a series of numbers in an arithmetical proportion, answering to as many others in a geometrical proportion, and in such a manner, that 0 in the arithmeticals is the index of 1 in the geometricals. Logarithms were invented for the ease of arithmetical calculations, where the numbers, or operations, are large.

The nature of logarithms depends upon these axioms: If a series of quantities increase, or decrease, according to the same *ratio*, it is called a *geometrical progression*, as the numbers 1, 2, 4, 8, 16, 32, which are multiplied by 2: if the series of quantities increase, or decrease, according to the same *difference*, it is called an *arithmetical progression*, as the numbers 3, 6, 9, 12, 15, 18, &c. which increase by 3, which is therefore called their common difference. Now, if underneath the numbers proceeding in a geometrical progression, be placed as many other numbers, proceeding in an arithmetical progression, these last are called the logarithms of the first; as in the following:

Terms - 1. 2. 4. 8. 16. 32. 64. 128. 256. 512.

Logarithms 0. 1. 2. 3. 4. 5. 6. 7. 8. 9.

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In

In this progression, 0 is the logarithm of 1, the first term : 1 the logarithm of the second, which is 2 ; and 2 the logarithm of the third term, 4, &c.

These indices, or logarithms, may be adapted to any series in a geometrical progression ; and, therefore, there may be as many different kinds of indices, or logarithms, as there can be different kinds of geometrical progressions ; as may be seen in the following series :—

Log.	0	1.	2.	3.	4.	5.	&c.
{	1.	2.	4.	8.	16.	32.	&c.
{	or 2 ⁰ .	2 ¹ .	2 ² .	2 ³ .	2 ⁴ .	2 ⁵ .	&c.
{	1.	3.	9.	27.	81.	243.	&c.
{	or 3 ⁰ .	3 ¹ .	3 ² .	3 ³ .	3 ⁴ .	3 ⁵ .	&c.
{	1.	10.	100.	1000.	10000.	100000.	&c.
{	or 10 ⁰ .	10 ¹ .	10 ² .	10 ³ .	10 ⁴ .	10 ⁵ .	&c.

The Geometrical Progression.

Here the same indices, or logarithms, serve for any of the six under-written geometrical series, from which it appears, that there may be an endless variety of sets of logarithms, adapted to the same common numbers, by varying the second term of the geometrical series, as this will change the original series of terms, whose indices are the numbers 1. 2. 3. &c. And by interpolation the whole system of numbers may be made to enter the geometric series, and receive their proportionable logarithms, whether they be integers or decimals.

The logarithm of any number is the index of such a power of some other number, as is equal to the given one involved to the power denoted by the index of the other number. Thus, if N be equal to r^n , then the logarithm of N is n , which may be either positive or negative, and r any number whatever, according to the different systems of logarithms. When N is one, then n is 0, whatever the value of r may be ; and, consequently, the logarithm of 1 is always 0 in every system of logarithms. But in the common logarithms, r is equal to 10 ; so that the common logarithm of any number is the index of that power of 10, which is equal to the said number : thus the common logarithm of $N=10^a$, is a the

index of the power of 10.—For example:—1000 being the third power of 10, has 3 for its logarithm: and if 50 be = $10^{1.69897}$, then 1.69897 is the common logarithm of 50; from which it will follow, that the following decimal series of terms will have the following logarithms respectively:

The Geometric Series. { 1000, 100, 10, 1, .1, .01, .001
 { or $10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}$
 Logarithms 3, 2, 1, 0, -1, -2, -3.

The logarithm of a number, which is contained between any two terms of the first series, is included between the two corresponding terms of the latter series; therefore, that logarithm will have the same index, whether positive or negative, as the smaller of these two terms, together with a decimal fraction, which will always be positive. Thus, the number 50 falling between the numbers 10 and 100, its logarithm will fall between 1 and 2, being equal to 1.69897 nearly; and the number .05 falling between .1 and .01, its logarithm will fall between -1 and -2, and is equal to -1 + .69897, the index of the less term, together with the decimal .69897. The index is sometimes called the characteristic of the logarithms, and is always an integer, either positive or negative, or else 0, and shews what place is occupied by the first significant figure of the given number, either above or below the place of units, being in the former case positive, and in the latter negative.

When the characteristic of a logarithm is negative, the sign — is commonly set over it to distinguish it from the decimal part, which being the logarithm found in the tables, is always positive; thus, the logarithm of .05 or $-2 + .69897$ is written thus, $\overline{2}.69897$. But when it is required to reduce the whole expression to a negative form, it is done by making the characteristic less by 1, and taking the arithmetical complement of the decimal; that is, beginning at the left hand, subtract each figure from 9, except the last significant figure, which is subtracted from 10; then will the remainder form a

L 2

logarithm,

logarithm, wholly negative : thus, the aforementioned logarithm $\overline{2}.69897$, or $-2 + 69897$ is expressed by -1.30103 , which is all negative. Sometimes it is convenient to express the logarithm as positive, which is done by joining to the tabular decimal, the complement of the index to 10 : thus, the above logarithm is expressed by 8.69897 , which is only increasing the indices in the scale by 10.

From the foregoing definitions of logarithms, considered either as the indices of a geometric series, or as the indices of the powers of the same root, it appears, that numbers may be multiplied together by the addition of their logarithms : and they may be divided by the subtraction of their logarithms. Also a number may be raised to any power by multiplying the logarithm of the root by the index of the power : and the extraction of roots may be performed by dividing the logarithm of the given number by the index of the root required to be extracted.

Logarithms considered in their theory, are of very ancient origin, and were known to most of the ancients ; but the celebrated John Napier, Baron of Merchiston, in Scotland, was the first who applied the use of them to Trigonometry ; but the form of their construction was not made known till the opinion of the mathematicians was had concerning them. His son, Robert Napier, in the year 1619, published a new edition of his father's work, with the construction of logarithms. And in the same year Mr. John Speldell published an Improvement of Napier's Logarithms.

Other Tables were soon after published by John Kepler, and some others ; all which tables were of that kind, called Hyperbolical ; because the numbers expressed are as between the asymptote and curve of the hyperbola.

Henry Briggs, Professor of Geometry in Gresham College, soon after published the logarithms of the first one thousand numbers on a new scale : viz.—In which the logarithm of the ratio of 10 to 1 is 1 ; whereas the logarithms of the same ratio

ratio in Napier's system is 2.30258, &c. And in 1624 he also published his *Arithmetica Logarithmica*, containing the logarithms of 30,000 natural numbers, to 14 places of figures, besides the index, which form was recommended to him by Napier, and which is the form in present use. In 1633, he published, to 14 places of figures, his *Trigonometria Britannica*, which contained the natural and logarithmic sines, tangents, secants, &c.

From the time of Robert Napier, to the present period, several mathematicians published logarithmic tables, with various improvements; the principal of which were Gunter, Wingate, Henrion, Miller, and Norwood, Cavalierius, Vlacq, and Rowe, Frobenius, Newton, Caramuel, Sherwin, Gardiner, and Dodson.

In addition to what has been said, it may be observed, that the indices or characteristic of logarithms correspond to the denominative part of the natural number, as the other member of the logarithm does the numerative part of the number; that is, the index shews the denomination of, or place of the left hand figure of the number, and consequently of all the rest. Thus, 0 affixed to a logarithm, denotes the first figure of the number to which the logarithm answers to be nothing distant from the place of units. The index 1 shews the first figure of the number to be distant 1 place from the place of units; that is, in the place of tens, and consequently the number itself to be either 10, or some number between 10 and 100; and the same may be observed of all the other indices. Therefore, all numbers that have the same denominative, but not the same numerative parts, as all numbers from 1 to 10, from 10 to 100, &c. will have logarithms whose indices are the same, but the other members different. And again, all numbers, which have the same numerative, but not the same denominative parts, will have different indices; but the rest of the logarithms will be the same. If a number be purely decimal, to its logarithm is affixed a negative index, shewing the distance of its first significative

ficative figure from the place of units. Thus the logarithm of the decimal .256 is $\overline{1}.408240$; that of the decimal .0256 is $\overline{2}.408240$. Instead of these negative indices, some use their complements to 10, set down with a point on each side, thus, .9, and .8. that is, such a figure is made the index as when subtracted from 9, leaves a remainder expressing the number of cyphers prefixed to the decimal, as before observed.

S E C T. II.

OF THE ARITHMETIC OF LOGARITHMS.

FROM what has been said, the learner may have acquired a tolerable idea of the nature of logarithms. The construction of the logarithmic canon requires more room than is consistent with the present treatise. The table of logarithms is generally printed in a table consisting of nine columns, besides the column of natural numbers and that of the differences, except that part of the table which contains the logarithms of numbers from 1 to 100.

To find the Logarithm of a Number, or the Number corresponding to a given Logarithm.

If the given number be under 100, the logarithm is found at once in the table, standing opposite the number: if the given number be above 100, and under 1000, its logarithm will be found opposite to it in the column under 0: if the number be between 1000 and 10000, the first three figures of the number are to be found in the column marked No, and the fourth figure at the top of one of the other columns, and in the column under the said fourth figure, and even with the first three figures, will be found the logarithm required, changing the index 2 into 3.

If

If the given number be greater than any in the common canon, but less than 10000000, cut off four figures on the left of the given number, and find the logarithms thereof as before directed; then multiply the common difference which stands against the logarithm in the column marked *Difference*, by the remaining figure or figures of the given number, and from the product cut off as many figures from the right hand side as you multiplied by, and add the remainder to the logarithm of the first four figures of the given number, fitting it with a proper index. Thus, if it were required to find the logarithm of the number 92375: the logarithm of the first four figures 9237 is 3.965531, then 47 multiplied by 5=235, therefore, cut off 5 and add 23, and the required logarithm (proportioning its index) is 4.965554.

To find the Number corresponding to any Logarithm.

Seek the logarithm in the table, beginning in the column under 0 (neglecting the index); and if the logarithm, or the two or three first figures thereof, cannot be found in this column, seek them in the other columns; and having found it on the next less logarithm thereto, lineally against it in the column of numbers are the three first figures of the number sought, to which joining the figure over the column of the logarithm, you have the required number; when the number consists of four places of figures, i. e. when the index of the logarithm is 3; but when the index is 1, the two last figures of the number are decimal, and only the two first numbers integral: when the index is 0, the first number only is an integral, &c.

Thus, to find the number corresponding to a logarithm is the converse of finding the logarithm of a number.

To multiply or divide Numbers by Logarithms.

RULE. To multiply two numbers together, add their two logarithms together, and their sum will be the logarithm of the product, adding to the affirmative indices what is carried from the decimal parts; and if the indices be one affirmative, and

and the other negative, the difference is to be taken for the index to the logarithm of the product.

The reason of this rule is evident, for as unity is to one of the factors, so is the other factor to the product. Thus, the logarithm of the product is a fourth equi-different term to the logarithm of unity and those of the factors; but the logarithm of unity being 0, the sum of the logarithms of the factors must be the logarithm of the factum, or product.

Therefore, as the factors of a square are equal to each other, that is, a square is the factum or product of its root multiplied into itself; the logarithm of the square will be double the logarithm of the root.

Therefore, unity is to the exponent of the power, as the logarithm of a root to the logarithm of the power.

From hence it appears that the logarithm of the cube is triple the logarithm of the root; the logarithm of the biquadrate, quadruple that of the root; the logarithm of the fifth power quintuple that of the root; that of the sixth power sextuple, &c.

To perform division by logarithms, subtract the logarithm of the divisor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient, first changing the sign of the logarithm of the index of the divisor, and if they be of different signs, the difference is to be taken, but if of the same kind, their sum.

For as the divisor is to the dividend, so is unity to the quotient; therefore the logarithm of the quotient is a fourth equi-different number to the logarithms of the divisor, the dividend, and the logarithm of unity. The logarithm of unity, therefore, being 0, the difference of the logarithm of the divisor, and that of the dividend, is the logarithm of the quotient. For the difference between 7 and 5, or the remainder, is 2, which is the logarithm of the quotient 4, which quotient is obtained by dividing 128 by 32. Also, subtracting 3 from 8, the remainder 5 is the logarithm of the quotient 32, which is obtained by dividing 256 by 8.

Examples

*Examples in Multiplication.***EXAMPLE 1.**

	<i>Num.</i>	<i>Log.</i>
Multiply	68	1.832509
By	12	1.079181
Products	816	2.911690

EXAMPLE 2.

	<i>Num.</i>	<i>Log.</i>
Multiply	9	0.954242
By	9	0.954242
	81	1.908484

EXAMPLE 3. Multiply 3.902, and 597.16, and .0314728, continually together.

EXAMPLE 4. And also multiply 35.86, and 2.1046, and 0.8372, and 0.0294, continually together.

<i>Num.</i>	<i>Log.</i>	<i>Num.</i>	<i>Log.</i>
3.902	0.5912873	35.86	0.5546103
597.16	2.7760907	2.1046	0.3231696
.0314728	2.4979353	0.8372	1.9228292
73.33533	1.8653133	0.0294	2.4683473
		1857618	1.2689564

In the last example the 2 carried from the decimals cancels the 2, and there remains 1 to be set down; and in the third example, the 2 cancels the 2, and the 1 carried from the decimals is set down.

*Examples in Division.***EXAMPLE 1.**

	<i>Num.</i>	<i>Log.</i>
Div.	24163—	4.3831509
By	4567—	3.6596310
Quo.	5.290782—	0.7235199

EXAMPLE 2.

	<i>Num.</i>	<i>Log.</i>
Div.	37.149—	1.5699471
By	523.76—	2.7191323
Quo.	0.07092752—	2.8508148

In the last example, the 1 carried from the decimals is added to the 2, which, with its sign, changed, becomes $\overline{3}$; consequently the remainder, or index of the quotient, is 2.

EXAMPLE 3.			EXAMPLE 4.		
	<i>Num.</i>	<i>Log.</i>		<i>Num.</i>	<i>Log.</i>
Div.	.06314—	$\overline{2.8003046}$	Div.	.7438—	$\overline{1.8714562}$
By	.007241—	$\overline{3.8597985}$	By	12.9476—	$\overline{1.1121893}$
Quo.	$\overline{8.719792}$	$\overline{0.9405061}$	Quo.	$\overline{05744694}$	$\overline{2.7592669}$

In the third example, there is 1 carried from the decimals and added to the $\overline{3}$, which makes it $\overline{2}$; and which taken from the other $\overline{2}$ leaves 0. In the fourth example, the 1 is taken from the $\overline{1}$, which leaves a remainder $\overline{2}$.

To find the Logarithm of a Fraction.

RULE. Subtract the logarithm of the numerator from that of the denominator, and to the remainder prefix the negative sign. Thus, suppose it was required to find the logarithm of the fraction $\frac{4}{7}$.

$$\begin{array}{rcl}
 & 0.845098 & \text{Logarithm of 7.} \\
 & \underline{0.477121} & \text{Logarithm of 4.} \\
 - & 0.367977 & = \text{Logarithm of } \frac{4}{7}.
 \end{array}$$

The reason of this rule is evident: for a fraction being the quotient of the numerator divided by the denominator, its logarithm must be the difference of the logarithms of these two: so that the numerator being subtracted from the denominator, the difference becomes negative. And the logarithms of proper fractions must always be negative, if the logarithm of unity be 0; because a proper fraction is less than an unit.

Or the logarithm of the denominator, though greater than that of the numerator, may be subtracted from that of the

numerator, regard being had to the sign of the index, which alone in this case is negative. Thus,

0.477121 Logarithm of 3.

0.845098 Logarithm of 7.

1.632023 = Logarithm of $\frac{1}{3}$.

This produces the same effect in any operation as the logarithm before found, viz.—0.367977, this being to be subtracted, and the other added.

Or the fraction may be reduced to a decimal, and its logarithm found; which logarithm differs from that of a whole number only in the index, which is to be negative. For an improper fraction, subtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction, as in the fraction $\frac{2}{3}$.

0.9542425 Logarithm of 9.

0.6989700 Logarithm of 5.

0.2552725 = Logarithm of $\frac{2}{3}$.

In the same manner the logarithm of any mixed number may be found, by reducing the mixed number into an improper fraction.

Or lastly, an improper fraction may be reduced to a mixed number, and its logarithm must be found as if it were a whole number; and its index taken according to the integral part.

In addition, subtraction, &c. of logarithms, with negative and affirmative indices, the same rules are to be observed as those given in algebra, for like and unlike signs.

In addition of logarithms of this nature, all the figures, except the index, are reckoned positive; and, therefore, the figure to be carried to the index from the other part of the logarithms, takes away so much from the negative index. Thus, if 1.863326 be added to $\overline{3}$.698972, the sum is $\overline{1}$.562298.

And in subtraction, if either one or both of the logarithms have negative indices, you must change the sign of the index of the subtrahend, after you have carried, to it what may arise from the decimal part, and then add the indices together; thus, if 1.863326 be subtracted from $\overline{1.562298}$, the remainder will be $\overline{3.698972}$. In multiplication, what is carried from the product of the other parts of the logarithms must be subtracted from the product of the indices; thus, if $\overline{2.477121}$ be multiplied by 5 , the product will be $\overline{8.385605}$. In division, if the divisor will exactly measure the index, proceed as in common arithmetic. Thus, $\overline{4.984782}$ divided by 2 , quotes $\overline{2.462391}$; but if the divisor will not exactly measure the index, add units to the index till you can exactly divide it, and carry these units to the next first number. Thus, if $\overline{8.385605}$ be divided by 5 , it quotes $\overline{1.677121}$.

Involution and Evolution.

Involution is performed by multiplying the logarithm of the number given by the proposed index of the power; and if the index of the logarithm be negative, the product will be negative; but what is carried from the decimal part of the logarithm will be affirmative; therefore the difference of this carried number and the product of the index will be the index of the product.

EXAMPLE 1. To find the cube or third power of 3.07146 .

EXAMPLE 2. To find the fourth power of $.09163$.

<i>Num.</i>	<i>Log.</i>	<i>Num.</i>	<i>Log.</i>
Root 3.07146 —	0.4873449	Root $.09163$ —	$\overline{2.9620377}$
	3		4
Pow. 28.97575 —	1.4620347	Pow. $.0000704938$ —	$\overline{5.8481508}$
<u> </u>	<u> </u>	<u> </u>	<u> </u>

Evolution is performed by dividing the logarithm of the power or given number by its power, and the quotient will be

be the logarithm of the root; and when the index of the logarithm is negative, and the divisor is not exactly contained in it, without a remainder, increase the index of the logarithm by such a number as will make it exactly divisible; and carry the units borrowed, as so many tens, to the decimals; then divide the results by the index of the root.

EXAMPLE 1. To find the cube root of 12345.

EXAMPLE 2. To find the cube root of .00048.

Num.	Log.	Num.	Log.
Pow. 12345—	3)4.0914911	Pow. .00048—	3)4.6812412
Root 32.11162—	1.3638304	Root .07829735—	2.8937471

In the last example, 2 is to be added to the 4, to make it become 6, in which the divisor 3 is contained 2 times; and the 2 borrowed is carried to the other figure 681, &c.; and the logarithm of the root is 2.8937471.

S E C T. III.

OF THE RULE OF PROPORTION BY LOGARITHMS.

To perform the Rule of Proportion by Logarithms.

RULE. Add the logarithm of the second and third terms together, and from the sum subtract the logarithm of the first; and the remainder is the logarithm of the fourth number.

EXAMPLE. Find a fourth proportional to the numbers 4, 68, and 3.

1.832509

1.838509	Logarithm of 68.
0.477121	Logarithm of 3.
<hr/>	
2.309630	Sum.
0.602060	Logarithm of 4.
<hr/>	
1.707570	Logarithm of 51, the answer.
<hr/>	

This rule is founded on the same reason as the rule of proportion in common arithmetic; for adding the logarithms of the second and third numbers together, and subtracting the logarithm of the first from the sum, is the same thing as multiplying the second and third numbers together, and dividing the product by the first.

Or the operation may be performed by the following rule, viz. Against the first term write the arithmetical complement of its logarithm, and against the second and third terms write the logarithms themselves; and the sum of these three logarithms, abating 10 in the index, will be the logarithm of the fourth term; thus, in resolving the aforesaid question, the operation will stand thus:

9.397940	Arithmetical complement of log. of 4.
1.838509	Logarithm of 68.
0.477121	Logarithm of 3.
<hr/>	
1.707570	Logarithm of 51. Answer.
<hr/>	

The resolution of problems of this nature is of eminent service of trigonometry, as will be seen hereafter.

CHAP. XII.

OF TRIGONOMETRY.

Definitions.

1. **TRIGONOMETRY** is the art of finding all the sides and angles of a triangle, from having any three of these, one of which, at least, must be a side. Or, to find the ratio of the sides, when the angles are given; and conversely the ratio of the angles, when the sides are given. And it is founded on the mutual proportion which subsists between the sides and angles of triangles, which proportions are known by finding the relations between the radius of the circle and certain lines drawn in and about the circle, called *chords, sines, tangents, secants, &c.*

For this purpose, the circumference of a circle is divided into 360 parts, called degrees; and every degree subdivided into 60 other parts, called minutes, and every minute into 60 seconds, and every second into 60 thirds, &c. and any angle is said to consist of so many degrees, minutes, and seconds, as are contained in the arc, that measures the angle; or that is intercepted between the legs or sides of the angle; the point of the said angle being at the centre of the circle.

2. The

2. The *complement* of an arc is the difference between the arc and a quadrant.

3. The *supplement* of an arc is the difference between the arc and a semicircle.

4. The *right sine* of an arc, commonly called the *sine*, is perpendicular falling from one end of the arc, to the radius drawn through the other end of the same arc, as DE (fig. plate 9) is the sine of the arc DB, and it is always equal half the subtense of double the arc. Thus, DE is equal half of DO, which is the subtense or chord of the arc DC and the arc DO is double the arc DB. Hence the sine of an arc of 30 degrees is equal to one half the radius, because the chord of 60 degrees is equal to the radius.

5. The *sine complement* of an arc is that part of the radius intercepted between the centre and the right sine, as CE and is also the sine of the complement of the arc to a quadrant; for CE is equal to FD, which is the sine of the arc DH.

6. The *cosine* of an arc is the same as the sine complement.

7. The *versed sine* is that part of the radius intercepted between the right sine and the circumference of the circle, as EB.

8. The *tangent* of an arc is a perpendicular drawn from the extremity of the radius to the secant, as BG, which is the tangent of the arc DB.

9. The *secant* of an arc is a line drawn from the centre of the circle, through one end of the arc, till it meets the tangent; as CG.

10. The *cosecant* and *cotangent* of an arc is the secant and tangent of that arc, which is the complement of the arc. And the chord of an arc, and the chord of its supplement to a circle, is the same; so likewise the sine and secant of an arc are the same as the sine and secant of its supplement or complement to a semicircle. Thus, the sine ED, the tangent BG, and secant CG,

the sine, tangent, and secant of the arc, DA , which is the supplement of the former arc DB .

11. The *finus totus* is the greatest sine, being the sine of an arc of 90 degrees, or one quarter of a circle, and is equal to the radius of the circle.

Thus, the sines always increase from B , at which place they are nothing, till they come to the radius CH , which is the greatest, being the sine totus. From hence they decrease all the way along the second quadrant from H to A , and at length vanish at the point A ; whereby we see that the sine of the semicircle BHA , is nothing. After this, the sines are negative, as they proceed along the next semicircle AOB , being drawn on the opposite side, or downwards, from the diameter AB .

As DE is the cosine of DH : the sine, cosine, and radius of any arc form a right-angled triangle; as, CDE , or CDF ; of which, the radius CD is the hypotenuse: and therefore the square of the radius is equal to the sum of the squares of the sine and cosine of any arc.

The sines, cosines, tangents, &c. of every degree and minute, in a quadrant, are calculated to a radius of 1, and ranged in tables for use. But to shorten the operation in calculations in trigonometry, we use the logarithms of them, instead of the natural numbers, which are called the artificial sines, tangents, &c. and these numbers so ranged in tables, form the trigonometrical canon; and contain every species of right-angled triangles; so that no triangle can be proposed, but one similar to it can be found there by comparison, with which the proposed one may be computed by analogy of proportion. Lastly, sometimes the proportion is not expressed in numbers; but the several sines, tangents, &c. are actually laid down upon lines of scales; from whence the line of sines, of tangents, &c. on the plane scale, the construction and use of which follow:—

The *plane scale* is a mathematical instrument of most extensive use, commonly two feet in length. The lines usually

drawn upon it, are the following:—1. Lines of equal parts.—2. Of Chords.—3. Rhumbs.—4. Sines.—5. Tangents.—6. Secants.—7. Semi-tangents.—8. Longitude.—and 9. Latitude. (Fig. 2.)

1. The lines of equal parts are of two kinds: viz.—simply divided, and diagonally divided. The first of these are formed by drawing three parallel lines, and dividing them into any number of equal parts, by short lines drawn across them; and in like manner subdividing the first division into ten other equal smaller parts, by which numbers or dimensions of two figures may be taken off. Upon some rules several of these scales of equal parts are ranged parallel to each other, with figures set to them, to shew into how many equal parts they divide the inch; as 20, 25, 30, &c. 2. The diagonal divisions are formed by drawing eleven long parallel lines, equidistant from each other, which are divided into equal parts, and crossed by other short lines, as the former; then the first of these equal parts have the two outermost of the eleven parallel lines divided into ten equal parts, and the points of division connected by diagonal lines, as shewn in Mensuration. The whole scale is thus divided into dimensions of three places of figures.

The other lines upon the scale are commonly used in Trigonometry, Navigation, Astronomy, Dialling, &c. &c. and are all constructed from the divisions of a circle, as follows:—

1. Describe a circle* with any convenient radius, and divide it in four equal parts, by two diameters, drawn at right angles to each other, (fig. 2.) Continue one diameter C D towards F, and draw the tangent line E A, parallel thereto; then draw the chords D A and D B.

2. To construct the line of chords, divide the quadrant A D into 90 equal parts: then on A, as a centre, with the

* Only half the circle is drawn in the figure for want of room; but in general a complete circle is formed.

compasses, transfer these divisions to the chord line $A D$, which mark with the corresponding numbers, and it will become the line of chords, which may be transferred to the ruler.

4. For the line of rhumbs, divide the quadrant $B D$ into eight equal parts: then with the compasses, from the centre B , transfer the divisions to the line $B D$, which will be the line of rhumbs.

5. For the line of sines, through each of the divisions of the arc $A D$ draw right lines parallel to the radius $D C$, which will divide the radius $A C$ into the sines, or versed sines; numbering it from C to A for the sines, and from A to C for the versed sines.

6. For the line of tangents, lay a ruler on C , and the several divisions of the arc $A D$; and it will intersect the line $E A$, which will become a line of tangents, transferring the numbers from the arc $A D$ to that line.

7. For the line of secants transfer the divisions from the tangent line to the line $F D$ with the compasses, and from C as the centre, marking the divisions with the corresponding numbers on the tangent line.

8. For the line of semi-tangents, lay a ruler on B and the several divisions of the arc $A D$, which will intersect the radius $C D$ in the several divisions of the semi-tangents, which are to be numbered according to the arc $A D$.

9. For the line of longitude, divide the radius $A C$ into 60 equal parts, through each of these draw lines parallel to the radius $C D$; the points where these lines intersect the arc $A D$ are to be transferred with the compasses from A as a centre to the chord $A D$, and numbered thereon, which will give the line of longitude.

10. For the line of latitude, the semicircle $A D B$ must be completed to a circle, then a ruler laid on the point D , and on the several divisions on the line of sines, $A C$ will intersect the next quadrant of the circle, in as many points; when from the opposite part of the circle to D , as the centre,

the intersections of the arc are to be transferred to its chord, and numbered according to the numbers on the line of sines.

The chief uses of the lines of sines, tangents, secants, and semi-tangents, are to find the poles, and centres of the several circles, represented in a projection of the sphere.

I have been more particular in describing the construction of this scale, as it is an instrument in most general use in mathematics: and by the foregoing directions the learner may construct any lines on the scale himself, where there happens not to be a mathematical instrument maker nigh at hand, and place them on a rule, as seen *fig. 3*.

S E C T. I.

OF PLANE TRIGONOMETRY.

THE three methods of resolving triangles, or cases in trigonometry, are:—1. By geometrical construction. 2. Arithmetical computation. And, 3. Instrumental operation. In the first method, the triangle is constructed, by drawing, and laying down the several parts, viz.—the sides from a scale of equal parts, and the angles from a scale of chords, or other instrument: then the unknown parts are measured by the same scales; and thus they become known.

In the second method, the terms of the proportion are stated according to rule; which terms consist partly of the numbers of the given sides, and partly of the sines, &c. of angles taken from the tables; the proportion is then resolved like all other proportions, in which a fourth term is to be found from three given terms, viz. by the Rule of Three.

In the third method of resolving the triangle, by instrumental operation, recourse must be had to the logarithmic lines, on one side of the two foot scales; extending the compasses from the first term to the second or third, which happens to be of the same kind with it; then that extent will reach

reach from the other term to the fourth term. In this operation for the sides of triangles, is used the line of numbers, and for the angles the line of sines or tangents, according as the proportion respects sines or tangents.

In every case in plane trigonometry, there must be given three parts, one of which, at least, must be a side. And every triangle that can be proposed, will fall under one of the three following cases:

CASE I.

When two of the three given Parts are a Side, and its opposite Angle.

CASE II.

When there are given two Sides, and their contained Angle.

CASE III.

When the three Sides are given.

RULE. For the first case, viz.—That the sides are proportional to the sines of their opposite angles: that is, as the one side given, is to the sine of its opposite angle, so is another side given to the sine of its opposite angle. Or, as the sine of a given angle is to its opposite side, so is the sine of another given angle to its opposite side. Thus, to find an angle, we must begin the proportion with a given side, that is opposite to a given angle; and to find a side, we must begin with an angle opposite to a given side.

EXAMPLE. In the triangle, BDC , (fig. 4.) having the side BD equal to 106, the side BC equal to 65, and the angle BDC , 31 degrees, 49 minutes, to find the angle BCD , and the side CD .

1. By geometrical Construction.

Draw a line BD equal to 106; at D , make an angle of $31^{\circ} 49'$ by drawing DC ; take 65 in the compasses, and with one

one foot at B, extend the other foot to C, in the line D-C, then draw the line B C, and it is done: for the angle C will be $120^{\circ} 43'$; the angle D $31^{\circ} 49'$; and the angle B, $27^{\circ} 28'$; and the side D C, 56.

2. *By arithmetical Computation.*

	<i>Log.</i>
As the side B C, 65	1.81291
Is to fine angle D, $31^{\circ} 49'$	9.72198
So is the side B D, 106	<u>2.02531</u>
	11.74729
	<u>1.81291</u>
To fine angle C, $120^{\circ} 43'$	9.93438

To find the side D C.

	<i>Log.</i>
As the fine angle D, $31^{\circ} 49'$	9.72198
Is to side B C, 65	1.81291
So is fine angle B, $27^{\circ} 28'$	<u>9.66392</u>
	11.47683
	<u>9.72198</u>
To the side D C, 56.88	<u><u>1.75485</u></u>

Or, it may be wrought as follows:—

180° 0'	The sum of three angles
<u>59° 17'</u>	Supplement of angle C
120° 43'	Angle C
<u>31° 49'</u>	Angle D
152° 32'	Their sum
<u>180° 0'</u>	
152° 32'	
<u><u>27° 28'</u></u>	Angle B.

Here it must be noted, that when the given angle is obtuse, the angle sought will be acute; but when the given angle is acute, and opposite to a less given side, then the required angle is doubtful, whether acute or obtuse; it ought therefore to be determined before the operation be performed.

For

For the above proportion gives $59^{\circ} 17'$ for the required angle; but as it is obtuse, its supplement to 180° must be taken, viz. $180^{\circ} 43'$.

3. By Gunter's Line, or instrumental Operation.

RULE. Extend the compasses from 65 to 106 on the line of numbers, and that extent will reach from $31^{\circ} 49'$ to $59^{\circ} 17'$ on the line of sines.

Secondly. The extent from $31^{\circ} 49'$ to $27^{\circ} 28'$ on the line of sines, will reach from 65 to 56.88 on the line of numbers.

CASE II.

When the three given Parts are two Sides and their contained Angle.

RULE. As the sum of the two given sides is to the difference of the sides, so is the tangent of half the sum of the two opposite angles or cotangent of half the given angle to the tangent of half the difference of those angles.

Then the half difference added to the half sum gives the greater of the two unknown angles, and subtracted, leaves the less of the two angles.

Thus, having all the angles, the remaining third side will be found by the former case.

EXAMPLE. Having the side B C, equal to 109, B D equal to 76, (fig. 5,) and the angle C B D, $101^{\circ} 30'$, to find the angle B D C, or B C D, and the side C D.

1. By geometrical Construction.

Draw the line B C equal to 109, and B D, so as to make an angle with B C, of $101^{\circ} 30'$, and make B D equal to 76; join B C and B D with a right line, and it is done; for the angle D being measured, is found to be equal to $47^{\circ} 36'$, the angle C $30^{\circ} 58'$, and the side D C 144.8.

2. Aritb-

2. *Arithmetically by Logarithms.*

Side B C	109	109	180° 0'	
Side B D	<u>76</u>	<u>76</u>	101° 30'	
Their Sum	<u>185</u>	their	<u>78° 30'</u>	Sum of the angle D & C.
		difference	<u>39° 15'</u>	Half the sum.

Then to find the angles D and C.

	<i>Log.</i>
As the sum of the sides B C and B D = 185	2.26717
Is to their difference 33	1.51851
So is tan. of $\frac{1}{2}$ sum of the angles C & D 39° 15'	9.91224
	11.43075
	2.26717
To tangent of $\frac{1}{2}$ the diff. of the angles C & D 8° 17'	9.16358
To $\frac{1}{2}$ the sum of the angles D and C 39° 15'	
Add half the difference of the angles C and D 8° 17'	
Gives the greater angle D	47° 32'
Subtracted, gives the lesser angle C	30° 58'

To find D C.

	<i>Log.</i>
As sine angle D 47° 32'	9.86786
Is to the side B C 109	2.03743
So is sine angle B 101° 30'	9.99119
	12.02862
	9.86786
To the side D C, required 144.8	2.16076

3. *By Gunter.*

First. The extent from 185 to 33 on the line of numbers will reach from 39° 15' to 8° 17' on the line of tangents. Secondly, the extent from the angle D 47° 32' to 78° 30', (the supplement

2. *Arithmetically by Logarithms.*

The two shortest sides of the triangle B D and C D added together, is 135, and their difference 35. The segments of the base B C are found in the following manner :

	<i>Log.</i>
As the side B C equal to 105	2.02119
Is to the sum of the sides B D and D C 135	2.13033
So is their difference 35	1.54407
To the difference of the segments of B C 45	<u>1.65321</u>

Thus, having the sum and difference of the segments of the base, it is only necessary to add half their sum $52\frac{1}{2}$ to half the difference $22\frac{1}{2}$, and it will give the greater segment, which is 75; and which subtracted from 105 leaves 30, the lesser segment.

To find the Angle B D A.

	<i>Log.</i>
As the hypotenuse B D 85	1.92942
Is to the radius	10.00000
So is the greater segment 75	1.87506
To the sum of the angle B D A	<u>9.94564</u>

Therefore, the angle B D A is equal to $61^{\circ} 56'$.

To find the Angle A D C.

	<i>Log.</i>
As the hypotenuse D C 50	1.69897
Is to the radius	10.00000
So is the less segment 30	1.47712
To the sine of A D C	<u>9.77815</u>

Therefore, the angle A D C is equal to $36^{\circ} 53'$, and the whole angle B D C equal to $98^{\circ} 49'$.

To find the angle B, it is only necessary to subtract the angle B D A, or $61^{\circ} 56'$, from 90° , and the remainder $28^{\circ} 4'$, is the angle B, and the angle C is equal to $53^{\circ} 7'$.

3. By Gunter.

1. The extent from 105 to 135 will reach from 35 to 45 on the line of numbers. Secondly, The extent from 85 to 75 on the line of numbers, will reach from the radius to $61^{\circ} 56'$, or the angle B D A, on the line of fines. Thirdly, The extent from 50 to 30 on the line of numbers, will reach from the radius to $36^{\circ} 53'$, the angle A D C on the line of fines.

The three foregoing cases of plane triangles, contain all the varieties of both right and oblique triangles. But there are some other theorems, suited to some particular forms of triangles, which are often more expeditious in use, than the foregoing general ones; particularly the following theorem, for right-angled triangles, being a case which frequently occurs.

CASE IV.

When in a right-angled Triangle there are given the Angles, and one Leg, to find the other Leg, or Hypothenuse.

RULE. As the radius is to the given leg A B, (*fig. 7,*) so is the tangent of the adjacent angle A, to the opposite leg B C; and so is the secant of the same angle A, to the hypothenuse A C.

EXAMPLE. In the triangle A B C, having the leg A B equal to 162, and the angle A equal to $53^{\circ} 7' 48''$, and consequently the angle C $36^{\circ} 52' 12''$; to find the sides B C and A C.

1. *Geometrically.*

Draw AB equal to 162, and erect the indefinite perpendicular BC ; make the angle A $53^\circ 7' 48''$; then the side AC will cut BC in the point C , and form the triangle ABC , which by measuring, AC is found equal to 270, and BC to 216.

2. *Arithmetically.*

	<i>Log.</i>
As radius	10.0000000
Is to AB 162	2.2095150
So is the tangent A $53^\circ 7' 48''$	10.1249372
	<u>12.3344522</u>
	10.0000000
To BC 216	<u>2.3344522</u>
So is the secant A $53^\circ 7' 48''$	10.2218477
To AC 270	<u><u>2.4313627</u></u>

3. *By Gunter.*

Extend the compasses from 45° at the end of the tangents, (the radius,) to the tangent of $53^\circ \frac{1}{2}$; and that extent will reach from 162 to 216 on the line of numbers for BC . Then extend the compasses from $36^\circ 52'$ to 90 on the sines; and that extent will reach from 162 to 270 on the line of numbers for AC .

There is also another method of frequent use in trigonometry; called, *making every side radius*, which is as follows:

Let ABC , (fig. 8,) be a given triangle; make the hypotenuse AC radius first; that is, with the extent of AC as a radius, and on A and C , as two centres, describe the two arcs CD and AE ; then each leg AB , BC , will represent the sine of its opposite angle: viz.—The leg BC will be the sine of the arc CD , or of the angle A ; and the leg AB the sine of the arc AE , or of the angle C .

Again,

Again, making either leg radius, the other leg will represent the tangent of its opposite angle, and the hypotenuse the secant of the same angle: thus, with the radius AB and centre A , describe the arc BK ; and BC will represent the tangent of that arc, or of the angle A , and the hypotenuse AC the secant of the same; or, with the radius BC and centre C , describe the arc BG ; then the other leg AB is the tangent of that arc BG , or of the angle C ; and the hypotenuse CA is the secant of the same.

Then the proportions are obvious; for the sides in this figure bear the same proportions to each other as the parts they represent.

S E C T. II.

OF SPHERICAL TRIGONOMETRY.

SPHERICAL Trigonometry teaches the resolution of a *spherical triangle*, having three given parts: and, like plane trigonometry, may be either right-angled, or oblique-angled; but before the learner can proceed to the analogies of a spherical triangle, it is necessary to be acquainted with the six following theorems:—

Theorem I.

In all right-angled spherical triangles the sine of the hypotenuse is to the radius, as the sine of a leg is to the sine of its opposite angle. And as the sine of a leg is to the radius, so is the tangent of the other leg to the tangent of its opposite angle.

Demonstrated thus. Let $EDAFG$, (fig. 9,) represent the eighth part of a sphere, where the quadrantal planes EDF G , and EDB C , are both perpendicular to the quadrantal

tal plane $A D F B$, and the quadrantal plane $A D G C$ is perpendicular to the plane $E D F G$; and the spherical triangle $A B C$ is right-angled at B , therefore $C A$ is the hypotenuse, and $B A, B C$, are the legs.

Draw the tangents $H F$ and $O B$ to the arches $G F$ and $C B$, and the sines $G M, C I$, on the radii $D F$ and $D B$; also draw $B L$ the sine of the arc $A B$, and $C K$ the sine of $A C$: then join $I K$ and $O L$. Now $H F, O B, G M$, and $C I$, are all perpendicular to the plane, $A D F B$. And $H D, G K$, and $O L$, lie all in the same plane $A D G C$; also $F D, I K, B L$, lie all in the same plane $A D G C$. Therefore, the right-angled triangles $H F D, C I K$, and $O D L$, having the equal angles $H D F, C K I, O L B$, are similar. And as $C K$ is to $D G$, so is $C I$ to $G M$; that is, as the sine of the hypotenuse is to the radius, so is the sine of a leg to the sine of its opposite angle. For $G M$ is the sine of the arc $G F$, which measures the angle $C A B$. Also as $L B$ is to $D F$, so is $B O$ to $F H$; that is, as the sine of a leg is to the radius, so is the tangent of the other leg to the tangent of its opposite angle. $Q E D$.

From this it follows, that the sines of the angles of any oblique spherical triangle, as $A C D$, (*fig. 10*,) are to one another directly as the sines of the opposite sides; therefore, in every right-angled spherical triangle, having the same perpendicular, the sines of the bases will be to each other inversely as the tangents of the angles at the bases.

Theorem II.

In every right-angled spherical triangle, as $A B C$, (*fig. 11*,) the proportion is, As radius is to the cosine of one leg, so is the cosine of the other leg to the cosine of the hypotenuse.

Therefore, if two right-angled spherical triangles $A B C, C B D$, (*fig. 10*,) have the same perpendicular $B C$, the cosines of their hypotenuses will be to each other directly as the cosines of their bases.

Theorem

Theorem III.

In any spherical triangle, the proportion is, As radius is to the sine of either angle, so is the cosine of the adjacent leg to the cosine of the opposite angle.

Therefore, in right-angled spherical triangles having the same perpendicular, the cosines of the angles, at the base, will be to each other, directly, as the sines of the vertical angles.

Theorem IV.

In any right-angled spherical triangles, the proportion is, As radius is to the cosine of the hypotenuse, so is the tangent of either angle to the cotangent of the other angle.

Thus, As the sum of the sines of two unequal arches is to their difference, so is the tangent of half the sum of those arches, to the tangent of half their difference; and, As the sum of the cosines is to their difference, so is the cotangent of half the sum of the arches to the tangent of half the difference of the same arches.

Theorem V.

In any spherical triangles, ABC , (*fig. 12 and 13,*) the proportion is, As the cotangent of half the sum of half the difference, so is the cotangent of half the base, to the tangent of the distance (DE) of the perpendicular, from the middle of the base.

Theorem VI.

In any spherical triangle ABC , (*fig. 12,*) As the cotangent of half the sum of the angles at the base, is to the tangent of half their difference, so is the tangent of half the vertical angle, to the tangent of the angle which the perpendicular CD makes with the line CF , bisecting the vertical angle.

The Solution of the Cases of right-angled spherical Triangles, (Fig. 11.)

Cate.	Given.	Sought.	Solution.
1	The hypo. A C, and one angle A.	The opposite leg B C.	As radius : sine hypothenuse A C :: sine A : sine B C. (by Theor. 1.)
2	The hypo. A C, and one angle A.	The adjacent leg A B.	As radius : cosine of A :: tangent A C : tangent A B. (by Theor. 1.)
3	The hypothenuse A C, and one angle A.	The other angle C.	As radius : cosine of A C :: tangent A : cotangent C. (by Theor. 4.)
4	The hypo. A C, and one leg A B.	The other leg B C.	As cosine A B : radius :: cosine A C : cosine B C. (by Theor. 2.)
5	The hypo. A C, and one leg A B.	The opposite angle C.	As sine A C : radius :: sine A B : sine C. (by Theor. 1.)
6	The hypo. A C, and one leg A B.	The adjacent angle A.	As tangent A C : tangent A B :: radius : cosine A. (by Theor. 1.)
7	One leg A B, and the adjacent angle A.	The other leg B C.	As radius : sine A B :: tangent A : tangent B C. (by Theor. 4.)
8	One leg A B, and the adjacent angle A.	The opposite angle C.	As radius : sine A :: cosine A B : cosine C. (by Theor. 3.)
9	One leg A B, and the adjacent angle A.	The hypothenuse A C.	As cosine A : radius :: tangent A B : tangent A C. (by Theor. 1.)
10	One leg B C, and the opposite angle A.	The other leg A B.	As tangent A : tangent B C :: radius : sine A B. (by Theor. 4.)
11	One leg B C, and the opposite angle A.	The adjacent angle C.	As cosine B C : radius :: cosine of A : sine C. (by Theor. 3.)
12	One leg B C, and the opposite angle A.	The hypothenuse A C.	As sine A : sine B C :: radius : sine A C. (by Theor. 1.)
13	Both legs A B and B C.	The hypothenuse A C.	As radius : cosine A B :: cosine B C : cosine A C. (by Theor. 2.)
14	Both legs A B, and B C.	Any angle as A.	As sine A B : radius :: tangent B C : tangent A. (by Theor. 4.)
15	Both angles A and C.	Any leg as A B.	As sine A : cosine C :: radius : cosine A B. (by Theor. 3.)
16	Both angles A and C.	The hypothenuse A C.	As tangent A : cotangent C :: radius : cosine A C. (by Theor. 4.)

NOTE. The 10th, 11th, and 12th cases are ambiguous, as it cannot be determined by the data, whether A, B, C, and A C, be greater or less than 90 degrees each.

The

The Solution of the Cases of oblique spherical Triangles.
(Fig. 12 and 13.)

Case.	Given.	Sought.	Solution.
1	Two sides AC, BC, and the angle A opposite to one of them.	The angle B, opposite to the other.	As sine BC : sine A :: sine AC : sine B. <i>Note.</i> When BC is less than AC, it cannot be determined whether B be acute or obtuse.
2	Two sides AC, BC, and the angle A opposite to one of them.	The angle ACB	Let fall the perpendicular CD upon AB, produced (if necessary) then radius : cosine AC :: tangent A : cotangent ACD.
3	Two sides AC, BC, and the angle A opposite to one of them.	The other side AB.	As radius : cosine A :: tangent AC : tangent AD. (by Theor. 1.) This and the last case are both ambiguous, when the first is so.
4	Two sides AC, AB, and the included angle A.	The other side BC.	As radius : cosine A :: tangent AC : tangent AB. (by Theor. 1.) Whence AD is also known.
5	Two sides AC, AB, and the included angle A.	Either of the other angles, as B.	As radius : cosine A :: tangent AC : tangent AD. (by Theor. 1.) Whence BD is known ; then as sine BD : sine AD :: tangent A : tangent B. (by Theor. 4.)
6	Two angles A, ACB, and the side AC betwixt them.	The other angle B.	As radius : cosine AB :: tangent A : cotangent ACD. (by Theor. 4.) Whence BCD is also known ; then as sine ACD : sine BCD :: cosine A : cosine B. (by Theor. 3.)
7	Two angles A, ACB, and the side AC betwixt them.	Either of the other sides, as BC.	As radius : cosine AC :: tangent A : cotangent ACD. (by Theor. 4.) Whence BCD is also known ; then as cosine BCD : cosine ACD :: tangent AC : tangent BC. (by Theor. 1.)
8	Two angles A, B, and a side AC opposite to one of them.	The side BC, opposite to the other.	As sine B : sine AC :: sine A : sine BC.
9	Two angles A, B, and the side AC opposite to one of them.	The side AB betwixt them.	As radius : cosine A :: tangent AC : tangent AD (by Theor. 1.) and as tangent B : tangent A :: sine AD : sine BD, whence AB is also known.
10	Two angles A, B, and the side AC opposite to one of them.	The other angle ACB.	As radius : cosine AC :: tangent A : cotangent ACD (by Theor. 4.) and as cosine A : cosine B :: sine ACD : sine BCD. (by Theor. 3.)
11	All the three sides AB, AC, and BC	Any angle, as A.	As tangent $\frac{1}{2} AB$: tangent $\frac{AC + BC}{2}$:: tangent $\frac{AC - BC}{2}$: tangent DE the distance of the perpendicular from the middle of the base, whence AD is known. Then as tangent AC : tangent AD :: radius : cosine A. (by Theor. 1.)
12	All the three angles A, B, and ACB.	Any side, as AC.	As cotangent $\frac{ABC + A}{2}$: tangent $\frac{ABC - A}{2}$:: tangent $\frac{ACB}{2}$: tangent of the angle between the perpendicular and a line bisecting the vertical angles, whence ACD is also known. Then as tangent A : to tangent ACD :: radius : cosine AC.

The following propositions concerning spherical triangles, will render them more intelligible.

1. A spherical triangle is either equilateral, isoscelar, or scalene, according as it has the three angles all equal, or two of them equal, or all three unequal; and *vice versa*.

2. The greatest side is always opposite the greatest angle, and the smallest side opposite the smallest angle.

3. Any two sides, taken together, are greater than the third.

4. If the three angles of a spherical triangle be all acute, or all right, or all obtuse, angles, the three sides will be accordingly all less than 90 degrees, or equal to 90 degrees, or greater than 90 degrees, and *vice versa*.

5. If from the three angles A, B, C , (*fig. 14*,) of a spherical triangle, ABC , as poles, there be described upon the surface of the sphere, three arches of a great circle DE, DF , and FE , forming by their intersections another spherical triangle DEF ; each side of the new triangle will be the supplement of the angle at its pole; and each angle of the same triangle will be the supplement of the side opposite to it in the triangle ABC .

6. In any triangle ABC , or AbC , right angled at A , (*fig. 15*,) the angles at the hypotenuse are always of the same kind as their opposite sides. And the hypotenuse is greater or less than a quadrant, according as the sides, including the right angle, are of different kinds, or of the same kind; that is to say, according as these same sides are either both obtuse, or both acute, or as one is obtuse, and the other acute, and *vice versa*. First, the sides including the right angle, are always of the same kind as their opposite angles. Secondly, the sides including the right angle, will be of the same or different kinds, according as the hypotenuse is less or more than 90 degrees; but one of them at least will be of 90 degrees, if the hypotenuse be so.

CHAP. XIII.

OF GEOGRAPHY.

SECT. I.

GEOGRAPHICAL DEFINITIONS.

1. **GEOGRAPHY** is the knowledge of the earth, or a description of the terrestrial globe, particularly of the surface and known habitable parts thereof, with all its different divisions.

2. The earth is a globular body, surrounded with an atmosphere of air, by which all terrestrial bodies are confined to its surface, being attracted thereto by the laws of gravity.

That the earth is of a globular form, has been demonstrated by a number of experiments, particularly by observations of the eclipses of the moon; in which it appears, that the shadow of the earth always appears circular, whichever way it is projected. Also, by the observation of ships at sea, which, after their departure from any coast, gradually disappear to an observer on land, from the bottom upwards; that is, the first part which disappears from the sight, is the keel, or lower part of the ship; then those parts which are higher up, and so on; the top of the mast being the part

that is last seen : this is owing to the convexity of the waters, which have the same globular figure as the earth *.

3. The earth also has a diurnal motion on its own axis, performing one revolution in 24 hours ; thereby occasioning the changes of the day and night, as will be seen in Astronomy.

4. The circumference of the globe is supposed to be divided into 360 parts, called degrees, and each degree subdivided into 60 minutes, and each minute into 60 seconds, &c. Every degree contains 60 geographic miles ; consequently, the circumference of the globe is 21,600 such miles ; and the diameter 6900 miles. But as 60 geographical miles are above 69 British measure, the circuit of the globe is therefore 24,840 English miles, and the diameter almost a third, or 7900 in round numbers.

5. The globe of the earth consists of land and water : the proportion of the land to the water is not accurately known, but it is generally believed to be near one third.

6. The waters are divided into three oceans, (besides the smaller seas :) viz.—The Atlantic, Pacific, and the Indian ocean. 1. The Atlantic or Western ocean divides Europe from America, and is 3000 miles wide. 2. The Pacific ocean divides America from Asia, and New Holland, and is 10,000 miles wide. 3. The Indian ocean lies between the East Indies and Africa, and is 3000 miles wide. The parts or

* The philosophers of the last age differed greatly in their descriptions of the true spherical figure of the earth, of which there were two general opinions. The one maintained that the earth was a prolate spheroid : of this number was Cassini. The other party maintained it to be an oblate spheroid : of this Sir Isaac Newton was the chief, who insisted that the polar, was shorter than the equatorial diameter by 36 miles. A party of philosophers from France was sent by the king of that country, to measure a degree on the polar circle, and also on the equator ; the result of their experiment turned out exactly in favour of Sir Isaac Newton's theory. But this inequality of the shape of the earth, as well as the inequalities occasioned by the mountains, &c. make no *sensible* difference in the form of the earth.

branches of these oceans, called seas, as the Mediterranean sea, &c. receive their names generally from the countries they border upon.

7. A *bay*, or *gulf*, is a part of the sea, almost surrounded by land; as the gulf of Mexico, the bay of Biscay, &c.

8. A *strait* is a narrow passage out of one sea into another; as the strait of Gibraltar.

9. A *lake* is a water surrounded by land; as the lake of Geneva.

10. *Rivers* are streams of water, issuing from springs in high grounds, and falling into the sea, or other rivers; and are wider near their mouths than towards their heads or springs: they are described in maps by black lines.

11. The land is divided into two great continents: viz. The eastern and western continents, besides islands. The eastern continent is subdivided into three parts: viz.—Europe, which is the north-west part; Asia, the north-east; and Africa, the south. The western continent consists of America only, divided into North and South America.

12. An *island* is a piece of land entirely surrounded by water.

13. A *peninsula* is a country, or piece of land, surrounded by water, except on one side, where it joins to some other land.

14. An *isthmus* is a narrow neck of land which joins a peninsula to some other country; as the Isthmus of Suez, which joins Africa to Asia; the Isthmus of Darien, which joins North and South America.

15. A *cape*, called sometimes a promontory, or head-land, is a point of land extending some way into the sea.

16. The surface of the earth is supposed to be divided by several imaginary circles, for the better determining the situation and boundaries of the several countries and parts of the world, of which the most considerable circles are the following:—1. The *equator*, called also the equinoctial line, which divides the globe of the earth into two equal parts, or

hemispheres, the one north and the other south. This circle is every where equally distant from the two poles; and upon this circle, the degrees of longitude are marked. 2. The two *tropical circles*: viz.—The *tropic of Cancer*, or the northern tropic, which encompasses the globe, at the distance of $23\frac{1}{2}$ degrees from the equator; and the *tropic of Capricorn*, or southern tropic, which encompasses the globe at the same distance on the south side of the equator. The space between these two tropics is called the *Torrid Zone*. 3. The two polar circles: viz.—The *arctic circle*, which furrounds the north pole; and the *antarctic circle*, which furrounds the south pole; each at the distance of $23\frac{1}{2}$ degrees from its respective pole. The space included between the tropic of Cancer and the arctic circle is called the northern *Temperate Zone*, and that space between the arctic circle and the north pole, is called the north *Frigid Zone*; and the corresponding spaces on the southern hemisphere have similar names, as the southern *Temperate Zone*, and the southern *Frigid Zone*. 4. The *meridional lines*, which are lines drawn at right angles to the equator, and coinciding at the poles. These lines run directly north and south: and when the sun appears full south of any place, he is then said to be on the meridian of that place; and it is then 12 o'clock at noon at that place. The latitude of places is always numbered on these lines.

17. The *longitude* is the distance of one country from another, and is either east or west, and measured on the equator. The longitude of a place is always measured from the capital of the country where the author or traveller is; thus, when a person in England mentions the longitude of any other place, it is always understood that the longitude is reckoned from London; that is, the degrees of longitude are measured on the equator, and from that part of the equator, where the meridian passing through London, intersects the equator, at that part of the equator which is cut by the meridian of the other place measured to.

18. The *latitude* of a place is the distance of that place from the equator, measured on the meridional line; and is either north or south.

19. The inhabitants of the earth are distinguished from each other by their relative situations; of which there are reckoned three sorts, *Periæci*, *Antiæci*, and *Antipodes*:—

1. The *Periæci* are those people who live at the same distance from the equator, but under opposite meridians: the length of their days and seasons is the same; but when it is mid-day with one, it is midnight with the other. 2. The *Antiæci* live under the same meridian, but opposite parallels, and live equally distant from the equator; the one being in the south latitude, and the other in the north. These have the sun at the same hour at noon; but the longest day of the one is the shortest day of the other, and their seasons of the year are different; for when it is summer with one, it is winter with the other. 3. The *Antipodes* are situated directly on opposite sides of the globe to each other, the feet of the one being directly opposite to the feet of the other. These lie under opposite meridians, and opposite parallels: it is noon-day with the one, when it is midnight with the other; the longest day with the one, is the shortest day with the other; and when it is summer with the one, it is winter with the other.

20. The inhabitants of the earth are sometimes distinguished from each other, (in geography,) by the direction of their shadows at noon-day; and are called *Amphiscii*, *Æscii*, *Heteroscii*, or *Periscii*. 1. The *Amphiscii* are those situated in the torrid zone, and have their shadows, one part of the year, directed towards the north at noon-day, and at another part of the year, towards the south, at noon-day, according to the part of the ecliptic the sun is in; consequently, the sun is vertical to these people twice a year. They are then called:—2. *Æscii*, shewing no shadow at noon-day. 3. The *Heteroscii*, are those who inhabit the temperate zones, and whose shadows at mid-day always fall one way: viz.—The shadows

of those in the northern temperate zone, falling always towards the north, at noon-day; and those in the southern zone, always south at noon-day. 4. The *Periscii* are those who inhabit either of the frigid zones. These have their shadows moved entirely round them every 24 hours, when the sun is in their hemispheres, and so far decline towards their pole, as not to set for several days.

21. The *horizon* is properly a double circle; one of the horizons being called the *sensible*, and the other the *rational horizon*. The former comprehends only that space which we can see around us, upon any part of the earth; and is very different, according to the difference of our situation. The other, called *rational*, is parallel to the former, but passing through the centre of the earth, and supposed to be continued as far as the celestial sphere itself; whereas the former is supposed to pass over the surface of the earth, where the spectator stands: but in geography, when the horizon is mentioned, the rational horizon is always understood. By reason of the round figure of the earth, every different part has a different horizon. The poles of the horizon, that is, the points directly above the head, and opposite the feet of the observer, are called the *zenith* and *nadir*.

22. The *zenith* is that pole of the horizon directly over the observer's head.

23. The *nadir* is the opposite pole of the horizon, or that directly under the observer's feet.

A TABLE,

SHOWING

The Number of Miles in a Degree of Longitude, in every Degree of Latitude, from the Equator.

Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.	Degrees of Latitude.	Miles.
1	59.96	31	51.43	61	29.04
2	59.94	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.86	34	49.74	64	26.30
5	59.77	35	49.15	65	25.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.45
8	59.40	38	47.28	68	22.48
9	59.20	39	46.62	69	21.51
10	59.08	40	46.00	70	20.52
11	58.89	41	45.28	71	19.54
12	58.68	42	44.95	72	18.55
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	58.00	45	42.43	75	15.52
16	57.60	46	41.68	76	14.51
17	57.30	47	41.00	77	13.50
18	57.04	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.00	51	37.73	81	9.38
22	55.63	52	37.00	82	8.35
23	55.23	53	36.18	83	7.32
24	54.81	54	35.26	84	6.28
25	54.38	55	34.41	85	5.23
26	54.00	56	33.55	86	4.18
27	53.44	57	32.67	87	3.14
28	53.00	58	31.70	88	2.09
29	52.48	59	30.90	89	1.05
30	51.96	60	30.00	90	0.00

Description and Use of the Globes and Armillary Sphere.

By means of maps, the true situations of the different places of the earth, with regard to one another, and every other particular relative to them, may be easily known; consequently the hour of the day, season of the year, &c. for any particular place may be discovered. But these problems, to be resolved by maps, would be tedious and complex; therefore, those machines called the celestial and terrestrial globes, and the armillary sphere, have been invented, by which many calculations are saved, and every problem in geography may be solved mechanically, in the most easy and expeditious manner.

If a map of the world be accurately delineated on a spherical ball, the surface thereof will represent the surface of the earth; for the highest hills bear no greater proportion to the bulk of the whole earth than so many grains of sand do to a common mathematical globe of twelve or eighteen inches diameter; the diameter of the earth being near 8000 miles, and no hill upon its surface is above three miles in perpendicular height.

The armillary sphere is a large hollow sphere of glass, having as many bright studs fixed on its inside, as there are visible stars in the heavens, and of the same magnitude, and at the same angular distances from each other. This sphere is a true representation of the heavens, to an eye supposed to be placed in the centre; for to an observer placed any where within the surface of an indefinite sphere, all objects will appear equally distant, though some be much nearer than others: and if a small globe, having a map of the earth upon it, be placed on an axis in the centre of this sphere, and the sphere be made to turn round its axis, it will represent the *apparent* motion of the heavens round the earth: but if the globe be turned round its axis, while the sphere remains fixed, it will represent the *true* motion of the earth.

If there be drawn a great circle upon this sphere, equally distant from its poles, and having the plane of the circle perpendicular to the axis of the sphere, it will represent the celestial equinox, which divides the heavens into two equal parts or hemispheres; and the two axes of the sphere will represent the two poles of the heavens.

If there be another great circle drawn upon the sphere, cutting the equinoctial at an angle of $23\frac{1}{2}$ degrees, in two opposite points, this circle will represent the ecliptic, or circle of the sun's apparent annual motion; one half of which is on the north side, and the other half on the south side of the equinoctial.

If there be made a large stud to move eastward in this ecliptic, and with such a motion as to go quite round it in the time that the sphere is turned round westward upon its axis 366 times; this stud will represent the sun, changing its place every day in the ecliptic, a 365th part; and going round westward in the same direction as the stars, but with a motion so much slower than that of the stars, that they will make 366 revolutions in the time that the sun makes only 365, about the axis of the sphere.

If the terrestrial globe in this machine be about one inch in diameter, and the diameter of the starry sphere about five or six feet, a small insect, placed upon the globe, would see only a very small portion of its surface; but it would see one half of the surface of the starry sphere, the convexity of the globe hiding the other half from its view. If the sphere be set in motion as before directed, and the globe also revolving on its own axis, the insect will see all the phenomena observed by the inhabitants of this world, in the diurnal rotation of the earth round its axis.

The exterior parts of this machine are several brass rings, which represent the principal circles in the heavens: viz.
1. The *equinoctial*: 2. the *ecliptic*, divided into the signs and degrees, and also into the months and the days of the

year, to shew in what point of the ecliptic the sun is on any given day in the year: 3. the *two tropics*: 4. the *arctic* and *antarctic circles*: 5. the *equinoctial colure*, which is a great circle passing through the north and south poles of the heavens, and through the equinoctial circle at the points where the equinoctial is cut by the ecliptic: 6. the *solstitial colure*, which is a great circle passing through the poles of the heavens, and at right angles to the equinoctial colure. Hence the solstitial colure passes through the equinoctial at the points where the equinoctial is at the greatest distance from the ecliptic. These points in the equinoctial are called the *solstitial points*.

In the north pole of the ecliptic is a nut, to which is fixed one end of a quadrantal wire, having at the other end a small sun, which is carried round the ecliptic, by turning the nut; and in the south pole of the ecliptic, another quadrantal wire is fixed, with a small moon upon it, which may be moved round by the hand. There is also a particular contrivance, for causing the moon to move in her own orbit.

On the axis of the small globe is fixed a flat celestial meridian, which may be set directly over the meridian of any place on the globe; and then turned round with the globe, so as to keep over the same meridian. This globe has also a moveable horizon, which turns upon two wires, which proceed from it on the east and west points of the globe, and entering the globe at the opposite points in the equator, which is a moveable brass ring, let into the globe in a groove. The whole fabric is supported on a pedestal, and may be elevated or depressed to any number of degrees, from 0 to 90.

Description of the Terrestrial Globe.

On the terrestrial globe are drawn all the principal circles before mentioned, as the equator, ecliptic, tropics, polar circles, and meridians. The ecliptic is divided into twelve signs, and each sign into thirty degrees. Each tropic is a $2\frac{1}{2}$ degrees

degrees from the equator; and each polar circle $23\frac{1}{2}$ degrees from its respective pole. There are also circles drawn parallel to the equator, at every 10 degrees distance from it, on each side towards the poles; these circles are called *parallels of latitude*. There are, also, several other circles, drawn perpendicularly to the equator, and intersecting each other at the poles; these circles are called *meridians*, and sometimes circles of *longitude*, or *hour circles*; and on large globes they are drawn through every tenth degree of the equator; but on globes of less than 12 inches diameter they are drawn through every fifteenth degree.

The globe is hung in a brass ring, called the *brazen meridian*, turning upon a wire in each pole, sunk into one side of the meridian ring. This meridian is divided into 360 degrees; one half of these degrees are numbered from the equator to the poles, to shew the latitude of places; the other half are numbered from the poles to the equator, to shew how to elevate either of the poles above the horizon. This ring divides the globe into two equal parts, called the *eastern* and *western hemispheres*; as the equator divides it into the *northern* and *southern hemispheres*.

The brazen meridian is let into two notches, made in a broad flat ring, called the *wooden horizon*; the upper surface of which divides the globe into two equal parts, called the *upper* and *lower hemispheres*. This horizon corresponds to the true rational horizon; and upon it are several concentric circles, which contain the months of the year, the signs and degrees answering to the sun's place for each month and day, the thirty-two points of the compass, and the circles of amplitude and azimuth, with some other circles.

There is a small horary circle, fixed to the north part of the brazen meridian, and having the wire in the north pole of the globe in its centre; on which wire is an index, which goes over all the twenty-four hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one at each pole.

There

There is a thin slip of brass, called the *quadrant of altitude*, divided into 90 degrees. This is occasionally fixed to the uppermost point of the brazen meridian, by a nut and screw, about which the quadrant turns round.

There is also, to some globes, a magnetic needle, which moves over a circle, divided into 360 degrees, also over the thirty-two points of the compass. This needle serves to fix the globe according to the meridian of the place, as the needle makes nearly a constant certain angle with the meridian, which angle is called the *variation of the compass*; and which being known, the globe may be rectified to the true meridian of the place: thus, at London, the variation of the needle is 23 degrees northward; therefore, by moving the frame of the globe about, till the needle settles itself near the twenty-third degree, reckoning westward from the north point, we then have the brass meridian of the globe coinciding with the true meridian of the place.

S E C T. II.

OF THE SOLUTION OF GEOGRAPHICAL PROBLEMS.

Directions for using the Globe.

IN using the globe, the east side of the horizon must always be kept towards you (except the globe require turning by the Problems,) for then the graduated side of the meridian will be towards you, the quadrant of altitude before you, and the globe divided exactly into two hemispheres, by the graduated side of the meridian.

When

When it is required to turn the globe and frame entirely round, as is the case in working some Problems, the ball of the globe will sometimes be moved from the degree to which it is set, by such motion; to avoid this, the feather end of a quill may be thrust between the ball of the globe and the meridian, which, without hurting the ball, will keep it from turning in the meridian, while you turn the horizon about.

PROBLEM I.

To find the Latitude and Longitude of any given Place upon the Globe.

Turn the globe on its axis, until the given place comes exactly under the graduated side of the brazen meridian, and observe what degree of the meridian the place then lies under, which is its *latitude*, and is either north or south.

The globe remaining in this position, the degree of the equator, which is under the graduated side of the brazen meridian, is the *longitude* of the place; and is either east or west, as the place lies on the east or west side of that which is accounted the *first meridian*. Thus, all the Atlantic Ocean, and the continent of America, is on the west side of the meridian of London; and the greater part of Europe and of Africa, together with all Asia, is on the east side of the meridian of London, which is reckoned the first meridian of the globe, by British geographers and astronomers; though the exact meridian for England is that of Greenwich.

PROBLEM II.

To find any Place on the Globe, having the Latitude and Longitude.

This is the reverse of the former problem, and is found by bringing the point of longitude in the equator to the brazen meridian: then count from the equator on the brazen

meridian

meridian to the degree of the given latitude; and under that degree of latitude will be the place required.

PROBLEM III.

To find the Difference of Longitude, or Difference of Latitude, between any two given Places.

Bring each of the two given places to the brazen meridian, and mark their latitudes; then, if both places are on the same side of the equator, the lesser latitude subtracted from the greater, will give the difference; but if they are on different sides of the equator, both latitudes must be added together. And the difference of longitude is found by bringing each place to the meridian, and reckoning on the equator the difference of degrees between the meridians of the two places, if it be less than 180 degrees, or half a circle; but if the difference be greater, it must be subtracted from 360, and the remainder is the difference of longitude.

PROBLEM IV.

To find all those Places that have the same Latitude and Longitude with any given Place.

Bring the given place to the brazen meridian; then all those places which lie under the said meridian will have the same longitude. Then turn the globe round on its axis, and all those places which pass under the same degree of latitude in the brazen meridian, that the given place does, have the same latitude.

PROBLEM V.

To find the Distance between any two Places on the Globe.

Lay the graduated edge of the brass quadrant of altitude, over both the places, and count the number of degrees intercepted

intercepted between them on the quadrant, which will be the distance in degrees; and which multiplied by 60, will give the distance in geographical miles; but multiplied by 69½ gives the distance in English miles. Or, the distance between the two places may be taken, with a pair of compasses, and that extent applied to the equator, will shew the number of degrees distant.

PROBLEM VI.

The Hour of the Day at any Place being given, to find what is the Hour at any other Place.

Bring the given place to the brazen meridian, and set the index to the given hour; then turn the globe, until the place where the hour is required, comes to the meridian; and the index will point to the hour at that place.

PROBLEM VII.

To find the Sun's Place in the Ecliptic, and his Declination, for any given Day in the Year.

Look on the wooden horizon for the given day, and against it there is placed the degree of the sign in which the sun is on that day at noon. Find the same degree of this sign in the ecliptic line upon the globe, and having brought it to the brazen meridian, observe what degree of the meridian stands over it; and that is the sun's declination, reckoned from the equator.

PROBLEM VIII.

To find all those Places in the north Frigid Zone where the Sun begins to shine constantly, without setting, on any given Day: which must be between the 21st of March and the 23^d of September. (See Fig. 3, Plate 15.)

Having brought the sun's place for the given day to the brazen meridian, and found his declination, (by Problem

VII.) count as many degrees on the meridian from the north pole as are equal to the sun's declination, and mark that degree from the pole on the meridian ; then turning the globe round on its axis, observe what places in the north frigid zone pass directly under that mark ; for they are the places required.

The same problem may be resolved for places within the south frigid zone, for the other half of the year.

PROBLEM IX.

To find the Place over which the Sun is vertical at any Hour of a given Day.

Find the sun's declination for the given day, (by Problem VII.) which mark on the brazen meridian ; then bring the place where you are, (suppose London,) to the brazen meridian, and set the index to the given hour ; then turn the globe on its axis until the index point to 12 at noon ; and the place on the globe which is directly under the point of the sun's declination, marked upon the meridian, has the sun that moment in the zenith, or directly vertical.

Note. The hour 12 at noon on the hour circle is the uppermost 12.

PROBLEM X.

Having the Day and Hour of a Lunar Eclipse, to find all those Places of the Earth to which it will be visible.

When the moon is eclipsed, she is always at the full, and, consequently, opposite to the sun ; therefore, whatever part of the earth the sun is vertical to, the moon must be vertical to the antipodes of that part ; consequently, the sun will be visible then to one half the earth, and the moon to the other half.

Therefore, find the place to which the sun is vertical at the given hour, (by Problem IX.) elevate the pole to the latitude

latitude of that place, and bring the place to the upper part of the brazen meridian: then as the sun will be visible to all those parts of the globe which are above the horizon, the moon will be visible to all those parts below it, at the middle of the eclipse.

PROBLEM XI.

To rectify the Globe for the Latitude, the Zenith, and the Sun's Place.

Find the latitude of the place, (by Problem I.) and if the place be in north latitude, raise the north pole as many degrees above the horizon, counting upon the meridian from the north pole to the horizon; but if the place be in south latitude, raise the south pole as many degrees: then turn the globe till the place comes under its latitude on the brazen meridian, and fasten the quadrant of altitude to the meridian, so that the chamfered edge of its nut may be joined to the zenith. Then bring the sun's place in the ecliptic for the given day, to the graduated side of the brazen meridian, and set the hour index to 12 at noon, and the globe will be rectified.

PROBLEM XII.

Having the Latitude of any Place between the two Polar Circles, to find the Time of the Sun's Rising and Setting, or the Length of the Day and Night, for any given Day in the Year.

Rectify the globe for the latitude, and the sun's place, by the foregoing problem; then bring the sun's place in the ecliptic to the eastern side of the horizon, and the hour index will shew the time of sun-rising; then turn the globe on its axis, until the sun's place comes to the western side of the horizon, and the index will then shew the time of sun-setting.

The hour of sun-setting being doubled, gives the length of the day; and the hour of the sun-rising doubled, gives the length of the night.

PROBLEM XIII.

Having the Latitude of a Place, and the Day of the Month, to find when the Morning Twilight begins, and Evening Twilight ends.

Rectify the globe, and bring the sun's place in the ecliptic to the eastern side of the horizon; then mark that point of the ecliptic which is in the western side of the horizon, which is the point opposite to the sun's place; and lay the quadrant of altitude over the said point, and turn the globe eastward, keeping the quadrant at the same mark, until the said point on the ecliptic is 18° high on the quadrant, and the index will point out the time when the morning twilight begins; for the sun's place will be 18° below the eastern side of the horizon. Then, to find the time when the evening twilight ends, bring the sun's place to the western side of the horizon, and the point opposite to it, which was marked, will be rising in the east; bring the quadrant over that point, and keeping it thereon, turn the globe westward, until the said point be 18° above the horizon on the quadrant, and the index will shew the time when the evening twilight ends; as the sun's place will be then 18° below the western side of the horizon.

When the sun does not go 18° below the horizon of any place, the twilight continues the whole night in that place: and between 49° of latitude, and the polar circles, the twilight continues for several nights together, in the summer season: and the nearer the place is to the polar circle, the greater is the number of these nights.

PROBLEM

PROBLEM XIV.

To find what Day of the Year the Sun begins to shine constantly without setting, on any given Place in the Frigid Zone, and how long it continues to do so.

Rectify the globe for the latitude of the place, and turn it about till some point of the ecliptic, between *Aries* and *Cancer*, (if the given place be in the north frigid zone,) coincides with the north point of the horizon, where the brazen meridian cuts it. Then find on the wooden horizon, what day of the year the sun is in that point of the ecliptic; for that is the day on which the sun begins to shine constantly on the given place, without setting. Then turn the globe until some point of the ecliptic, between *Cancer* and *Libra*, coincide with the north point of the horizon, where the brazen meridian cuts it; and find on the wooden horizon on what day the sun is in that point of the ecliptic; which is the day the sun leaves off constantly shining on the said place, and rises and sets to it, as to other places on the globe. The number of natural days, or complete revolutions of the sun about the earth, between the two days above found, is the time that the sun keeps constantly above the horizon without setting; for all that portion of the ecliptic which lies between the two points which intersect the horizon in the very north, never sets below it; and there is just as much in the opposite part of the ecliptic that never rises; therefore, the sun will keep as long constantly below the horizon of every place upon the globe in winter, as he is above it in summer.

PROBLEM XV.

Having the Latitude, the Sun's Place, and his Altitude, to find the Hour of the Day, and the Sun's Azimuth, or Number of Degrees that he is distant from the Meridian.

Having rectified the globe, and brought the sun's place to the given height upon the quadrant of altitude, which must

be

be on the eastern side of the horizon, if the time be in the forenoon; and on the western side if it be afternoon; then the index will shew the hour of the day; and the number of degrees in the horizon intercepted between the quadrant of altitude and the south point, will be the sun's true azimuth at that time.

PROBLEM XVI.

To find what Hour of the Day it is, in any Part of the World.

Rectify the globe for the latitude of the place; and having set the index to the hour of the day, turn the globe, and bring the places of which the hour is required, successively to the brazen meridian, and the index will point to the several hours. For example: if the place be London, and the hour 12 at noon, the globe being rectified for London, and London brought to the meridian, and the index set to the hour 12, turn the globe, till Naples comes to the brazen meridian, and the index then will point to the hour 1; Naples being 15° eastward of London. Then continue to turn the globe 15° further, and Petersburg, Constantinople, and Grand Cairo, will come under the brazen meridian, or very near it, then the index will point to the hour 2; these three cities having the noon-day sun about two hours before us in London. And turning the globe 15° further, the index will point to the hour 3; and all the places under the meridian will have the sun vertical to them. And thus for every 15° of longitude eastward, the inhabitants of those places have the sun an hour sooner. On the contrary, all the inhabitants situated to the westward of London, have the sun later in the same proportion; that is, an hour later for every 15° of western longitude.

Most of the foregoing Problems may be resolved by a map as well as a globe, though the operation may be somewhat more

more tedious, particularly by Plate 10, where the two hemispheres of the world represent the surface of a terrestrial globe in plano.

WINDS.

Winds are generally divided into two parts, according to the different parts of the earth on which they blow; being the tropical winds, or those which blow between the two tropics, and those which blow without the tropics.

The tropical winds generally extend to 30° on each side the equator, and are of three kinds: 1. The *general trade-winds*. 2. The *monsoons*. 3. The *sea and land breezes*.

1. The trade-winds blow from north-east on the north side of the equator, and from the south-east on the south side of the equator, and near the equator almost due east; but under the equator, and from 2 to 5° on each side of the equator, the winds are variable; and sometimes it is calm for a month together.

2. The monsoons are periodical winds, which blow six months in one direction, and the other six months in the opposite direction. At the change, or shifting of the monsoons, are generally violent storms of wind, thunder, lightning, and rain, which always happen about the equinoxes. The monsoons extend about two hundred leagues from land, and are chiefly in the Indian seas.

3. The sea and land breezes are also periodical winds, which blow from the land in the night, and best part of the morning; and from the sea from ten in the forenoon till six in the evening. These do not extend above two or three leagues from shore.

Near the coast of Guinea, in Africa, the wind blows almost constantly from the west.

On the coast of Peru, in South America, the wind blows constantly from the south-west.

Between the third and tenth degree of south latitude, the south-east trade-wind continues from April to October; but during the rest of the year, the wind blows from the north-west. Between Sumatra and New Holland this wind blows from the south from March to September; but from September to April blows in the opposite direction. Between Africa and Madagascar its direction is influenced by the coast; for it blows from the north-east from October to April; and from the south-west the rest of the year.

In the Indian ocean, to the northward of the third degree of south latitude, the north-east wind blows from October to April, and the opposite wind the rest of the year. This wind blows nearly from the south in the summer months, from the isle of Borneo, along the coast of Malacca, as far as China; and in the winter months it blows from the north by east.

In the temperate zones the winds are very irregular, and no certain rule can be formed of their changes. But when winds are violent and continue long, it is generally found that they extend over a large tract of country; particularly if they blow from the north or east. By the multiplication and comparisons of meteorological tables, the following theorems have been deduced.

In Virginia, the prevailing winds are between the south-west, west, north, and north-west; but the most frequent is the south-west. At Ipswich, in New England, the prevailing winds are the same, but the most frequent is the north-west. At Cambridge, in the same province, the most frequent is the south-east. The predominant winds of New York are the north and west. And in Nova Scotia, north-west. And at Hudson's Bay, west.

It appears from these observations, that the westerly winds are the most frequent over the whole eastern coasts of North America; but in the southern provinces the south-west wind is predominant; and the north-west wind becomes gradually more frequent as we approach the frigid zone.

In

In Egypt, from May to September, inclusive, the wind blows almost constantly from the north, varying in a few points from east to west, in the months of June and July.

In the Mediterranean sea the wind blows nearly nine months of the year from the north; and at the equinoxes there is always an easterly wind in that sea. But in the straits of Gibraltar the winds are either from east or west.

In Italy the prevailing winds differ considerably, according to the situation of the places: at Rome and Padua they are northerly; at Milan easterly.

The prevailing wind in Spain and Portugal is the west; particularly on the western coasts of these countries; but at Madrid it is north-east.

In France, along the whole south coast of that country, the wind blows most frequently from the north, or north-east and north-west. On the western coast of the Netherlands, as far north as Rotterdam, the prevailing wind is the south-west.

From the register, kept for the space of ten years, by order of the Royal Society at London, the average of the winds at that place, blow in the following order:—

Winds.	Days.	Winds.	Days.
South-west	112	South-east	92
North-east	58	East	26
North-west	50	South	18
West	53	North	16

It appears from this register, that the south-west wind blows at an average more frequently than any other winds, during every month of the year, but particularly in July and August; and the north-east blows most constantly during the months January, March, April, May, and June, and most seldom during February, July, September, and December; and the north-west blows more frequently from November to March; and more seldom in September and October than in any other months.

Tides.

The tide is that rise and fall of the water observed on all maritime coasts.

It is observable, that on the shores of the ocean, and in all bays, creeks, harbours, &c. which have a free communication with the ocean, the waters rise up above a certain mean rate twice a day, and as often sink below: this is what is called the *flood* and *ebb*; or an high and low water. The whole interval between high and low water is called a *tide*: the water is said to *flow* and to *ebb*; and the rising is called the *flood tide*, and the falling the *ebb tide*.

This rise and fall of the waters is very variable in quantity. Thus, at Plymouth it is sometimes twenty-one feet between the greatest and least depth of the water in one day, or between high and low water; and sometimes it is only twelve feet.

The greatest flow of tide in any place is called a *spring tide*, and the least flow is called a *neap tide*; and the different heights of the tide gradually increase every day from a neap to a spring tide; and then gradually decrease from a spring to a neap tide.

The whole time between the spring and neap tide is about fifteen days; and two of these intervals will make an exact lunation, or change of the moon. For the spring tide is observed to happen at a certain interval of time (generally between two and three days) after the new or full moon; and the neap tide at a certain interval after the half moon. Thus, the high water happens at new and full moon, when the moon has a certain determined position with respect to the meridian of the place of observation, preceding or following the moon's southing a certain interval of time; which is always constant with respect to that place, but very different in different places.

The

The interval between two succeeding high waters is very variable. It is least of all about new and full moon, and greatest when the moon is at her quadratures. As two high waters happen every day, we may call the double of their interval, a tide day. Now, this tide day is shortest about new and full moon, being then about 24 hours and 37 minutes; but longest at the moon's quadratures, being then 25 hours and 27 minutes.

The tides, being in similar circumstances, are greatest when the moon is at her least distance from the earth; and least, when she is at her greatest distance from the earth.

The same may be remarked with respect to the sun's distance. Thus, the greatest tides are observed during the winter months in Europe, or when the sun is at his least distance.

The tides in every part of the ocean increase as the moon, by changing her declination, approaches the zenith of that place.

The tides which happen while the moon is above the horizon are greater than those of the same day, when the moon is below the horizon.

These are all the regular phenomena of the tides. They are of the utmost importance to all commercial nations, and have therefore been much attended to by all navigators and astronomers.

SECT. III.

THE GRAND DIVISIONS OF THE EARTH.

THE principal divisions of the earth, as before mentioned, are, into land and water.

The land is divided into two great continents, besides islands, viz. :—The eastern and western continents. The eastern continent is subdivided into the following parts, viz. :—Europe on the north-west, Asia on the north-east, and Africa on the south, being joined to Asia by the Isthmus of Suez, which is 60 miles over. The western continent consists of North and South America, joined by the Isthmus of Darien, between 60 and 70 miles in breadth.

Europe is again subdivided into the following principal parts, and is situated between the tenth degree west longitude and the sixty-fifth degree east longitude, and between the thirty-sixth and seventy-second degree of north latitude. It is bounded on the north by the Frozen Ocean, on the east by Asia, on the south by the Mediterranean Sea, which divides it from Africa; and on the west by the Atlantic Ocean, which separates it from America; being 3000 miles in length, from Cape St. Vincent in the west to the mouth of the river Oby in the north-east; and 2500 miles broad from north to south, from North Cape in Norway to Cape Cayha in the Morea, the most southern point of Europe. It contains the following states and kingdoms :

A TABLE

A TABLE OF EUROPE.

Nations.	Length.	Breadth.	Chief Cities.	North Latitude.	Longitude from Greenwich.	Difference of Time from Greenwich.
				D. M. S.	D. M. S.	H. M. S.
England	360	300	London	51 31 0	0 5 37 W	0 0 12 aft.
Scotland	300	150	Edinburgh	55 57 57	3 12 15 W	0 12 49 aft.
Ireland	285	150	Dublin	53 21 12	6 6 30 W	0 24 16 aft.
Norway	1000	300	Bergen	60 11 0	5 45 0 E	0 24 0 bef.
Denmark	240	180	Copenhagen	55 40 45	12 35 15 E	0 50 21 bef.
Sweden	800	500	Stockholm	59 20 35	18 3 55 E	1 12 16 bef.
Russia	1500	1100	Peterburgh	59 56 0	30 19 15 E	2 1 17 bef.
Poland	700	680	Warsaw	52 14 0	21 0 30 E	1 24 2 bef.
Prussia	609	350	Berlin	52 32 30	13 26 15 E	0 53 45 bef.
Germany	600	500	Vienna	48 12 40	16 22 30 E	1 5 30 bef.
Bohemia	300	250	Prague	50 4 30	14 45 0 E	0 59 0 bef.
Holland	150	100	Amsterdam	52 22 45	4 45 30 E	0 19 2 bef.
Flanders	200	200	Brussels	50 51 0	4 21 45 E	0 17 27 bef.
France	600	500	Paris	48 50 14	2 20 0 E	0 9 20 bef.
Spain	700	500	Madrid	40 25 0	3 25 45 W	0 13 43 aft.
Portugal	300	100	Lisbon	38 42 25	9 9 59 W	0 36 40 aft.
Switzerland	260	100	Bern	40 0 0	7 40 0 E	0 28 0 bef.
Italy	750	400	Rome	41 53 54	12 29 15 E	0 49 50 bef.
Hungary	300	200	Buda	47 20 0	19 22 0 E	1 17 0 bef.
Turkey in Europe	1400	730	Constantinople	41 0 24	28 53 49 E	1 55 35 bef.

Besides the foregoing states, Europe contains several islands, of which the following are the principal.

Islands.

<i>Islands.</i>		<i>Chief Towns.</i>	<i>Subject to</i>
In the North- ern Ocean	Iceland	Skalholt	Denmark
In the Baltic Sea	Zealand, Funen, Alsen, Falster, Langeland, La- land, Femeren, Mona, Bornholm	-	Denmark
	Gothland, Aland, Ru- gen	-	Sweden
	Ofel Dagho	-	Russia
	Usedom, Wollin	-	Prussia
	Ivica	Ivica	Spain
Mediterranean Sea	Majorca	Majorca	Spain
	Minorca	PortMahon	Spain
	Corfica	Bastia	France
	Sardinia	Cagliari	K. Sardinia
	Sicily	Palermo	K. of S. Sicily
Gulf of Venice	Lusitana, Corfu, Ce- phalonia, Zant, Leu- cadia	-	Venice
Archipelago & Levant Seas	Candia, Rhodes, Ne- gropont, Lemnos, Te- nedos, Scyros, My- telene, Scio, Samos, Patmos, Paros, Cerigo, Santorin, &c. being part of Ancient and Modern Greece	-	Turkey

Asia is situated between 25 and 180 degrees of east longitude, and between the equator and 80 degrees north latitude; being about 4740 miles in length, from the Dardanelles on the west to the eastern shore of Tartary; and about 4380 in breadth, from the most southern part of Malacca to the most northern cape of Nova Zembla. It is bounded by the Frozen Ocean on the north, on the west it is separated from Africa by the Red Sea, and from Europe by the Levant or Mediterranean, the Archipelago, the Hellespont, the Sea of Marmora, the Bosphorus, the Black Sea, the river Don, and a line drawn from it to the river Tobel,

and from thence to the river Oby, which falls into the Frozen Ocean; on the east it is bounded by the Pacific Ocean or South Sea, which separates it from America; and on the south, by the Indian Ocean: thus, it is almost surrounded by the sea. The principal divisions are as follow:

A TABLE OF ASIA.

Nations.	Length.	Breadth.	Chief Cities.	North Latitude.	East Longitude from Greenwich.	Difference of Time from Greenwich.
				D. M. S.	D. M. S.	H. M. S.
Tartary { Independent	Boundaries		Samarcand	39 50 0	69 0 0	4 36 obef.
{ Mogulean	of these		Tibet	37 0 0	8 0 0	5 40 obef.
{ Chinese	countries		Chynian	48 0 0	12 0 0	4 08 obef.
{ Russian	variable.		Tobolski	58 12 18	68 12 45	4 38 42 obef.
Persia	1300	1100	Isfahan	32 25 0	52 40 0	3 31 20 obef.
India	2000	1000	Siam or Pegu	14 18 0	100 50 0	6 43 20 obef.
{ Empire of Mogul }	2000	1500	Delhi	28 20 0	79 25 0	5 16 obef.
China	1440	1260	Pekin	39 54 30	116 14 15	7 45 27 obef.
Georgia	210	140	Teflis	43 0 0	46 15 0	3 10 obef.
Turcomania	360	300	Erzerum	39 56 35	48 35 45	3 14 21 obef.
{ Diarbec or Mesopotamia }	560	310	Bagdat	33 20 0	43 46 30	6 obef.
Natolia	750	388	{ Burfa or Smyrna	38 28 7	35 20 0	2 21 20 obef.
Palestine	210	90	Jerusalem	31 55 8	27 19 45	1 49 15 obef.
Syria	270	160	Aleppo	35 45 23	37 20 0	2 29 20 obef.
Part of Arabia	1300	1200	Mecca	21 45 0	40 55 0	3 52 obef.

Asiatic Islands in the Indian and Pacific Ocean.

<i>Islands.</i>	<i>Towns.</i>	<i>Belonging to</i>
New Holland	Sydney Cove	English
The Japanese Isles	Jeddo and Meaco	Dutch
The Ladrões	Guam	Spain
Formosa	Tai-ouan-fou	China
Anian	Kiontcheow	China
The Philippines	Manilla	Spain
The Molucca or Clove Isles	Victoria Fort	Dutch
The Banda or Nutmeg Isles	Lantor	Dutch
Amboyna	Amboyna	Dutch
Celebes	Macassar	Dutch
Gilolo	Gilolo	Dutch
The Sunda Isles	Borneo, Caytoogee	Several Nations
	Sumatra, Achen & Bencoolen	English & Dutch
	Java, &c. Batavia and Bantam	Dutch
The Andaman and Nicobar Isles	Andaman, Nicobar	Several Nations
Ceylon	Candia	English
The Maldives	Caridon	English
Bombay	Bombay	English
The Ale Isles in the Sea of Kamschatka, discovered by the Russians	-	Russia
New Guinea, New Britain, New Ireland, New Hebrides, New Caledonia, New Zealand, and the Friendly, Sandwich, and Society Islands, are uncolonized.		

Africa, the third grand division of the globe, is generally represented as bearing some resemblance to the form of a pyramid, whose vertex or point is the Cape of Good Hope, and its base the shores of the Mediterranean Sea. It is a peninsula of great extent, joined to Asia by the Isthmus of Suez: its greatest length from north to south, from Cape Bona in the Mediterranean, to the Cape of Good Hope, is 4600 miles; and the breadth, from Cape Verd to Cape Guardafui, is 3500 miles. It is bounded on the north by the Mediterranean Sea, which separates it from Europe; on the east by the Isthmus

Isthmus of Suez, the Red Sea, and the Indian Ocean, which divides it from Asia; on the south by the Southern Ocean; and on the west by the great Atlantic Ocean, which separates it from America.

Very few travellers have penetrated into the interior part of this quarter of the world; consequently we still remain ignorant of the bounds, and even of the names of many of the inland parts; but, according to the best accounts, it is divided according to the following table:—

Nations.	Length.	Breadth.	Chief Cities.	Latitude.	Longitude from Greenwich.	Difference of Time from Greenwich.
				D. M. S.	D. M. S.	H. M.
Barbary { Morocco,						
	500	480	Fez	33 40 0N	6 0 0W	0 24 aft.
	600	400	Algiers	36 49 0N	2 12 0E	0 9 bef.
	400	250	Tunis	36 40 0N	10 0 0E	0 39 bef.
	400	300	Tripoli	32 50 0N	21 30 0E	1 26 bef.
	700	240	Polemota	32 53 0N	13 5 0E	0 52 bef.
	600	250	Grand Cairo	30 2 0N	31 18 0E	2 5 bef.
Egypt						
Biledulgerid	2500	350	Dara		8 0 0W	0 32 aft.
Zaara	2400	660	Tegefa	21 40 0N	6 0 0W	0 24 aft.
Negroland	2200	840	Madinga			0 38 aft.
Guinea	1800	360	Benin	7 40 0N	5 4 0E	0 20 bef.
Upper Ethiop. { Nubia	940	600	Nubia	17 0 0N	33 0 0E	2 12 bef.
	900	800	Gondar	13 10 0N	35 0 0E	2 20 bef.
	540	130	Doncala	15 6 0N	39 0 0E	2 36 bef.
<p>The middle parts, called Lower Ethiopia, are very little known to Europeans, but are computed at one million two hundred thousand miles.</p>						
Loango	510	300	Loango	5 0 0N	11 0 0E	0 44 bef.
Congo	640	420	St. Salvador	5 0 0S	15 0 0E	1 1 bef.
Angola	460	250	Loando	8 30 0S	14 30 0E	0 58 bef.
Benguela	430	180	Benguela	11 0 0S	14 30 0E	0 58 bef.
Mataman	450	240	No towns			
Ajan	900	300	Biava	1 0 0N	45 0 0E	3 0 bef.
Zanguebar	1400	350	Mozambique	15 0 0S	40 0 0E	2 40 bef.
Monomotapa	560	660	Monomotape	—	—	1 18 bef.
Monemugi	900	660	Chicova	—	—	1 44 bef.
Sofola	480	300	Sofola	20 0 0S	36 40 0E	2 26 bef.
Terra de Natal	600	330	No town			
Caffraria, or Hottentots	708	660	{ Cape of } { G. Hope }	33 55 0S	18 23 0E	1 13 bef.

<i>Islands.</i>	<i>Towns.</i>	<i>Belong to</i>
Babel Mandel	Babel Mandel	
Zocora in the Indian Ocean	Calanfia	
The Comora Isles, ditto	Joanna	
Madagascar, ditto	St. Austin	
Mauritius, ditto	Mauritius	French.
Bourbon, ditto	Bourbon	Ditto.
St. Helena, in the Arkan.Ocean	St. Helena	English.
Ascension Isle, ditto	—	Uninhabited.
St. Thomas	St. Thomas	
St. Matthew, ditto	—	Uninhabited.
Anaboa, Prince's Islands	Anaboa	Portuguese.
Fernamcogo, ditto		
Cape Verd Islands, ditto	St. Domingo	Portuguese.
Goree, ditto	Fort St. Michael	French.
Cmaries, ditto	{ Palma, St. Christopher }	Spain.
Madeiras, ditto	Santa Cruz	Portuguese.
The Azores, or Western Isles, which are at an equal distance from Eu- rope, Africa, and Ame- rica, ditto	Angra, St. Michael	} Portuguese.

America, the great western continent, called the **New World**, runs north and south through every habitable climate upon the earth; extending from the eightieth degree of north latitude to the fifty-sixth degree of south latitude; and its breadth, where it is known, extends from the thirty-fifth to the one hundred and thirty-sixth degree of west longitude from London; being near 9000 miles in length, and 3690 in breadth. Extending into both the hemispheres, it has consequently two summers and two winters, and has all the variety of climates to be met with on the face of the earth. On the east it is bounded by the great Atlantic Ocean, which divides it from the eastern continent. On the west it has the Pacific Ocean, or great South Sea, which separates it from Asia. It is composed of two parts, called North and South America, joined together by a narrow neck of land, called the

the Isthmus of Darien, in the kingdom of Mexico, 1500 miles long, and at one part being only 60 miles in breadth, so that to effect a communication between the two oceans is by no means difficult. In the great gulf, which is formed between the isthmus and the northern and southern continents, lie a multitude of islands, denominated the West Indies, in contradistinction to the islands of Asia, beyond the Cape of Good Hope, which are called the East Indies. The grand divisions of North America are as follows:

North America.

Countries.	Length.	Breadth.	Chief Towns.	Latitude.	Longitude from Greenwich.	Difference of Time from Greenwich.
New Britain	Not known			D. M. S.	D. M. S.	H. M. S.
Canada	800	200	Quebec	46 55 0 N	69 53 0 W	4 39 30 aft.
Nova Scotia	350	250	Halifax	44 45 0 N	64 30 0 W	4 18 0 aft.
United States	1390	700	Philadelphia	39 57 0 N	75 8 0 W	5 0 0 aft.
East Florida	500	440	{ St. Augustine	30 8 0 N	81 10 0 W	5 25 0 aft.
West Florida			{ Pensacola	30 32 0 N	87 20 0 W	5 49 0 aft.
Louisiana	Not known		New Orleans	30 0 0 N	87 5 0 W	5 48 0 aft.
N. Mexico and California	1000	1600	{ Santa Fee	35 32 0 N	105 0 0 W	7 0 0 aft.
			{ St. Juan			
Mexico, or New Spain	2000	600	Mexico	19 54 0 N	100 5 0 W	6 40 0 aft.

South America.

Terra Firma	1400	700	Panama	8 48 0 N	80 28 0 W	5 22 0 aft.
Peru	1800	500	Lima	12 1 0 S	76 49 0 W	5 7 0 aft.
Amazonia	1200	960	Little known			
Guyana	780	480	{ Surinam	6 0 0 N	55 30 0 W	3 42 0 aft.
			{ Cayenne	4 56 0 N	52 15 0 W	3 29 0 aft.
Brazil	2500	700	{ St. Sebastian	22 59 0 S	44 16 0 W	2 57 0 aft.
			{ St. Salvadore	12 0 0 S	38 0 0 W	2 32 0 aft.
Paraguay, or La Plata	1500	1000	{ Assumption and Buenos Ayres	34 10 0 S	60 40 0 W	4 3 0 aft.
				34 35 0 S	58 31 0 W	3 54 0 aft.
Chili	1200	500	St. Jago	33 40 0 S	77 0 0 W	5 8 0 aft.
Terra Magellanica, or Patagonia	700	300	{ Uncolonised by Europeans.			

Grand Divisions of South America.

<i>Nations.</i>	<i>Length.</i>	<i>Breadth.</i>	<i>Chief Towns.</i>	<i>Distance bearing from London.</i>	<i>Belongs to</i>
Terra Firma	1400	700	Panama	4650 S. W.	Spain.
Peru	1800	600	Lima	5520 S. W.	Ditto.
Amazonia	1200	900			
Guziana	760	680	{ Surinam } { Cayenne }	3840 S. W.	Dutch. French.
Brazil	2500	700	St. Sebastian	6000 S. W.	Portugal.
La Plata	1500	1000	Buenos Ayres	6040 S. W.	Spain.
Chili	1200	500	St. Jago	6600 S. W.	Ditto.
Terra Magellanica, or Patagonia	1400	460			

The

The principal Islands in North America belonging to the Europeans.

<i>Islands.</i>		<i>Length.</i>	<i>Breadth.</i>	<i>Chief Towns.</i>	<i>Belongs to</i>
The West India Islands, lying in the Atlantic, between N. and S. America.	In the gulf of				
	Atlantic. St. Lawrence				
	{ Newfoundland	350	200	Placentia	England
	{ Cape Breton	110	80	Louisburg	Ditto
	{ St. John	60	30	Charlotte	Ditto
	{ The Bermuda	400 in	—	St. George	Ditto
	{ Isles	number			
	{ The Bahama Isles			Nassau	Ditto
	Jamaica	140	60	Kingston	Ditto
	Barbadoes	21	14	Bridgetown	Ditto
	St. Christopher's	20	7	Basse Terre	Ditto
	Antigua	20	20	St. John's	Ditto
	Nevis	6	3	Charlestown	Ditto
	Montserrat	5	4	Plymouth	Ditto
	Barbuda	20	12	—	Ditto
	Anguilla	30	10	—	Ditto
	Dominica	28	13	—	Ditto
	St. Vincent	24	18	Kingston	Ditto
	Granada	30	15	St. George	Ditto
	Cuba	700	90	Havannah	Spain
	Hispaniola	450	150	St. Domingo	France
	Porto Rico	100	49	Porto Rico	Spain
	Trinidad	90	60	St. Joseph	England
	Margarita	40	23	—	Spain
	Martinico	60	30	St. Peter	France
	Guadaloupe	45	38	Basse Terre	England
	St. Lucia	23	12	—	France
	Tobago	32	9	—	Ditto
	St. Bartholomew	—	—	—	Sweden
	Defeada	—	—	—	France
	Marigalante	—	—	—	Ditto
	St. Eustatia	7	4	The Bay	Dutch
	Curassou	30	10	—	Ditto
	St. Thomas	5	3	—	Denmark
	St. Croix	30	10	Basse End	Ditto

I shall here subjoin a table of the superficial content of the several parts of the globe in square miles, accounting 6p miles to a degree on the equator.

	Square Miles.	Islands.	Square Miles.	Islands.	S.M.
The Globe	199,512,595	Cuba	38,400	Funen	62
Seas and un- known Parts }	160,522,026	Java	38,250	Yvica	25
The habitable World }	38,990,567	Hilpaniola	36,000	Minorca	20
Europe	4,456,700	Newfoundland	35,500	Rhodes	20
Asia	10,768,822	Ceylon	27,730	Cephalonia	20
Africa	9,654,807	Ireland	27,457	Amboyna	40
America	14,110,874	Formosa	17,000	Orkney	20
Perfian Emp. }	1,650,800	Anian	11,900	Pomona	24
under Darius }		Gilolo	10,400	Scio	30
Roman Emp. }	1,610,000	Sicily	9,400	Martinico	26
in its great- est height }		Timor	7,800	Lemnos	20
Ruffian	4,161,685	Sardinia	6,600	Corfu	194
Chinese	1,749,000	Cyprus	6,300	Providence	168
Great Mogul	1,116,000	Jamaica	6,000	Man	160
Turkish	950,000	Flores	6,000	Bornholm	160
Perfent Perfia	800,000	Ceram	5,400	Wight	150
ISLANDS.		Bretun	4,000	Malta	150
Borneo	228,000	Socotra	3,600	Barbadoes	140
Madagascar	168,000	Candia	2,220	Zant	120
Sumatra	129,000	Porto Rico	3,200	Antigua	100
Japan	118,000	Corfica	2,720	St. Chrift-	
Great Britain	72,926	Zealand	1,935	pher	80
Celebes	68,400	Majorca	1,406	St. Helena	80
Manilla	56,500	St. Jago	1,400	Caernfey	50
Iceland	46,000	Negropont	1,300	Jersey	43
Terra del Fu- ego }	42,075	Teneriff	1,272	Bermudas	40
Mindanao	19,200	Gothland	1,000	Rhodes	36
		Madeira	950		
		St. Michael	910		
		Skye	900		
		Lewis	880		

There are also several other considerable islands, chiefly in the South Seas, the exact dimensions of which are not certainly known, but they may be ranged according to their magnitude in the following order: New Holland being nearly equal in size to the whole continent of Europe.

New

New Holland.	Otaheite, or King George's
New Guinea.	Island.
New Zealand.	Friendly Islands.
New Caledonia.	Marquesas.
New Hebrides.	Easter, or Davis's Island.

S E C T. IV.

OF THE DIFFERENT GOVERNMENTS OF THE WORLD.

MANKIND were no sooner united into civil societies, than they discovered an inclination to oppress each other. That system of equality, in which they were left by nature, gave the strongest, and the most crafty, the advantage over his weaker and undesigning neighbours. From hence arose the necessity of forming conjunctions of several individuals, or families together, who should implicitly follow the dictates or commands of some chosen superior, or leader. And, to prevent the altercations, strife, and consequently bloodshed, that inevitably followed the nomination of every new leader, or prince, they caused the office to be made hereditary. Consequently, absolute and hereditary monarchy was the first original form of government; as appears from sacred writ; where Nimrod is represented by his courage and dexterity to have acquired a superiority of fame and power above his contemporaries; and he founded, at Babylon, the first monarchy whose origin is mentioned in history.

In the year 1496 before Christ, the Greeks were the first people who, by the advice and public-spirited endeavours of Cecrops, and Cranaus his successor, formed a regular council. For Amphictyon, one of those disinterested characters

rafters who live for the good of the community of which he is a member, endeavoured to find an expedient to unite the several independent kingdoms of Greece into one body; and thus to put a stop to those fatal consequences of intestine division, and civil discord, which rendered them a prey to each other, and an easy conquest to the invader. He, therefore, engaged the kings, or leaders, of twelve different cities, to unite together for their mutual security and welfare. Two deputies from each city assembled twice a year at Thermopylæ, and formed the Amphictyonic Council. In this assembly the general interests of the states were discussed. Amphictyon, in order to render those several connexions more durable, connected them with religious charge, intrusting the care of the temple at Delphi, with the riches that accrued to this place from those who consulted the oracles, to the care also of these deputies. This assembly was the first political establishment of a plurality of power, that we have any authentic account of in history; and gave an energy of action to Greece, which enabled them to defend their liberty and independence against the great force of the Persians.

This was the first deviation from absolute monarchy, recorded in profane history; from that time, various have been the modes and forms of government in different nations; though, if we except some part of the Roman history, Greece, and a few nations of less note, the monarchical form of government was the most prevailing for the next two thousand years.

Athens is an instance of the pernicious effects of division in a state; and also displays the benefits of unanimity. Theseus, king of Attica, about the year before Christ 1234, perceiving the danger to which his country was exposed by this twelve-fold division, endeavoured to form a conjunction of the states; for this purpose, he detached the leaders of the different tribes as much as possible from the people they governed; he abolished the different courts established in different parts of Attica

Attica, and appointed one council hall, common to all the Athenians. He established a common form of religion, with certain religious ceremonies to be performed at Athens, the more effectually to strengthen civil allegiance: and by inviting strangers from all parts of the world, by the promise of privileges and protection, he raised the city to the highest pitch of fame and popularity. The splendour of Athens eclipsed that of all the other states of Greece.

This monarchy soon gave place to an overbearing influence. Theseus had formed his kingdom into three distinct classes: the nobles, the artisans, and the husbandmen. And to prevent the increasing power of the nobles, he granted many immunities and privileges to the two other classes. This system of politics, in a few years, gave the two inferior classes an opportunity of acquiring considerable property; and, consequently, they became considerable members of the state; and, by their riches and independence, upon the death of Codrus, a prince of great merit, in the year B. C. 1070, they had power and influence enough to abolish the regal power, under pretence of finding no one worthy of filling the throne of Codrus, who had devoted himself to death for the safety of his people. Thus they proclaimed Jupiter king, declaring none else was fit to govern Athens. This was the first instance of a republican form of government in Greece.

From this period, so various have been the modes and forms of government, that it is impossible to distinguish them all. Governments are generally divided into three distinct forms, each of which has its partisans, viz. the monarchical, aristocratical, and democratical.

The monarchical form of government is, where a nation is governed by a king, or monarch; and is divided into two parts, called absolute, and limited, monarchy. Absolute monarchy is, where the sovereign is entirely unrestrained, having the legislative as well as the executive power. A limited

ed monarchy is, where the sovereign is restrained by certain laws, beyond which he cannot pass.

An aristocracy is, where the legislative and executive authority is vested in the hands of a select number of persons, generally titled nobility; and in whom the office is mostly hereditary.

A democracy is that government in which the legislative and executive authority is vested in a certain number of individuals, who hold their office by election; and generally elected by the majority of the nation at large.

From the various modifications of these different forms of government, all the governments of the earth are formed; some approaching nearer to one, and some to another form. For there is hardly a government existing, that is entirely either an absolute monarchy, a perfect aristocracy, or a complete republic.

S E C T. V.

OF RELIGION.

RELIGION is coeval with the origin of mankind: without it the present order of the universe would be entirely overturned; and mankind, from their natural depravity, be rendered worse than the most voracious of the brute creation.

The distinguishing religions in antiquity were Judaism, and Polytheism, or Paganism.

But in modern times the prevailing religions may be divided into the four following, viz. the Jewish, Christian, Mahometan, and Pagan.

Before

Before treating of the four foregoing systems, it may be necessary to premise the following general axiom, viz. That all systems of religion contribute more or less to the welfare of society. From hence we deduce the following theorem; that all religion must have somewhat in its origin of a divine nature, however it may be transformed, corrupted, or misapplied, by the ignorance or artifice of its propagators.

In considering the Jewish code of religion, it does not appear as a complete system of religion, adapted to all countries and ages, but seems particularly designed by the all-wise Creator, for the people to whom it was sent; for the age they lived in, being over-run by idolatry; the circumstances in which they had lived in Egypt, and the means by which they were to form their new settlement in the land of Canaan.

From hence they were enjoined the observation of the sabbath, in honour to that Being who created the heavens and the earth, with all the host of heaven; which host, sun, moon, stars, &c. were worshipped by the Egyptians as *eternal beings*. To prevent their communication with the neighbouring idolatrous nations, they were proscribed the use of certain animals for food, and permitted others; that, by being forbidden the use of those animals for food, such as the hog, &c. which the Gentile nations considered as the greatest luxury, a perpetual bar might be kept up between the Jews and Gentiles. And by being permitted to eat other animals, such as goats, sheep, oxen, &c. which were worshipped in Egypt, and from which the Egyptians religiously withheld all violence, the Jews would soon overcome any religious prejudices they might have acquired from the Egyptian idolatry.—The restitution of property, in the year of Jubilee, which would answer no purpose in another state, was designed to preserve the order of rank, and that division of property, originally established.

In condescension to their rude and gross notions of Deity,

the Creator permitted them, in their wanderings through the Wilderness, to have a tabernacle, or portable temple, in which he sometimes deigned to display some rays of his glory.

From this general view of the Jewish religion, it appears happily adapted to promote the welfare of its followers. In comparing it with other religions, it is necessary to reflect on the peculiar purposes for which it was established; which were principally two; first, to preserve the Jews a separate people; and secondly, to guard them from the idolatry with which they were every where surrounded. The religion of the Jews was not formed, nor designed to be propagated through all the earth; that would have been inconsistent with the purposes for which it was instituted: therefore we see the Jewish religion, though near four thousand years old, wants that essential attribute for propagation, to be found in all other religions, viz. a difference of sentiment, and, consequently, a division, and subdivision into different sects.

The Christian religion is to be considered as an improvement of the Jewish. The effects of the Jewish religion were indeed beneficial, but were confined almost to them alone; whereas the effects of the Christian religion are extended to all mankind; representing them with true philanthropy as children of the same God, and heirs of the same salvation. It levels all distinctions of rich and poor, native and foreigner, as accidental and insignificant distinctions with that impartial Being, who rewards or punishes according to the demerits of his creatures.

The precepts of the Christian religion are more happily calculated to promote the happiness of mankind, than those of any other religion. Its whole design is to inspire mankind with mild, benevolent, and peaceable dispositions. Its distinguishing rule, by which it excels all other religions, is, *to do unto others, as we would they should do unto us*; and such is its purity, that it does not allow an impure thought. It requires its followers to abandon their vices, however dear; and

to join the cautious wisdom of the serpent with the innocent simplicity of the dove. And to prevent perseverance in immorality, it offers a pardon for the past, provided the offender forsake his vicious practices. The practice and belief of the Gospel have a peculiar tendency to raise the mind of man above the trifling pursuits of time; and to render its followers incorruptible by wealth, honour, or pleasures. It not only requires the Christian to abstain from injuring his neighbour, but even enjoins him to forgive any unmerited injuries which he himself suffers, upon the principle of his being forgiven by his offended Creator. It represents the Deity and his attributes in the fairest light, so as to render our ideas of him consistent with the correct principles of reason and philosophy. The rites of this gospel are few and simple; easy to perform, expressive, and edifying. It inculcates no duties, but what are founded on the principles of human nature, and on the relation in which man stands to God, as his Creator, Redeemer, and Sanctifier. The assistance of the Spirit of God is there promised to those who labour to discharge the duties which it enjoins. It teaches us that worldly afflictions are casual accidents; incident to both bad and good men: *a doctrine highly encouraging to virtue, consoling in affliction, preventing despair, and encouraging in difficulty.*

Such are the precepts and spirit of the Christian religion. And even those who have refused to give credit to its history, and follow its doctrines, have acknowledged the excellency of its precepts. Bolingbroke, one of its most zealous opposers, says, that "no religion ever yet appeared in the world, of which the natural tendency was so much directed to promote the peace and happiness of mankind, as the Christian; and that the Gospel of Christ is one continued lesson of the strictest morality, of justice, benevolence, and universal charity." Thus we can pronounce, with confidence, that the precepts of a religion, which is so happily formed to promote all that
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is just and beneficial to mankind, cannot but be in the highest degree divine. By reviewing the effects which it has produced, we shall be more confirmed in our assertion.

Christianity has produced the most beneficial change in the circumstances of domestic life. It has greatly contributed towards the abolition of slavery, and towards the mitigation of the rigours of servitude. We meet with no laws in Christian countries so inhuman as those practised at Rome; where masters were allowed to remove their sick or infirm slaves to an island in the Tiber, there to perish without any assistance. The rigours of slavery are eased and abolished; not by any particular precept of the Gospel, but by the gentle and humane spirit which breathes through the general tenour of the whole system. And though it may be objected, that a trade in slaves is at present carried on by people who presume to call themselves Christians, and sanctioned by the legislature of some Christian states; yet it must be remembered, that the spirit of the Christian code condemns the practice; and the true Christian will not engage in it.

Christianity is also gradually softening barbarous nations into humanity. The influence of selfishness has been checked and restrained. And even war, with all the pernicious improvements, by which mankind has sought to render it more terrible, has assumed much more the spirit of mildness and peace, than ever entered into it under the influence of Paganism.

These are a few of the excellencies of the Christian system. Its last distinction I shall mention, is that of its extending its benefits to those nations who have not received its doctrines and precepts. The virtues ascribed to Julian the apostate, are, no doubt, owing to his acquaintance with Christianity; and after the propagation of Christianity through the Roman empire, even while the purity of its doctrines was despised, it had a remarkable effect on the manners of those unconverted

verted Pagans, who, in their religious doctrines and worship, became less immoral and absurd.

Upon the whole, we must conclude, that Christianity is infinitely superior to every other religious system, both in point of its religious doctrines, and the effects it has produced upon society. It is an universal religion; formed to exert its happy influence in all ages, and among all nations; and has a tendency to dispel the shades of barbarity and ignorance; to promote the cultivation of the powers of the human understanding; and to encourage every virtuous refinement in manners.

As the Christian religion is destined to be of an universal nature, and to be disseminated into all parts of the world; so, in order for its more effectual propagation, its all-wise Founder has ordained that it shall be divided into different sects and parties; that the leaders of each being governed by a mutual emulation, might endeavour to propagate their respective opinions, and thereby form a grand junction for propagating a religion, the fundamentals of which would be ultimately the same.

The two principal sects into which the Christian religion is divided, are the Protestant and Romish churches.

The Romish church differs from the Protestant chiefly in the following particulars: 1. In believing every thing that was defined by the Council of Trent, concerning original sin and justification. 2. In believing transubstantiation, or the conversion of the material bread and wine, given at the sacrament, into the real body and blood of Jesus Christ. 3. In the belief of a purgatory; and that souls are kept prisoners there after their departure from the body; and that they receive help by the prayers of the faithful. 4. That the saints reign together with Christ, and are to be worshipped as mediators for man. 5. That the images of Christ, the Virgin Mary, and other saints, shall be retained, and due honour
and

and veneration be given unto them. 6. That the power of indulgences was left by Christ to the church. 7. That the holy church of Rome is the mother and mistress of all churches; and that the bishop of Rome, or pope, is the successor of St. Peter, the prince of the apostles, and vicar of Jesus Christ on earth; and that he is infallible and invincible.

These are the chief tenets which distinguish the church of Rome from the Protestant church. The implicit obedience which the followers of this church pay to their leaders, has been a source of a very black corruption and error; of which their numerous persecutions of the Protestants are an ample proof. But, on the other hand, it must be allowed, that there is no religion so zealous of propagating its doctrines. Their missionaries have been sent to all parts of the earth, some of whom, by their perseverance and abstemiousness, were as great an honour, as others by their profligacy were a disgrace, to the cause in which they were concerned.

The church of Rome is now divided into two sects. The already described, which prevails over most parts of Italy, Spain, and France, and several other parts of the continent of Europe; and the Greek church, which differs from the former in not allowing the pope's supremacy, not worshipping idols, though they have many in their churches, and in not enjoining their priests to celibacy.

The Protestant religion is divided into numerous sects and parties. The two principal of which are the Lutherans and the Calvinists.

The Lutherans maintain, that man is a free agent, perfectly capable of performing good or evil: that according to his actions he shall be rewarded or punished hereafter: that he is left at perfect liberty to choose the good or evil: and that God has no predilection for any particular persons. That the sacrament of the Lord's supper is nothing but a mere ordinance.

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The Calvinists, on the contrary, assert, that man is not a free agent, that he has no power to perform any good action without the Spirit of God assisting him : that God, according to his fore-knowledge, has elected a certain number of individuals to be saved : that he is the former of every good thought ; and saves the elect, not from any goodness in themselves, but merely from his own unmerited mercy ; consequently, that Christ did not die for all the world : that the sacrament is a spiritual rite ; that the bread and wine is consubstantiated (not transubstantiated) in a spiritual manner into the body and blood of Christ.

Besides these two divisions of the Protestant religion, one or both of which prevail in most Protestant countries, on the continent of Europe, there are a great number of inferior sects in England, America, Holland, Germany, and other parts, and some of them very numerous ; as the Quakers, Baptists, Dissenters, Methodists, &c. which are too well known to need any description. Suffice it to say, each of them differs from the established church, on account of some trifling errors, which they imagine to have detected in the national church.

The next division of religion that deserves our notice is that of Mahomet, which still makes such a conspicuous figure in the world, extends over a large tract of country, and is professed by very powerful nations. Like the Jewish religion, it is not merely a system of religious doctrines, and moral precepts ; but it forms both the civil legislature, and religious systems of the nations by which it is professed. It also appears to be rather calculated for one particular period, in the progress of mankind from barbarity to refinement, than for all ages, and all parts of the world.

In viewing this system of religion, there are many parts of it which seem copied from the Christian, the Jewish, and the Pagan religions. It is difficult to tell which is the greater, the purity of some parts of this doctrine, or the absurdity of other parts.

The greatest absurdities, or that which tends most effectually to promote impurity of manners, are the Prophet's ideas of heaven and hell. Paradise, or the place of future rewards, he makes to abound with rivers, trees, fruits, and shady groves; wine, without its intoxicating quality, is to be there served out to believers, who, as they enjoy perpetual youth, their powers of enjoyment are to be enlarged and invigorated, according to the delights they are to enjoy. Mahomet celebrates the pearls and diamonds, robes of silk, palaces of marble, dishes of gold, numerous attendants, wines, and dainties, with the whole train of sensual luxury, reserved for the faithful in these regions. Seventy-two black-eyed damsels of resplendent beauty, blooming youth, virgin purity, and exquisite sensibility, will be created for the use of the meanest believer. A moment of pleasure will be prolonged to one thousand years; and the faculties will be increased a hundred fold, to render him worthy of his felicity. There are also certain more refined enjoyments; as, believers are to see the face of God morning and evening—a pleasure which is to exceed all the other pleasures of Paradise.

In hell, the place of future punishments, the wicked are to drink nothing but boiling, stinking water; eat nothing but briars and thorns, and the fruit of a tree that grows in the bottom of hell, whose branches resemble the heads of devils, and whose fruit shall be in their bellies like burning pitch as they are, to breathe nothing but hot winds, and dwell for ever in continual burning, fire, and smoke.

Thus Mahometism appears to be a strange mixture of absurdities, with a few truths and valuable precepts inconspicuously intermixed. A great part of it is incompatible with virtue, and the progress of knowledge and refinement. It substitutes trifling superstitious ceremonies in the room of genuine piety and virtue; and presents such a prospect of futurity, as renders purity of heart no necessary qualification for seeing God.

However, Mahometism forms in some measure a regular system

system of religion, as it has borrowed many of its precepts and doctrines from both Judaism and Christianity, which are, however, greatly degraded and corrupted. It has, nevertheless, considerably contributed towards the support of civil government in those countries in which it is established.

It is divided into a numerous party of sects, which, however, differ so little from each other, as scarcely to deserve mention in this place.

Paganism next deserves our notice. It was the most prevailing religion of antiquity, and may be principally divided into two parts:—1. The Pagan religion of the ancient barbarous nations; and, 2. The Polytheism of the more civilized Greeks and Romans.

The Paganism of the ancient nations presents us with a most shocking picture of ignorance, superstition, and absurdity. We there behold the most absurd doctrines concerning a future state. Various nations have imagined, that the scenes and objects of the world of spirits are only a shadowy representation of the things of the present world. According to them, not only the souls of men inhabit those regions, but all the inferior animals and vegetables, and even inanimate bodies that are killed or destroyed here, are supposed to pass into that visionary world, and existing there in unsubstantial forms, execute the same functions, or serve the same purposes, as on earth. By this belief they were stimulated, on the death of a king or other great personage, to provide for his accommodation in the world of spirits, by burying with his corpse, meat and drink for his subsistence, slaves for his attendance, and wives for his enjoyment. His faithful subjects vied with each other in their offerings upon this occasion; one brought a servant, another a wife, a third a son or daughter, to accompany their monarch in his future state. Similar practices, on the same occasion, prevailed in New Spain, in the island of Java, in the kingdom of Benen, and among the inhabitants of Hindostan. A like belief also prevailed among the Japanese. They not only bribed their

priests to solicit for them a place in the blissful mansions of futurity; but looking upon the present life with disgust and contempt, when set in competition with the joys of futurity, they used to dash themselves from precipices, or cut their throats, in order to get to Paradise as soon as possible. Various other superstitions subsisting among rude nations, might here be adduced, as instances of the perversion of the religious principles of the human heart, which render them injurious to virtue and happiness. Innumerable are the ways of torture which have been invented and practised on themselves by men ignorantly striving to obtain the favour of Heaven. These are sufficient proofs of religious sentiments having been so ill directed by the influence of imagination, and unenlightened erring nature, aided by the corrupt designs of artful priests.

The Polytheism of the Greeks and Romans, though more favourable to virtue and civilization, than the Pagan notions of antiquity, is yet a very imperfect, not to say a pernicious, code of religion: the vicious characters of their deities, the absurd notions they entertained concerning the government of the universe, and a future retribution, the absurdities of their religious rites and ceremonies, the frivolous practices with which they were intermixed, must altogether have a great tendency to pervert both the reasoning and moral principles of the human mind: however, it cannot be denied, that this system was friendly to the encouragement of arts; particularly to such as depend on the vigorous exertion of a fine imagination, as music, poetry, sculpture, architecture, and painting: all these arts appear to have been considerably indebted for that perfection to which they attained, to the splendid and fanciful system of mythology which was received by those people, particularly by the Greeks.

The effect of this religion, to reform the lives of its votaries, was very imperfect. Sacrifices and prayers, temples and festivals, not purity of heart and integrity of life, were the means prescribed for obtaining the favour of their deities.

There

There were also other means of gaining admission into the Elyfian fields, or the seat of the councils of the Gods; but none of these means appear to have been those commanded by the Christian religion. And whatever might be the effects of the religion of Greece and Rome in general, upon the civil and political establishments, and on the manners of the people, yet it must be confessed to have been but ill adapted to impress the heart with such principles as might in all circumstances direct to a firm, uniform tenour of righteous conduct.

From this view of religion, it appears, that though some particular forms, such as those of the Christian, have had a greater influence in reforming the manners of their followers; yet as they all have often contributed to form the mind to virtue, it must be acknowledged, that they have always, and under all their forms, been infinitely more beneficial than hurtful to mankind.

When we view the different systems in a comparative light, with respect to their influence on the welfare of society, no one will hesitate to prefer the Polytheism of the Greeks and Romans to the ruder ideas of the more ancient Pagans; and Mahometism to the Polytheism of the Greeks and Romans: Judaism is, however, greatly preferable to Mahometism; and Christianity to all of them.

SECT. VI.

OF EUROPE.

EUROPE, though the least quarter of the world, is by far the most eminent in modern history; and is at present the most distinguished part of the globe for the literature, arts, and sciences, to which it has given birth and encouragement, and for the learned men it has produced. It is also the most civilized quarter of the globe. Here are no public markets for buying and selling the human species, as are found in Asia and Africa. The Christian religion also prevails here almost universally. Its languages are as mixed as its inhabitants, but all derived from the six following: viz. the Celtic, Slavonian, Teutonic, Greek, Latin, and Gothic. It extends about 3000 miles in length, and 500 in breadth; and is divided into several kingdoms and states, as seen in the Table, page 133.

The **BARBARIC ISLES**, lying on the western part of Europe, consist of Great Britain (which comprises England, Wales, and Scotland), Ireland, and the Isles of Man, Jersey, Guernsey, Alderney, Sark, and Wight. England lies between 50 and 56 degrees north latitude, and between 2 degrees east, and 6 degrees 20 minutes west longitude; and is divided into forty counties. Its constitution is that of a limited monarchy, consisting of king, lords, and commons, with certain prerogatives and privileges annexed to each.

The legislative authority, or power of making laws and raising



raising money, is vested in these three branches of the government; and each branch has a negative voice.

The crown is made hereditary in the Hanover line, by several acts of Parliament, provided they do not profess Popery, marry Papists, or subvert the constitution.

The peers are created by the crown; but their honours are hereditary, and cannot be taken from them any more than their lives or estates, unless forfeited by the commission of high treason; and they can only be tried by the House of Peers, being subject to no other jurisdiction. This House is the last resort in all civil cases, and the highest court in the kingdom.

Any bill for making a new law, or altering an old law, may be brought in first in the House of Peers; but a bill relating to the revenues, or public taxes, must be brought into the House of Commons first; and it cannot be altered by the Peers, though it may be rejected.

The House of Peers can apprehend and commit any man for a reflection on their judicature.

The Commons are composed of 658 members: viz. 80 knights, every county in England sending two, and elected by the freeholders; 50 citizens, two being sent from each of the 25 cities in England (London sending four, and Ely none); 334 burgesses, from 167 boroughs, sending two each; five burgesses, from the boroughs of Abingdon, Banbury, Bewdley, Higham Ferrars, and Monmouth; four representatives, from the two universities; 16 barons, from the five Cinque Ports, Hastings, Dover, Sandwich, Romney, and Hythe, and their three dependants, Rye, Winchelsea, and Seaford; 12 knights, from the 12 counties of Wales; 12 burgesses, from the 12 boroughs in Wales (Pembroke sending two, and Merioneth none); 30 knights, from the shires of Scotland; 15 burgesses, from the Scotch boroughs; and 100 members from Ireland.

Wales is situated on the west and north-west of England, to which it joins; and is divided into 12 counties: it is a principality;

principality; and always considered as the right of the King's eldest son, who is therefore titled Prince of Wales. It was peopled in the year 410, by the ancient inhabitants of England, who fled thither from the persecution of the Picts and Scots.

IRELAND is situated between six and ten degrees of west longitude, and between 51 and 55 degrees of north latitude. Bounded on the east by St. George's Channel, or the Irish Sea, which divides it from Great Britain. It is divided principally into four provinces: viz. Ulster, on the north; Leinster, on the east; Munster, on the south; and Connaught, on the west.

The climate of these islands is in general mild for the latitude, but very changeable, the weather never continuing a month the same, owing to the exhalations from the surrounding sea, which render the air humid. But the soil is in general fruitful, and has been of late years greatly improved.

These islands have several very good mines of tin, copper, iron, and lead; gold has also been found in Scotland, in solid pieces, in the brooks, after a great torrent.

The chief manufacture of England is woollen cloth, which is accounted the staple trade of the kingdom; as linen cloth is that of Ireland.

DENMARK, including Norway, is the most northern kingdom of Europe, and includes Denmark Proper, the territories in Germany, Norway, part of Lapland, and several islands in the Baltic Sea, and in the German Ocean; and extends from 53 degrees of north latitude, to the farthest habitable part of the Arctic Circle. Denmark Proper is bounded on the north by the Cattegat or Schaggerac; on the south by Germany; on the west by the German Ocean; and on the east by the Sound.

The established religion is Lutherism. The king is absolute, though in general mild in his government. It is divided

vided into two parts, called North Jutland, and South Jutland.

The air of this country is sharp, but the exhalations from the sea abate its severity. The summers are very short, and hot, but the soil is in general fruitful, for a northern latitude, except on the tops of mountains. The manufactures of this country are chiefly hardware; and their artists and mechanics, in every branch, are generally skilful.

NORWAY is bounded on the south by the Cattegat, on the west and north by the Northern Ocean, and on the east by the mountains which separate it from Sweden; and is divided into the north, south, and middle divisions. The air of Norway is generally healthy and dry in the inland parts of the country, but on the sea-coast it is moist. In winter it is excessively cold, the whole country being covered with snow; it is also very hot in the summer. Their trade consists of copper, timber, iron, marble, mill-stones, fish, fowls, tallow, tar, oil, alum, vitriol, &c. Their language is the same as that used in Denmark; and their religion is that of Lutherism.

ICELAND is situated in the Northern or Atlantic Ocean; being 726 miles in length from east to west, and 300 in breadth: extending from sixty-three to sixty-eight degrees of north latitude; and from fourteen to twenty-nine degrees of west longitude. It has a milder climate than any other country in the same latitude. It is a very mountainous country, but well watered, with several large rivers. In this country there are some large springs of boiling hot water, the principal of which is Geyser, near Skalholt. The water issues from this spring several times a day, with a violent noise, like that of a great torrent, sometimes rising to the height of 60 fathoms, and seldom less than 90 feet.

There are also several burning mountains in this country, of which the most remarkable are, Hecla, Kotlegau, and Oraife, the eruptions of which have sometimes done considerable damage. The inhabitants live chiefly by fishing, and

breeding cattle, attending very little to agriculture. Their commerce is monopolized by a Danish company of merchants, and consists chiefly of salt-meat, butter, tallow, oil, wool, skins, furs, and feathers. The revenue arising from this country to the king of Denmark amounts to 30,000 crowns *per annum*.

GREENLAND is the most northern boundary of the king of Denmark's dominions; and is the farthest part of the globe northward which has been discovered. East Greenland extends beyond 76 degrees of north latitude; and between 10 and 11 degrees of east longitude. There are no inhabitants here, except a few convicts transported from Russia, and who gain their liberty by procuring skins, furs, tusks of morse, &c. for the sovereign of Russia.

West Greenland extends beyond 60 degrees of north latitude; and between 5 and 50 degrees of west longitude. There are a few natives who inhabit this country, and many of whom have lately been converted to Christianity, by the Danish and Moravian missionaries.

SWEDEN extends from 55 degrees 20 minutes to 69 degrees 30 minutes north latitude; and from the 12th to the 30th degree of east longitude. It is bounded on the south by the Baltic, the Sound, and the Cattegat Sea; on the north by Danish Lapland; by Russia on the east; and by the mountains of Norway on the west: and principally divided into seven provinces: viz.—1. Sweden, properly so called, lying between Norway and the Gulf of Bothnia; 2. Gothnia, or Gothland; 3. Livonia, on the south of Finland Gulf; 4. Ingria, on the north-east of Livonia; 5. Finland, on the east side of the Gulf of Bothnia; 6. Swedish Lapland, in the northern parts; 7. The islands of Gothland, Öland, Åland, Hogland, and Rugen.

Note. The provinces of Livonia and Ingria, with Kexholm and Karelia in Finland, and the islands of Dagö and Ösel, are under the government of Russia.

The

The natural soil of this country is in general barren, but has been greatly improved of late years, by the industry of the inhabitants, assisted by the affluent part of the nation, so that they have now fruitful harvests. Their manufactures are chiefly in silver, copper, and iron; and vast quantities of these metals, with timber, tar, hemp, flax, hides, furs, fish, &c. constitute the chief articles of their trade.

Their religion is the same as that of Denmark and Norway. The language is also partly the same, being only a dialect of the Teutonic language. The government of this country is a limited monarchy.

RUSSIA, the largest empire upon the globe, and greater than all the rest of Europe besides, extends in length from the Baltic Sea on the west, to within a few miles of America on the east, upwards of 6000 miles; and above 8400 miles in breadth from north to south. It is bounded on the west by Sweden and the Baltic; on the east by China, and the Pacific Ocean, which separates Asia from America; on the north by the Frozen Ocean; and on the south by Prussia, Poland, Turkey, Persia, and Tartary. Its measured length from the isle of Dagho to its eastern bounds is near 170 degrees. Thus it contains several different climates. In the southern parts, the longest day is scarcely sixteen hours, while in the northern parts it is nearly three months. In the southern provinces it is very hot; and extremely cold in the northern parts. The soil beyond the 60th degree of north latitude scarcely ever produces corn to any perfection; and beyond the 70th degree scarcely any species of fruit is found; but in the middle provinces the soil is fruitful, and produces good pasture for cattle, and excellent grain. The southern provinces being hot, have all the fertility of a warm country, where there is a sufficient depth of soil. There is a great variety of inhabitants in this extensive country: viz.—the Tartars, Kamtschatdales, Samocides, Laplanders, &c. There is considerable variety in the manners of the natives

of these different countries. In some of the northern parts they live in caverns, not five feet in height; in other parts they lead a wandering life: the natives of some parts practise agriculture; but in others live on the spontaneous productions of the soil.

The religion of some parts is next to Paganism, the natives idolizing inanimate objects, as a sheep's skin; but in some parts they make no public profession of religion. The established religion of Russia is the Greek Church.

The European part of Russia, called Muscovy, is divided into the following provinces: viz.—

In the northern division: Lapland, Samoieda, Bellamornskoy, Mescen, Dwina, Syrianea, Permia, Rubeninski, Belafeda.—In the middle division: Pereslaf, Belozero, Wologda, Jerslaf, Tweer, Moscow, Belgorod.—In the eastern division: Bulgar, Kansan, Lit, Novgorod, Don Cossacks.—In the western division: Great Novgorod, Rus, Finland, Kexholm, Karelia, Ingria.—In the southern division: Livonia, Smolensko, Zernigof, Seefsk, Ukrain. Their articles of commerce and manufacture are the same as those of Sweden and Denmark; they have, moreover, silk, cotton, teas, gold, &c. which they bring from China and India, in caravans, by way of the Caspian Sea.

The language is derived from the Slavonian, to which are added many words from the Greek; their alphabet consists of forty-two characters, which are principally Greek. The people of high rank generally speak French and High Dutch, but their priests speak the modern Greek.

POLAND, before its late dismemberment, was bounded on the north by Livonia, Muscovy, and the Baltic Sea; on the east, by Muscovy; on the south, by Hungary, Turkey, and Little Tartary; and on the west, by Germany; extending from 47 degrees 40 minutes, to 56 degrees 30 minutes, north latitude; and from 16 to 34 degrees east longitude. It was divided into the provinces of Great and Little Poland,

Polish

Polish Prussia, Samogitia, Courland, Lithuania, Masovia, Podlachia, Polesia, Red Russia, Podolia, and Volhinia. The soil of Poland is in general very fruitful, and the air mostly temperate, except in the northern parts, where it is very cold. Their pasture land is so fruitful, that the height of the grass often conceals the cattle from the view of a passenger at two hundred yards distance. Great numbers of beasts, as horses, asses, oxen, buffaloes, bears, foxes, wolves, &c. run wild in the forests. There are several mines in the country, of gold, silver, copper, lead, iron, &c.

The greatest curiosities in this country are the salt mines, of which that of Wielitska is the largest in the world, and has been wrought above 600 years. It is 743 feet below the surface of the ground, and 1115 feet in breadth, and 6691 in length; and appears like a spacious plain, with vaulted roofs, supported by columns of salt, which have been left standing. Many public lights are placed in this mine, for general use, which reflect a most luminous appearance from every part of the mine. Here are also great numbers of huts for the accommodation of the miners, and their families, many of whom are born and spend their lives in this place, without ever making their appearance on the surface of the earth. Through the midst of the mine is the great road, which passes to the mouth of the mine: this road is generally crowded with carriages full of salt. A stream of fresh water also runs through the mine.

The wild men who have been seen of late years in the woods of Poland form another curiosity.

The Poles at present seem almost annihilated, and their country divided among the Austrians, Prussians, and Russians.

PRUSSIA is bounded on the north by Samogitia; on the south by Poland Proper and Masovia; on the east by part of Lithuania; and on the west by Polish Prussia and the Baltic: but if we take it in its full extent, this kingdom
consists

consists of various territories, different parts of Germany, Poland, Swisserland, and other northern countries.

The principal divisions of this kingdom are, Regal Prussia, situated in Poland ; and Upper Saxony, containing Brandenburg, Prussian Pomerania, and Swedish Pomerania, Magdeburg, and Halberstadt in Lower Saxony ; Glatz in Bohemia ; Minden, Ravensburg, Lingen, Cleves, Meures, and Mark, in the dutchy of Westphalia ; East Friesland, Lippe, Gulick, and Tacklenburg, in the circle of Westphalia ; the margraviate of Anspach, in the circle of Franconia ; Gelder in the Netherlands ; Neufchatel in Swisserland ; and part of Silesia.

Prussia carries on a considerable trade, and the balance in favour of Prussia is reckoned greater than that of any other European state ; great quantities of glass, iron works, cloth, camlet, silk, linen, paper, powder, copper, and brass, are annually exported.

Amber is here found in great quantities, from which the crown of Prussia receives 26,000 dollars annually ; also great sums from the bitumen, of which several kinds are found in the Baltic Sea.

The religions of Prussia are those of the Lutherans and Calvinists ; but all religions are tolerated. His Prussian Majesty is absolute through all his dominions. The Prussian army, even in times of peace, consists of 180,000 men, which are reckoned the best disciplined troops in the world ; but in time of war it has been augmented to between 3 and 400,000 men.

GERMANY is bounded on the north by the German Ocean, Denmark, and the Baltic ; on the east, by Poland, Hungary, and Bohemia ; on the south, by Swisserland and the Alps ; and on the west, by France and the Low Countries. It extends from 45 degrees 4 minutes, to 54 degrees 40 minutes, north latitude ; and from 6 degrees to 19 degrees 45 minutes east longitude. Germany is a great empire, having several appendant sovereignties under it, under different modifications

of government, some of whom scarcely exceed an English manor in extent. It is divided into nine circles, three of which lie in the north, three in the middle, and three in the south: viz. — Upper Saxony, Lower Saxony, Westphalia; Upper Rhine, Lower Rhine, Franconia; Austria, Bavaria, and Suabia. These circles are subdivided into principalities, dutchies, marquisates, electorates, palatinates, counties, baronies, abbies, bishoprics, &c.

The climate of Germany is in general healthy and agreeable, except in the most northern and southern parts. And the soil is particularly fruitful; for though only a small proportion of the country is cultivated, yet provisions are in general cheaper than in most other countries of Europe. They have also a greater quantity of domestic animals and wild beasts, as boars, hares, rabbits, foxes, badgers, goats, &c. &c. than other European countries. They also abound in most of the species of tame fowl, as well as wild fowl.

There are several mines in Germany of silver, copper, iron, lead, quicksilver, sulphur, nitre, &c. and coal pits are found in every part of the empire.

Germany is also in great esteem in all other European countries for its mineral springs and baths, the most remarkable of which are those of Aix-la-Chapelle, Spa, Pyrmont, Embs, Wisbaden, Schwalbach, Wildungen, and Brakel, which last is enclosed, as the waters are so strong as to be capable of intoxication.

The manufactures of Germany consist of velvets, silks, cotton and woollen stuffs, linen, fustian, ribands, lace, tapestry, &c. They also make beautiful porcelain, and lacquered ware, and every kind of hard ware.

The Germans have a considerable commerce, owing to their central situation, and the balance of trade is greatly in their favour. The established religion is either Romish, Lutheran, or Calvinist, being different in the different parts of the empire; but most other religions are tolerated at present.

The

The German language is a dialect of the Teutonic, and is called the High Dutch, being the mother tongue of the whole empire; but every different province has a different dialect.

The government of Germany is in the hands of about 300 civil and ecclesiastical princes, each of whom is absolute in the government of his own state; and the whole of them form a great confederacy, governed by political laws, at the head of which is the Emperor, whose power in the collective body is only executive. The Emperor is elected; but the empire for some centuries has belonged to the House of Austria, as being the most powerful of the German princes. The nine electors of the empire have each a particular office in the Imperial court: they have the sole election of the emperor; and are as follow:—1. The Archbishop of Mentz, who is high chancellor of the empire, when in Germany.—2. The Archbishop of Treves, who is high chancellor of France and Arclat (a dignity merely nominal).—3. The Archbishop of Cologne.—4. The King of Bohemia, who is cup-bearer.—5. The Elector of Bavaria, who is grand sewer.—6. The Elector of Saxony, who is great marshal of the empire.—7. The Elector of Brandenburg (now King of Prussia), who is great chamberlain.—8. The Elector Palatine.—9. The Elector of Hanover (King of Great Britain), who claims the post of arch-treasurer.

The revenue of the Emperor, as such, is about 5 or 6000 pounds sterling, per annum, arising from the siefs in the Black Forest. The Austrian revenues are immense, amounting to 12,000,000 pounds sterling.

The military force of this country amounts to near half a million of men; the secular princes bringing upwards of 379,000, the ecclesiastical 7450, and the Emperor, as the head of the House of Austria, 90,000.

Hungary, Bohemia, and the provinces of Transylvania, Slavonia, Croatia, and Morlachia, may be considered as part

of the German empire, having been brought under the dominion of the house of Austria. The established religion of these countries is the church of Rome.—Bohemia lies between 48 and 52 degrees north latitude, and between 12 and 19 degrees east longitude. Bounded on the north by Saxony and Brandenburg; on the east by Poland and Hungary; on the south by Austria and Bavaria; and on the west by Bavaria.—Transylvania lies between 45 and 48 degrees north latitude, and between 22 and 25 degrees east longitude. Bounded on the north by Hungary and Poland; on the south by Wallachia; on the east by Moldavia; and on the west by Hungary.—Sclavonia lies between 45 and 47 degrees north latitude, and between 16 and 22 degrees east longitude. Bounded by the river Drave on the north; by Austria on the south; by the Danube on the east; and by the Save on the west.—Croatia lies between 44 and 47 degrees north latitude, and between 15 and 17 degrees east longitude. Bounded on the north by the Save; on the south by Morlachia; on the east by Bosnia; and on the west by Carniola.—Morlachia lies between 44 and 46 degrees north latitude, and between 16 and 17 degrees east longitude. Bounded on the north by Carniola; on the south by Dalmatia; on the east by Bosnia; and on the west by the Gulf of Venice.

SWISSERLAND is bounded on the north by Suabia; on the east by the lake of Constance, Tirol, and Trent; on the south by Italy; and on the west by France; extending from 45 to 48 degrees north latitude, and from 6 to 11 degrees east longitude: and is divided into thirteen cantons: viz. Bern, Fribourg, Basil, Lucern, Soloturn, in the west division; Schaffhausen, Zurich, Appenzel, in the east division; and Zug, Schwyz, Glaris, Uri, and Unterwald, in the middle division. Seven of these cantons profess the Romish religion: Fribourg, Lucern, Soloturn, Zug, Schwyz, Uri, and Unterwald; the other six are Protestants. The climate of this country is very various, on account of the inequality of the surface of the ground, being situated among the Alps; the highest

mountains in Europe: so that it is common for the inhabitants to be reaping on one side of the mountain, while those on the other side of the same mountain are sowing. The frosts in winter are very severe; and in the summer the heat is, in some parts, intense.

The commerce of Switzerland consists of their cattle, horses, cheese, butter, hides, skins, and the productions of their own manufacture, the principal of which are silks, brocades, linen, lace, woollens, stuffs, hats, paper, leather, porcelain, toys, watches, and clocks.

Each canton forms a separate republic; but when any controversy arises, it is referred to the general diet, which sits at Baden, where each canton has a vote, and sends two deputies.

The NETHERLANDS lie between 50 and 54 degrees north latitude, and between 2 and 7 degrees east longitude. They are bounded on the north by the German ocean; on the east by Germany; on the west by the British channel; and on the south by France and Lorrain. The Netherlands are divided into seventeen provinces; the seven northerly ones are called Holland, or the United Provinces, and the other ten are called Flanders, or the Austrian and French Netherlands.

The provinces of Holland, are Holland, Zealand, Friesland, Groningen, Overijssel, Gelderland and Zutphen, and Utrecht.

The air of these provinces is very moist and foggy; their harbours are generally frozen up four months in the year; and the soil is very unfavourable for vegetation; but the industry of the inhabitants has greatly improved it, by making canals and ditches to drain the land.

Their commerce is carried on to such an extent, that there is hardly a commodity of traffic on the face of the globe but may be bought here, and almost as cheap as in the places where it is produced.

The

The religion of this country is Calvinism; but all professions and societies are tolerated, of which there are great numbers.

The government of Holland is a democracy, and has continued for upwards of two hundred years; notwithstanding they had a prince under the title of a stadtholder, whose powers had very little of the regal nature.

The ten other provinces of the Netherlands, called Flanders, have been divided among the Austrians, French, and Dutch, but are now chiefly claimed by the French, and contain the ten following provinces: viz.—Brabant, Antwerp, Malines, Limburgh, Luxemburg, Namur, Hainault, Cambresis, Artois, and Flanders.

The soil in most of these provinces is extremely fruitful, and the air generally healthy, except in Brabant, and some parts of the sea-coasts.

The commerce of these provinces consists chiefly of their own manufacture, viz.—fine linens, cambricks, laces, and woollen manufactures.

FRANCE extends from 42 to 51 degrees north latitude; and from 5 degrees west, to 8 degrees east longitude. It is bounded on the north by the Netherlands and the English channel; on the east by Germany, Switzerland, and Italy; on the south by the Mediterranean sea and Pyrenean mountains; and on the west by the Bay of Biscay. France was formerly divided into 12 provinces; but at the late revolution it was divided into 84 departments, each department being divided into districts, and each district into cantons. The eighty-four departments are as follows:—1. Straits of Calais. 2. North. 3. Lower Seine. 4. Somme. 5. Aisne. 6. Ardennes. 7. Channel. 8. Calvados. 9. Eure. 10. Oise. 11. Marne. 12. Meuse. 13. Moselle. 14. Lower Rhine. 15. Finisterre. 16. North coast. 17. Isle and Vilaine. 18. Mayenne. 19. Orne. 20. Eure and Loire. 21. Seine and Oise. 22. Paris. 23. Seine and Marne. 24. Aube. 25.

Upper Marne. 26. Meurthe. 27. Vosges. 28. Upper Rhine. 29. Morbihan. 30. Lower Loire. 31. Mayenne and Loire. 32. Sarthe. 44. Loire and Cher. 34. Loirette. 35. Yonne. 36. Cote d'Or. 37. Upper Soanne. 38. Doubes. 39. Vendée. 40. Two Sevrés. 41. Vienne. 42. Indre and Loire. 43. Indre. 44. Cher. 45. Nièvre. 46. Soanne and Loire. 47. Jura. 48. Lower Charente. 49. Charente. 50. Upper Vienne. 51. Creuze. 52. Allier. 53. Rhone and Loire. 54. Ain. 55. Gironde. 56. Dordogne. 57. Corréze. 58. Puy de Dôme. 59. Upper Loire. 60. Mère. 61. Landes. 62. Lot and Garonne. 63. Lot. 64. Cantal. 65. Lozère. 66. Ardèche. 67. Drome. 68. Upper Alps. 69. Lower Pyrenees. 70. Gers. 71. Upper Garonne. 72. Tarne. 73. Aveyron. 74. Herault. 75. Gard. 76. Lower Alps. 77. Upper Pyrenees. 78. Arriège. 79. Aude. 80. East Pyrenees. 81. Mouths of Rhone. 82. Var. 83. Corfica. 84. Mount Blanc.

The climate of France is reckoned; upon the whole, to be more settled than that of any other country in Europe. In the north the winters are very cold; but in the interior parts the air is very temperate and healthy; and in the south it is so mild, that invalids retire thither from all the northern countries, to avoid the rigour of their own climates.

The commerce of France consists of wines, brandy, vinegar, drugs, oils, fruits, of which they have great variety, silks, cambricks, laces, paper, parchment, hardware, toys, &c. and their trade is very considerable and lucrative both to the East and West Indies; but particularly to the European countries.

The national religion was always Romish. And their monarchs were always limited till the three last sovereigns of France. The executive power is now vested in three consuls, the chief of whom is created consul for life, with the power of nominating a successor.

SPAIN lies between 36 and 44 degrees north latitude; and between 10 degrees west, and 3 degrees east longitude. It is bounded on the north by the Bay of Biscay and the Pyrenean mountains; on the south by Gibraltar straits; on the east by the Mediterranean sea; and on the west by Portugal and the Atlantic Ocean. It is divided into the following kingdoms or provinces: Galicia, Asturia, Biscay, Navarre, Arragon, Catalonia, Valencia, Murcia, Granada, Andalusia, Old Castile, New Castile, Leon, and Estremadura.

Spain enjoys a dry, clear, temperate air, except during the equinoctial rains; and in the southern provinces during the summer months, where it is very hot. The soil is as fruitful as the soil of any part of Europe; but the natives are very indolent. In many parts the choicest fruits grow spontaneously. They also have a great variety of aromatic herbs. Seville is celebrated for its oranges; and Murcia produces mulberry-trees in such abundance, that the silk exported from this part amounts to 200,000 pounds per annum.

The chief articles of commerce in Spain are gold and silver, which they derive from their settlements in South America. The principal manufactures are silk, wool, iron, copper, and hardware.

The national religion of Spain is the profession of the church of Rome. The Inquisition always reigned in this country, till, by a late edict, it was put under some restrictions.

The constitution of Spain is the most absolute monarchy in Europe. And the revenue from Old Spain only, amounts to upwards of 6,000,000 sterling: what the exact amount of the whole revenue is, is not accurately known.

The military force of Spain is never less than 70,000 men in time of peace; and in time of war the king has raised near 200,000.

PORTUGAL joins to Spain, and is bounded by it on the north and east; and on the south and west by the Atlantic Ocean.

Ocean. It extends from 30 degrees 30 minutes to 43 degrees north latitude, and from 7 to 10 degrees west longitude.

The climate of Portugal is more temperate than that of Spain, on account of its vicinity to the sea. Their commerce consists chiefly of wines, fruits, salt, linen, woollen, and some coarse silk. Their religion is that of the church of Rome; and the Inquisition has greater power here than in any other country. The constitution is, like that of Spain, absolute monarchy.

Italy, bounding Sicily. Lies between 37 and 47 degrees north latitude, and between 7 and 19 degrees east longitude. On the east, south, and west, it is washed by the Adriatic and Mediterranean seas; and on the north it is separated from the rest of Europe by the Alps. It contains the following countries: Piedmont, Montserrat, part of Milan, Sardinia Isle, Naples, Sicily, Milanese, Mantua, Tuscany, the Duke of Parma's territories, Genoese territories, Oneglia, the Duke of Modena's territories, Venetian territories, Pope's dominions, Corsica Isle, Malta Isle, and some other small Islands. All these countries are distinct from each other; having different forms of government, different trade, and separate interests.

Italy has a fine soil, and temperate, but warm climate; the soil however is greatly neglected, owing to the indolence of the inhabitants.

The religion, universally professed throughout Italy, is that of the church of Rome; but people of all other religions generally live unmolested in most parts of Italy. The commerce and manufactures are various, according to the different places; but wines, fruits, and oil, constitute the chief articles. The commodities to be met with in this extensive tract of country are almost innumerable, it being the seat of so many remains of antiquity, particularly of ancient Rome; hence, there are innumerable remains of the arts, the
works

works of ancient artists: the burning mountains also constitute one of their greatest natural curiosities. The Italian language is derived from the Latin; with an intermixture of words from the Goths, and other barbarous nations; but every separate state has a different dialect.

To describe the form of government of each state, would be to enter into too minute a detail, as they are different in every state.

TURKEY extends into both Europe and Asia.

European Turkey extends from 17 to 40 degrees east longitude, and between 37 and 49 degrees north latitude. It is bounded on the north by Russia, Poland, and Sclavonia; on the east by the Black Sea, the Hellespont, and the Archipelago; on the south by the Mediterranean; and on the west by the Mediterranean, and Venetian and Austrian territories.

Turkey in Europe contains some of the most genial climates in the world; and is divided into the following provinces: Crim and Little Tartary, Budzaic Tartary, Bessarabia, Moldavia, Wallachia, Bulgaria, Servia, Bosnia, Romania, Macedonia, Janna, Livadia, Epirus, Albania, Dalmatia, Ragusa, Corinth, Argos, Sparta, Olynthia, Arcadia, Elis.

The soil of Turkey is extremely fruitful, where the least industry has been employed: and all the fruits common to all the warm climates are produced here in great perfection; and many valuable drugs are natives of this country.

The commerce and manufactures of Turkey are chiefly silks, drugs, dying stuffs, in their natural state; with cottons, carpets, leather, velvets, soap, &c.; but though the Turks are situated in the most advantageous part of both Europe and Asia for traffic, yet they shamefully neglect it.

The religion which the Turks universally profess, is Mahometism; but they are divided into as many sects as the professors of Christianity. The high priest, or Mufti, is an officer

officer is then brought, that whenever he comes into court, the Grand Seignior rises from his seat and meets him. Most other nations are governed here by paying an annual tax.

The government of Turkey is that of an absolute monarchy, and in this empire there is no hereditary succession by law to any property; yet the rights of individuals are rendered secure by being annexed to the church, by which means even Jews and Christians may secure their property in lands to the latest posterity. The revenue of Turkey amounts to upwards of twenty-five millions per annum, but does not produce four millions to the emperor's treasury; the rest being expended in collecting, &c. The forces of the Turkish empire are of two parts: the one has certain lands for their maintenance, and the other is paid out of the treasury. The former amount to 200,000 troopers; the latter, called the *hulac-guard*, are about 12,000; and the janizaries, or foot-guard, 25,000; besides 200,000 foot soldiers in different parts of the empire.

SECT. VII.

OF ASIA.

ASIA forms the most remarkable quarter of the globe in ancient history. It was here that the first man was created—here the patriarch Noah was preserved during the flood—and from this quarter the world was repopled a second time. In Asia lived all the patriarchs recorded in Scripture—
and

and this was the scene of all the transactions recorded in holy writ—and, finally, it was here Jesus Christ appeared, and wrought the salvation of mankind—and from hence the Christian religion was propagated.

This quarter of the globe enjoys the most serene air and fruitful soil of all the quarters, and produces the most delicious fruits, odoriferous shrubs, spices, and valuable drugs, gums, &c.

Idolatry and Mahometism are almost universal in this quarter of the globe, except in a few European settlements. The languages in use in this quarter are chiefly the Arabic, Persian, Malayan, Chinese, Japanese, Tartarian, Russian, and Turkish.

Asia is bounded on the west by the Red Sea, the Mediterranean, the Archipelago, the Black Sea, and Europe; on the north by the Frozen Ocean; on the east and south by the Pacific and Indian Oceans. It is situated between the equator and the frigid zone, and between 25 and 180 degrees of east longitude; it is about 4800 miles in length, and 4300 in breadth, and contains the following countries.

TURKEY in Asia, being the other part of the Turkish empire, is about 1000 miles in length from east to west, and 800 in breadth from the northern parts to the deserts of Arabia. It is bounded on the north by the Black Sea and Circassia; on the east by Persia; on the south by Arabia and the Levant sea; and on the west by the Archipelago and the Hellespont.

This part of Turkey was the principal scene of all the transactions recorded in ancient writ, sacred and profane.

TARTARY is an extensive country taken in its full extent, and extends from Muscovy on the west, to the Pacific Ocean on the east; and from the nations of China, India, Persia, and Turkey, on the south, to the impenetrable regions of the north. It extends from the thirtieth degree of north latitude to the frozen regions of the north pole; and from 50 to 190 degrees east longitude; and contains Russian, Chinese, Mo-

gulean, and Independent Tartary, which are its four grand divisions, 4000 miles in length, and 2400 in breadth.

Through such an extensive tract of country the soil and climate must necessarily partake of a great variety.

Their manners, language, &c. must also be as various.

CHINA lies on the eastern borders of the continent of Asia, and is divided from Chinese Tartary on the north, by a prodigious wall, and, in some places, by inaccessible mountains; on the east it is bounded by the Yellow Sea and Pacific Ocean, which separates it from America; on the south by the Chinese sea, and the kingdom of Tonquin; and on the west by Tibet. It extends from 21 to 44 degrees north latitude, and from 94 to 133 degrees east longitude.

In such an extensive country there must no doubt be a variety of climates. The southern parts are very hot, and have violent rains, while the northern parts are very cold, and their rivers frozen for some months during the winter; but the middle parts are temperate and pleasant. The soil also partakes of a great variety, though there is no part of this extensive country but is fruitful, either from nature or art; for such is the industry of the Chinese, that they suffer very little, if any land, to lie uncultivated.

The Chinese have a considerable trade with every European nation, and with North America, exporting silks, cotton, gold and silver stuffs, painted gauzes, teas, china-ware, paper, and Indian ink, for which they receive ready money; despising the manufactures of every other country but their own.

There are a great number of natural and artificial curiosities in China. Among the latter are reckoned the famous wall which divides China from Tartary, extending over mountains and valleys, of 1500 miles in length, and from 20 to 25 feet in height, and broad enough for six horsemen to travel abreast. It has stood near 1800 years, and is now almost entire. 2. Their canals are works of great magnitude,
infinitely

infinitely exceeding those in Europe. 3. The bridge over the river Saffrany, which consists of a single arch, whose span is 400 cubits, and its height 500. 4. The Cien tao, or road of pillars, which is a road broad enough for four horses to travel abreast, and near four miles in length, defended by an iron railing; and unites the summits of several mountains, in order to avoid the winding of the roads. It rests upon strong stone pillars for the most part. 5. The bridge of chains, which is a bridge built upon a number of strong iron chains, and hangs over a very deep valley, in the neighbourhood of King-Tung. 6. The triumphal arches of China, of which there are above 1100; 200 of them are very magnificent; they were erected in memory of their great princes, legislators, &c. 7. The tower of Nan-King, called the Porcelain Tower, being wholly covered with the most beautiful china; upwards of three hundred feet in height, nine stories high; each story decreasing gradually to the top. The whole forms the most correct and grand piece of architecture to be met with in the East.

Among the natural curiosities may be reckoned their waterfalls and volcanoes.

Their religion is that of Paganism; the deities are men that have been eminent in arts and sciences. They also worship inanimate beings, as mountains, woods, and rivers; but they acknowledge but one Supreme Being.

INDIA, or HINDOSTAN, is an extensive country taken in its full extent. Bounded on the north by Tibet and Usbeck Tartary; on the south by the Indian Ocean; on the east by China and the Pacific; and on the west by Persia and the Indian Ocean. It extends from 1 degree to 40 degrees north latitude, and from 66 to 109 degrees east longitude: and is principally divided into three parts:—1. The peninsula of India beyond the Ganges, on the east; 2. the main land, or empire of the Great Mogul, on the north; 3. the peninsula within the Ganges, or on this side of it, on the west.

A great part of the sea-coast of India belongs to the English

East India Company, where there are many large and rich settlements, from which we receive great quantities of East India commodities.

As the country extends through so many degrees of latitude, there is a great difference in the climates of the different parts. In the northern parts the air is very dry and healthy; but in the southern parts near the sea, in low lands, the air is very hot and moist: they divide the year into the dry and wet seasons.

The soil, in general, throughout the whole country, is very fruitful, producing all the variety of plants, drugs, and fruits, to be met with in the other tropical climates. There are also mines of gold, diamonds, rubies, topazes, and other precious stones.

In the European settlements the religion is Christianity; but in the northern and inland parts they are either Mahometans or Pagans: and divided into several kingdoms, each of which is governed by one or more absolute monarchs.

PERSIA extends from 25 to 45 degrees north latitude, and from 45 to 67 degrees east longitude. It is bounded on the east by the Mogul's dominions; on the north by Usbeck Tartary, the Caspian Sea, and Circassia; on the south by the Indian Ocean and Gulf of Persia; and on the west by Arabia and the Turkish empire.

The climates of this country are very various. In the northern parts, and near the mountains, which are covered with snow, the air is very cold; in the midland parts it is serene, pure, and healthy; but towards the southern parts there are sometimes hot suffocating winds, which blow over a sandy desert from south and east; a blast of which has sometimes struck the unwary traveller with death in an instant. The soil is various, being in some parts very barren, but where it is well watered it is very fruitful.

The principal commodities of traffic are silks, camlets, carpets, leather, embroidery, gold and silver threads, mohair, &c.

The

The national religion of Persia is that of Mahometism, and the sect of Ali.

ARABIA extends from 35 to 60 degrees east longitude, and from 12 degrees 30 minutes to 30 degrees north latitude. It is bounded on the north by Asiatic Turkey; on the south by the Indian Ocean; on the east by the Euphrates and Gulf of Bassora; and on the west by the Red Sea.

Arabia is divided into three parts, viz. Arabia Petræa, or the Stony; Arabia Deserta, or the Desert; and Arabia Felix, or the Happy.

Arabia the Stony is the wilderness in which the children of Israel sojourned 40 years: and in it may be seen the mountains of Horeb and Sinai, mentioned in sacred writ.

Arabia the Desert principally consists of a large sandy desert; it has, however, a few spots of fruitful land, covered with verdure, which are interspersed in different parts of the desert. It is over this desert that some of the eastern nations bring their commodities of traffic from the East, travelling in large caravans.

Arabia the Happy is, in general, barren; but some of the vallies between the mountains, and those plains which are well supplied with water, are very fruitful. From this part great quantities of drugs are exported to Europe, and also Turkey coffee.

The Arabs are, in general, a wandering people: many of their tribes live wholly in tents, and subsist partly by robbing the caravans which travel through the desert, and partly by the produce of their country, and the flesh of their cattle; raising no grain of any kind for domestic use.

Their religion is that of Mahometism; but many of the tribes are still Pagans. Their language is said to exceed even the Greek itself in copiousness. The Arabians have never yet been subdued by any military force, though several attempts have been made for that purpose.

SECT. VIII.

OF AFRICA.

THE continent of Africa is in the form of a peninsula, surrounded on each side by water, except where it joins to Asia by the Isthmus of Suez. Several countries, famous in antiquity for the arts and sciences, were situated in the northern parts of this quarter. And in the early days of Christianity several Christian churches were founded here; but at the present period Mahometism and idolatry degrade this most fertile quarter of the globe. That most inhuman commerce, trafficking in men, also is carried on here by the European nations.

The ancients believed the greater part of this quarter of the globe to be uninhabited, as also the greater part of Asia, and, indeed, all that part of the globe lying between the tropics; but modern travellers have discovered, that the tropical countries are in general the most fertile and best populated; and of these the southern and interior parts of Africa are found the most eligible, both for vegetation and population. Its sea-coasts are the only parts with which we are particularly acquainted; but travellers are now busily employed in making discoveries in the internal parts.

Africa is bounded on the west by the Atlantic Ocean; on the north by the Mediterranean; on the east by the Red Sea; and on the south by the Southern Ocean. It lies between 37 degrees north, and 36 degrees south latitude, the equator running nearly through the middle thereof; and between 17 degrees west, and 51 degrees east longitude. In length, from north to south, it is about 4600 miles; and in breadth, from east to west, 3500 miles.

EGYPT

EGYPT is bounded on the north by the Isthmus of Suez ; on the east by the Red Sea ; on the south by Nubia ; and on the west by the interior parts of Africa. It lies between 30 and 36 degrees east longitude ; and between 20 and 32 degrees north latitude ; and is divided into Upper and Lower Egypt.

The climate, during the summer season, is excessively hot ; when the south winds often raise such a cloud of sand as to obscure the light of the sun, and cause epidemical diseases.

The soil is exceedingly fruitful, owing to the annual overflowing of the Nile. This river, so famous in ancient history, has its rise in Abyssinia, at between 11 and 12 degrees of north latitude, and pursues a northern course for above 1500 miles ; when it divides into two branches, about six miles below Grand Cairo ; one branch extending eastward, and the other westward. It begins to rise in the beginning of summer, and increases three or four inches in height each day, for the first week : the next fortnight it increases in a still greater proportion : and it is near four months before it is reduced into its channel again. The principal cities and towns are built on eminences on the banks of the Nile, and, during the inundation, correspond with each other by means of boats. When the Nile rises to the height of 49 feet, it produces a plentiful season, but if it exceed that height it is productive of great mischief, sweeping away both houses and cattle.

In Egypt they generally have three crops in a year : the first, of lettuces and cucumbers ; the second, of corn ; the third, of melons, and all the fruits common to hot climates.

Their pastures are the richest in the world, the grass being usually as high as the cattle.

Their trade consists of great quantities of flax and cotton, both prepared and unmanufactured ; leather of different kinds ; also a great variety of drugs, and roots for dying.

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The

The common language spoken here is the vulgar Arabic, as it is under the dominion of the Turks.

BAREARY extends from Egypt to the Atlantic Ocean, and from the Mediterranean Sea to the Libyan Deserts, being 750 miles in breadth, and near 2000 in length. Containing the countries of Morocco and Fez, which form one distinct empire: and the states of Algiers, Tunis, Tripoli, and Barca, composing several distinct states, united together in confederacy, under the Turkish government.

Its soil is exceedingly fruitful, producing excellent corn, cattle, and pasture, and all the variety of tropical fruits; and vast quantities of fish and fowl; also a great variety of tame and wild animals.

The commerce of this country is chiefly carried on by caravans: their exports consist of leather, mats, handkerchiefs, carpets, elephant's teeth, ostrich feathers, copper, tin, wool, fruits, gum, drugs, &c. for which they receive timber, artillery, gunpowder, &c.

Their religion is that of Mahometism. Their language varies according to the different parts of the country. That spoken in the inland parts, is either an African language, or a corrupt Arabic. The latter is also spoken in most of the sea-port towns: but in some parts they use a mixed language, such as is spoken in most of the Mediterranean ports.

Most of the Barbary states subsist by piracy: and their sailors fight desperately when they meet a vessel belonging to any power with whom they are at war.

The government is that of an absolute monarchy. The emperor is in general both judge and executioner: and he acknowledges the Grand Seignior of Turkey to be his superior. When there is a vacancy in the government, every soldier in the army has a vote in choosing a new emperor, which is often attended with great bloodshed.

The parts of Africa, from the tropic of Cancer to the Cape of Good Hope, are very little known, except the sea-coast thereof. The natives in general are black, except those

those of Abyssinia, who are of a tawney complexion, and are a mixture of Jews, Christians, and Pagans. The religion of the other countries in this part is generally that of Paganism, and the form of government every where monarchical, except in a few settlements formed by the Europeans, on the sea-coast. Few of their princes, however, possess an extensive degree of territory. As the natives are ignorant of all the arts, utility, and refinement, the different kingdoms are therefore unconnected with each other; and are generally at war.

The soil of Africa is in general very fruitful: though in some parts it is perfectly barren, particularly where there is very little water; the heat of the sun reducing the soil to a perfect sand; such are the countries of Anian and Zaara; but the countries of Mandingo, Ethiopia, Congo, Angola, Batua, Truticui, Monomotapa, Cafati, and Melunenrugi, are extremely fruitful, and very rich in gold and silver.

On the western coast the English trade is carried on at James's Fort, and other settlements, near and up the river Gambia, where woollen and linen cloths, hardware, and spirituous liquors are exchanged for the persons of the natives. Many of the negroes will sell their own families for those superfluities. The natives are often trepanned by foreigners, or their own countrymen, and then sold to the Europeans: and many more are sold by the princes of the different states, being captives taken in war. Gold and ivory form the principal branches of commerce, next to that of the slaves.

The Portuguese possess the greater part of the east and west coast of Africa, from the tropic of Capricorn to the equator. The Dutch have some settlements towards the southern parts of the continent; and Cape Town, at the Cape of Good Hope, is now a free port, and is well fortified, and where the ships bound for India usually put in, and trade with the natives, or Hottentots, for their cattle and

other provisions, for which they give them spirituous liquors. There are several islands near the coast of Africa, lying in the Eastern or Indian Ocean, or in the Western or Atlantic Ocean, of which the chief are :

1. **ZOCOTRA**, situated in 53 degrees east longitude, and 12 degrees north latitude ; 30 leagues east of Cape Guardafui, on the continent of Africa ; being 80 miles in length, and 54 in breadth, and has two good harbours. It is a populous, plentiful country, governed by a prince who is tributary to the Porte.

2. **BABELMANDEL**, situated in the strait of the same name, at the entrance of the Red Sea, in 44 degrees 30 minutes east longitude, and 12 degrees north latitude, being a small sandy island, not five miles round.

3. The islands of **Joanna**, **Mayotta**, **Mohilla**, **Angezeia**, and **Comora**, situated between 41 and 46 degrees east longitude, and between 10 and 14 degrees south latitude : the chief of these is **Joanna**, to which the others are tributary, being 30 miles long, and 15 broad : affording excellent fruits and provisions. The natives are a friendly set of people, and profess the Mahometan religion.

4. **MADAGASCAR**, the largest of the African islands, situated between 43 and 51 degrees east longitude, and between 10 and 26 degrees south latitude ; three hundred miles south-east of the continent of Africa, being near 1000 miles in length, from north to south ; and between 2 and 300 miles in breadth. Between this island and the Cape of Good Hope, or the continent of Africa, the sea rolls with great force, and is exceedingly rough. In this channel, all European ships pass in their voyage to and from India, except the water be too rough. Madagascar is a fertile country, abounding in all the variety of fruits and vegetables to be met with in the same climate. The air is temperate also, and healthy. It is inhabited by both blacks and whites, professing different religions ; but principally Mahometism and Paganism ; and governed by several petty princes.

5. **MAU-**

5. **MAURITIUS**, or Maurice, situated in 56 degrees east longitude, and 20 degrees south latitude; about 400 miles east of Madagascar. It is of an oval form, and about 150 miles in circumference, with a large fine harbour. The climate is healthy and pleasant; and the island is well watered with several rivers: though the soil is not so fruitful as that of the former, it nevertheless feeds a great number of cattle, sheep, deer, and goats.

6. **BOURBON**, situated in 54 degrees east longitude, and 21 degrees south latitude; about 300 miles east of Madagascar, and about 90 miles in circumference. Surrounded for the most part with blind rocks, a few feet under water. The climate is in general healthy, though hot. It affords very good pasture and cattle.

There are several other small islands about Madagascar, and on the eastern coast of Africa.

7. **ST. HELENA**, situated in 6 degrees west longitude, and 16 degrees south latitude; being 1200 miles west from the continent of Africa, and 1800 east from South America. The whole island is situated on a rock, and is about 21 miles in circumference. There is but one landing-place in the island, which is at the east side thereof. It is very fertile, diversified by hills and vallies, and abounds in all the conveniences and comforts of life. There are about 200 families, mostly descended from English parents.

8. **ASCENSION**, situated in 7 degrees 40 minutes south latitude, and 600 miles north-west of St. Helena. It is a mountainous barren island, and uninhabited; about 20 miles round.

9. **ST. MATTHEW**, situated in 6 degrees 1 minute west longitude, and 1 degree 30 minutes south latitude; and uninhabited.

10. **CAPE VERD ISLANDS** are situated between 23 and 26 degrees west longitude, and between 14 and 18 degrees north latitude. They are about 20 in number; but the principal are St. Jago, Bravo, Fogo, Mayo, Bonavista, Sal,

St. Nicholas, St. Vincent, Santa Cruz, and St. Antonio. They mostly belong to the Portuguese and Spaniards. The air in general is very hot, and in some unwholesome. They are inhabited by Europeans and their descendants.

11. GOREE, situated in 14 degrees 43 minutes north latitude, and 17 degrees 20 minutes west longitude. It is a small spot not exceeding two miles in circumference; but an important situation for trade.

12. The CANARIES, or FORTUNATE ISLANDS, are seven in number, and situated between 12 and 19 degrees west longitude, and between 27 and 29 degrees north latitude. These islands have a pure temperate air, and abound in most delicious fruits, from whence they have those rich wines called Canary, of which they export 10,000 hogheads annually.

13. The MADEIRAS are three islands, situated in 32 degrees 27 minutes north latitude, and between 18 degrees 30 minutes and 19 degrees west longitude. These islands are mostly famous for producing the Madeira wine, of which no less than 20,000 hogheads are annually exported.

14. The AZORES, or WESTERN ISLANDS, are situated between 25 and 32 degrees west longitude, and between 37 degrees and 40 degrees north latitude. Being 900 miles west of Portugal: and lying in the midway between Europe and America. Of these St. Michael is the largest, being near 100 miles in circumference, and containing 50,000 inhabitants. Tercera is the most important of these islands, on account of its harbour, which is very spacious, and affords good anchorage. There are seven other of these islands: their names are Santa Maria, St. George, Graciosa, Fayal, Pica, Flores, and Corvo.

SECT. IX.

OF AMERICA.

AMERICA, or the Great Western Continent, frequently called the New World, extends from the 80th degree of north latitude, to the 56th degree of south latitude; and where the breadth is known, from the 35th degree, to the 136th degree west longitude; extending near 9000 miles in length, and 2600 in breadth. As it extends into both hemispheres, it has two summers and two winters. It is washed by the two great oceans, the Atlantic and Pacific; having the former on the east, and the latter on the west; by these seas it has a direct communication with the other three quarters of the world. It is composed of two great continents, North and South America, connected together by the kingdom of Mexico, which is an isthmus of 1500 miles long, and in one part only 60 miles broad.

AMERICA is the best watered of any part of the globe; even those vast tracts of country situated beyond the Apalachian Mountains, at an immense distance from the ocean, are watered by inland seas, as the Lakes of Canada, which give rise to several large rivers, as the Mississippi, the Missouri, the Ohio, and on the north, the river St. Lawrence, all of them being navigable to their heads, which is a great advantage for commerce.

SOUTH AMERICA is better watered, if possible, than North America, having the two largest rivers in the world: viz.—
the

the river of Amazons, and the river of La Plata; the former having a course of about 3000 miles.

A country of such vast extent on both sides of the equator must necessarily have all the varieties of soils and climates to be met with in every other part of the globe. It also produces most of the metals, minerals, plants, fruits, trees, and wood, to be met with in the other parts of the world, and many of them in greater quantities and higher perfection.

This country likewise produces diamonds, pearls, emeralds, amethysts, and other precious stones; also cochineal, indigo, anatto, logwood, brazil, fustic, pimento, lignum vitæ, rice, ginger, cocoa or chocolate, sugar, cotton, tobacco, the balsams of Peru, Tolu, and Chili, Jesuit's bark, mechoacan, saffrafras, saraparilla, cassia, tamarinds, and a great variety of other woods, roots, and plants, many of which were not known before the discovery of America; besides hides, furs, and ambergris.

Though the Indians still live in quiet possession of many large tracts of country, in the inland parts, yet America, so far as is known, is generally claimed by four powers: viz.—the Spaniards, English, Portuguese, and American settlers, being the descendants of Europeans, and who have the largest share of country, except the Spaniards, who possess the largest and most extensive portion of all, extending from New Mexico and Louisiana, in North America, to the Straits of Magellan, in South America; except the large province of Brazil, which belongs to Portugal, Surinam, claimed by the Dutch, and Cayenne, the property of the French, all in South America. The United States of America possess all that tract of country which is bounded by the Mississippi, the river St. Lawrence, and the Lakes of Canada, on the north and west; and washed by the Atlantic Ocean on the east; and on the south by the Gulf of Mexico.

The AMERICAN ISLANDS, commonly called the West Indies,

Indies, was the first of America discovered by the Europeans, and are situated in the gulf called the Caribbean Sea, between the continents of North and South America, extending from the coast of Florida, to the river Oroonoko: they are divided between five European nations: viz.—the English, French, Spaniards, Dutch, and Danes.

As all these islands lie between the tropics, their climates and soil are pretty much alike: the heat would be intolerable, if it were not for the trade-winds which blow during the fore part of the day, and the sea and land breezes. Their seasons are divided into the wet and dry: in the wet seasons, the rain pours down with such impetuosity as to overflow the rivers, and lay the low country under water.

The principal trade of the West Indies consists of sugar and rum: they also export cotton, indigo, chocolate, coffee, and dying and physical drugs, spices, and hard woods; for which they receive from Europe, manufactures; from the African islands, wine; and from the neighbouring continent, lumber and provisions.

The BAHAMA ISLANDS, which are said to be 500 in number, lie to the south of Carolina, between 21 and 27 degrees north latitude, and between 73 and 81 west longitude. There are, however, not above twelve of them of any magnitude, the rest being little better than rocks or banks, and almost uninhabited, except Providence Island.

The BERMUDAS, or SUMMER ISLANDS, lie in the Atlantic Ocean, about 500 leagues east from Carolina, in 32 degrees north latitude, and in 65 degrees west longitude: there are said to be about 400 in number; but containing not more than 20,000 acres.

The islands of NEWFOUNDLAND, CAPE BRETON, and ST. JOHN, lie at the mouth of the river St. Lawrence; and are celebrated for the quantity of fish found on their coasts, which is supposed to increase the national stock upwards of 300,000 annually: in this branch of commerce 3000 small craft are employed, and 10,000 hands.

BRITISH AMERICA, or the territories on the continent belonging to the English, are New Britain, Canada, or the province of Quebec, and Nova Scotia, or Arcadia; bounded on the east and south by the Atlantic Ocean and the American States; on the north and west, their boundaries have never been defined, but are blended with the lands of the Indian nations. New Britain contains Labrador, and New North and South Wales. Canada contains the towns of Quebec, Trois Rivières, and Montreal, all situated on the river St. Lawrence.

Nova Scotia contains the towns of Halifax, Annapolis, and St. John's.

THE UNITED STATES OF AMERICA are bounded on the west by the Indian nations; on the north, by British America; on the west, by the Atlantic; and on the south, by Spanish America, containing the following states or colonies: New Hampshire, Massachusetts, Rhode Island, Connecticut, New York, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, South Carolina, Georgia, Vermont, Western Territory, and Kentucky.

The United States, in the year 1776, were only 13 in number: Vermont, Kentucky, and the Western Territory, have been added since. The Western Territory is of such extent, that the Congress have determined to divide it into ten new states.

From the latest accounts, it appears, that the population of the United States amounts to upwards of 3,083,600 persons, who are composed of almost all nations, languages, characters, and religions: the greater part, however, have descended from the English.

The language generally spoken through all these states is the English, in which all their civil and ecclesiastical matters are performed, and their records kept.

There are, however, great numbers of Dutch, French, Germans, Spaniards, Jews, and Swedes, who retain in a great degree each their native language, and have their respective

pective places of worship; and in general live comfortably and unmolested, as to principles of conscience.

NEW ENGLAND is bounded on the north by Canada; on the east by Nova Scotia and the Atlantic Ocean; on the south by the Atlantic; and on the west by New York; and is divided into five states: viz. New Hampshire, Massachusetts, Rhode Island, Connecticut, and Vermont, which are subdivided into counties, and those counties again subdivided into townships.

New England is a fine country for pastures; the valleys are generally intersected with brooks of water, the banks of which are covered with a tract of rich meadow land.

The state of **NEW YORK** is bounded on the south-east by the Atlantic Ocean; on the east by Connecticut, Massachusetts, and Vermont; on the north by Canada; on the south and south-west by Pennsylvania and New Jersey, being 350 miles in length, and 300 in breadth: and containing about 44,000 square miles, equal to 28,160,000 acres. The river St. Lawrence divides this state from Canada. The settlements formed in this state are chiefly upon two oblongs, extending from the city of New York east and north. The east is Long Island, which is 140 miles in length; the other, extending north, is about 40 miles in breadth. This state exports to the West Indies biscuits, peas, Indian corn, apples, onions, boards, slaves, horses, sheep, butter, cheese, pickled oysters, beef, and pork: but the principal part of their trade is wheat, of which, in the year 1775, they exported 677,700 bushels, and 2555 tons of bread, besides 2828 tons of flour, for which they receive in exchange the commodities of the West India islands.

NEW JERSEY is 160 miles in length, and 52 in breadth. Bounded on the east by Hudson's river and the sea; on the south by the sea; on the west by Delaware bay and river, which divides it from Pennsylvania; and on the north by a line drawn from the mouth of Mahakkamak river, in lati-

titude 41 degrees 24 minutes, to a point in Hudson's river, in latitude 41 degrees; containing about 8380 square miles.

This state has a great variety of soil, from the worst to the best. But it contains a greater portion of barren land than any other state, there being nearly one fourth of this state unfit for cultivation. But those parts which are fruitful are equal in fertility to any part of the United States.

The state of **PENNSYLVANIA** is bounded by Delaware river on the east; by the state of New York on the north; by the parallel of latitude, 39 degrees 43 minutes 18 seconds, on the south; and by a meridian line drawn from the said parallel at five degrees of longitude, from a point on Delaware river west. This state lies in the form of a parallelogram. The north side of Pennsylvania is the best soil, and most populated, owing to the great number of new roads which have lately been made.

The state of **DELAWARE** is bounded on the north by the territorial line (which is a circle), described with a radius of 12 English miles, and whose centre is in the middle of the town of Newcastle, which divides it from Pennsylvania; on the east by Delaware river and bay; on the south by a line drawn due east and west from Cape Henlopen in 38 degrees 30 minutes north latitude, to the middle of the peninsula, which line divides the state from Worcester county, in Maryland; and on the west by Maryland: containing about 1400 square miles; being 92 miles in length, and 16 in breadth.

The state of **MARYLAND** is bounded on the north by Pennsylvania; on the east by the Delaware state; on the south and south-east by a line drawn from the ocean over the peninsula (dividing it from Accomac county in Virginia), to the mouth of Patomac river, and from thence up the river to its first source; from thence, by a due north line, till it intersects the southern boundary at Pennsylvania: being 134 miles in length, and 110 in breadth. The soil of Maryland, where it is good, will produce from 12 to 16 bushels of

wheat, or from 20 to 30 bushels of Indian corn, per acre. Their trade is chiefly with the other states, the West Indies, and some parts of Europe: to which places they annually export 30,000 hogheads of tobacco, besides great quantities of pig-iron, lumber, flax-seed, and provisions.

The state of VIRGINIA is bounded on the east by the Atlantic; on the north by Pennsylvania and the river Ohio; on the west by the Mississippi; and on the south by North Carolina: being 750 miles in length, and 224 in breadth.

The state of KENTUCKY is bounded on the north-west by the river Ohio; on the west by Cumberland river; on the south by North Carolina; and on the east by Sandy river, and a line drawn full south from its source to the northern boundary of North Carolina: being 250 miles in length, and 200 in breadth. The fertility of the soil is such, that the land, in common, will produce 30 bushels of wheat, or rye, an acre. The best lands are too rich for wheat, and will produce from 50 to 80 bushels of good corn per acre, and few soils yield more and better tobacco.

The state of NORTH CAROLINA is bounded on the north by Virginia; on the east by the Atlantic; on the south by South Carolina and Georgia; and on the west by the Mississippi: being 758 miles in length, and 110 in breadth.

The state of SOUTH CAROLINA is bounded on the east by the Atlantic; on the north by North Carolina; on the south and south-west by the Savannah river. The western boundary is not ascertained. It is reckoned 200 miles in length, and 125 in breadth.

The state of GEORGIA is bounded on the east by the Atlantic; on the south by Florida; on the west by the river Mississippi; on the north and north-east by South Carolina: being 600 miles in length, and 250 in breadth.

The WESTERN TERRITORY includes all that part of the United States on the north-west of the river Ohio; bounded on the west by the Mississippi river; on the north by the lakes;

on the east by Pennsylvania; and on the south and south-east by the river Ohio; containing 411,000 square miles, equal to 263,040,000 acres, from which deducting 43,040,000 acres occupied by the water, there remain 220,000,000 acres, which are to be sold by Congress, for the discharge of the national debt.

Territories of Spain in North America.

The dominions of Spain, in North America, extend from 81 degrees to 120 degrees west longitude, and from 8 to 43 degrees north latitude. Bounded on the north by the United States and the Indian nations; on the west by the Pacific Ocean; on the east by the Gulf of Mexico and the Atlantic; and on the south terminating in the Isthmus of Darien. They contain the following countries, viz. East Florida, West Florida, New Mexico, California, and Old Mexico.

The soil of this extensive tract of country is very various, but in general fertile: and it is in the most mountainous parts that the mines of gold and silver are found. The air is in general warm and pleasant; but the northern parts have very cold winds; and the southern parts, lying within the torrid zone, are exceedingly hot.

South America.

SOUTH AMERICA, from the northern coast of Terra Firma, and the Isthmus of Darien, to the Straits of Magellan, belongs to the Spaniards, except the province of Brazil, which belongs to the Portuguese; and the settlements of the Dutch in Surinam, and those of the French in Cayenne.

BRAZIL

BRAZIL, belonging to the Portuguese, extends from the equator to 35 degrees south latitude, and from 35 to 60 degrees west longitude. Bounded on the north by the mouth of the river Amazons and the Atlantic; on the east by the Atlantic; on the south by the mouth of the river Plata; and on the west by the unknown country of the Amazons.

CAYENNE is the only settlement in the southern continent of America retained by the French, and is situated between the equator and 5 degrees north latitude; and between 52 degrees 15 minutes, and 55 degrees 30 minutes west longitude. It extends 240 miles along the coast of Guiana, and near 300 miles inland. Bounded by Surinam on the north; by the Atlantic on the east; by Amazonia on the south; and by the territories of the Indians on the west.

SURINAM, or Dutch America, lies between 5 and 7 degrees north latitude; and is bounded on the north by Cayenne; on the west by Terra Firma; on the south by the Indian nations; and on the east by the river Oroonoko.

The dominions of Spain, in South America, contain the following extensive countries, viz. Terra Firma, Peru, Chili, Paraguay, Amazonia, and Patagonia; extending, as before observed, through the whole continent of South America.

The climate and soil of Spanish America vary greatly, from the hot burning sand and smoking swamp in the northern parts in the torrid zone, to the cold region of the southern part, near Cape Horn.

The islands of South America are Terra del Fuego, the Falkland Islands, and the island of Juan Fernandes; the latter of which gave rise to the famous story of Robinson Crusoe, from one Alexander Selkirk, mariner, native of Scotland, who was put ashore in this island by his captain, in the year 1697, and discovered by Woodes Rogers in the year 1709, who took him on board, and brought him to Europe; after having been on this uninhabited island for twelve years.

The number of inhabitants in the known parts of the world, computed at a medium from the best calculations, are about nine hundred and fifty-three millions; viz.

Europe contains	153 millions
Asia - - -	500
Africa - - -	150
America - - -	150
	<hr/>
	953
	<hr/>

CHAP. XIV.

OF ASTRONOMY.

SECT. I.

OF THE PRIMARY PLANETS.

BEFORE I proceed to the description of the **Primary Planets**, it will be necessary to take a view of the solar system, with the order and economy of the motions and courses of those planets.

The systems which have been most generally received in astronomy, are the *Ptolemaic*, the *Copernican*, *Pythagorean*, and the *Tychonic*.

The

Fig. 1.

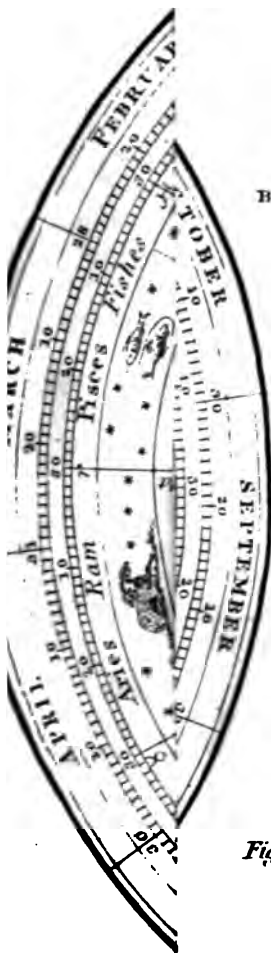


Fig. 3.

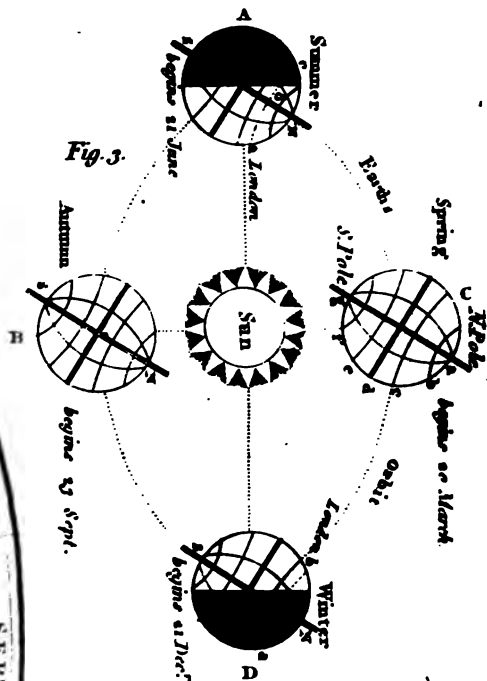
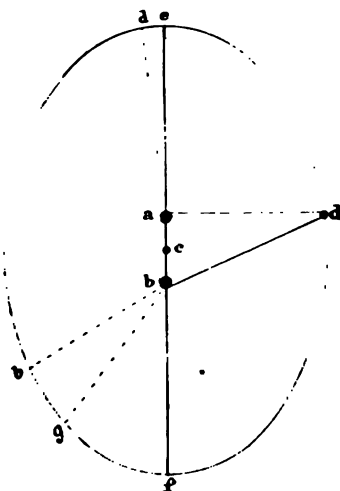


Fig. 2.





The *Ptolomaic* system, so called from its inventor Ptolomy, supposes the earth to be placed at rest in the centre of the universe, and the heavens revolving about it from east to west, in the space of twenty-four hours, and by this motion carrying the sun, stars, and planets, completely round the earth in that space of time.

It was this system which Aristotle, Hipparchus, and most of the philosophers of antiquity defended so strenuously, and was followed by the whole world for many ages, and longer retained in many learned universities. But latter improvements and more evident demonstrations have now utterly exploded it.

In the *Tychonic* system, invented by Tycho, the Dane, the earth is also supposed to be fixed in the centre of the universe, and all the heavenly bodies performing a revolution round it in the space of twenty-four hours, as in the Ptolomaic system; but with this difference, that it allows a monthly motion to the moon round the earth, and also the proper motions of the satellites about Jupiter and Saturn. It also supposes the sun to be the centre of all the primary planets: the primary planets being carried round the sun in their respective periods, while the sun, with all the aforesaid planets, revolve round the earth every twenty-four hours. But this system was so inconsistent with observations, that it had but few followers. It was therefore altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the sun. This improved hypothesis is called the *semi-tychonic* system.

But these systems have now given place to that called the *Copernican* system, which undoubtedly is the most ancient in the world. It was first introduced into Greece and Italy by Pythagoras, and from him called the Pythagorean system. It was adopted by Philolaus, Plato, Archimedes, and all the most ancient philosophers, but was at length lost under the Peripatetic philosophy, and restored again about the year 1500, by Nic. Copernicus.

This

This system has been proved by the most eminent demonstrations to be the only true one. I shall therefore confine myself to the description of this alone, and the phenomena that arise from it.

Fig. 1. Plate 13. is a representation of this system, where the seven concentric circles, marked Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Georgian Sidus, represent the orbits of these seven primary planets, performing each its annual rotation round the sun, which is placed in the centre. The next two circles represent the twelve signs of the zodiac, with all its divisions into thirty degrees in each sign. And lastly, the two next outer circles shew the twelve calendar months of the year, with their divisions into days. Of each of which in their order.

A planet, in the literal sense of the word, signifies a wanderer, or wandering star: and is therefore used in contradistinction to the stars which we call fixed. It is a celestial body, globular and opaque, and revolving around the sun, or some other planet, as a centre, at least as a focus to its orbit, which always has a moderate degree of eccentricity, so that it is never much farther from the sun, or centre of motion, at one time than at another, in proportion to the diameter of its orbit.

The planets are either primary or secondary.

Primary planets, sometimes called planets, by way of eminence, are those seven above described, which move round the sun, as their common centre, or focus of their orbits.

Secondary planets, or satellites, are such as move round some primary planet, as their focus, in the same manner as the primary planets move round the sun. Such are the moon, which moves round the earth, and the four moons of Jupiter, the seven moons of Saturn, and the six of the Georgian sidus.

The primary planets are divided into superior and inferior: the superior planets are those that perform their revolutions round the sun at a greater distance from the sun, than the earth

earth is, as Mars, Jupiter, Saturn, and the Georgian. The inferior planets are those included within the orbit of the earth, as Venus and Mercury; as may be seen in the figure.

The planets were formerly represented by the same characters which the chemists made use of, to represent their metals by, on account of a supposed analogy between the planets and those metals; and these characters are still used, to avoid confusion.

Thus, *Mercury*, anciently called the messenger of the Gods, was signified by the character ☿; which stood for the metal mercury, and also bore a rude resemblance to the heathen deity of that name, being a man with wings on his head and feet.

Venus, the next planet in order, so named from the Goddess of Love, was characterized by ♀, for the figure of a woman; and denoted the metal copper.

Tellus, or the Earth, was characterized by ⊕; and is the third planet distant from the Sun.

Mars, or the God of War, was denoted by ♂, and represented iron; and supposed to bear a resemblance to a man holding out a spear.

Jupiter, the chief of the heathen gods, was marked ♃, to represent thunderbolts; and signified the metal tin.

Saturn, the father of the gods, was represented by ♄, to resemble an old man, supporting himself with a staff; the same character being used for the metal lead.

The *Georgian*, or Herschel, is denoted by ♃, the initial of Dr. Herschel's name, with a cross for the Christian planet.

The *orbit* of a *planet* is the path it describes in going round the Sun; they are represented in the figure by concentric circles. The Earth's orbit is called the ecliptic.

Kepler was the first astronomer who discovered that the orbits of the planets were not circular, but of an elliptic form, in the form of the figure (fig. 2), having the Sun in one of the foci thereof: and he further discovered these two

primary laws, from hence called *Kepler's Laws*: viz. that a radius drawn from the centre of the Sun to the centre of the planet, always describes equal areas in equal times; or, which is the same thing, in unequal terms, it describes areas proportional to those times: and the squares of the periodical times of the planets are as the cubes of the mean distances of the planets from their centres.

Thus, let $d e f$ (fig. 2) be the orbit of a planet, or an ellipse, in which c is the centre, and $a b$ the two foci of the ellipse, in one of which foci the sun is placed, as suppose b : then if the area $b b g$ be equal to the area $b g f$, the planet will be as long in describing that part of its orbit from b to g , as from g to f . A straight line $e f$ drawn through the two foci of the orbit is called the line of the apses, and the point e , being the farthest from the Sun, or centre of motion, is called the higher apsis, or *aphelion*; and the point f , being that point of the line next the Sun, or centre of motion, is called the lower apsis, or *perihelion*; therefore, when the planet is in its aphelion, its motion is the slowest of all, and when in the perihelion the quickest.

The axis of a planet is an imaginary line, supposed to be drawn through the centre, about which the planet performs a diurnal rotation.

The method of describing an ellipse is this: having fixed two pins, or points, $a b$, with a piece of thread doubled, and a pen, or pencil d in the double of the thread, describe the ellipse $d e f$, keeping the thread extended to the full length by the pencil, d . And note: the two fixed points a and b , which hold the thread, are always the two foci of the ellipse; and the nearer these two foci are together, the nearer the ellipse will approach to the form of a circle: and the farther they are distant from each other, the farther the ellipse departs from a circular form. The distance $a c$, or $c b$, that is, the distance from the centre to either focus, is called the eccentricity.

The orbits of the planets are not all in the same plane; neither are any two of them in the same plane: but the plane of the ecliptic, or Earth's orbit, intersects the plane of the orbit of every other planet in a right line, which passes through the Sun, called the line of the nodes; and the points of intersection of the orbits are called the nodes.

The different inclinations of the axis of a planet to the plane of its orbit are the cause of the different seasons of the year in that planet; and the diurnal rotation of the planet round its axis produces the successive changes of the day and night. Thus, in the Earth's orbit (fig. 3), let A represent the Earth in the summer season, on the 21st of June. NS is the axis of the Earth, N being the north, and S the south pole, about which the Earth performs her revolution every twenty-four hours. Let *c a* represent the path described by the city of London, in the rotation of the globe round its axis. Now the reason why the days are longer than the nights at this season of the year in London, will be evident, from a bare inspection of the figure. For the axis of the Earth being inclined to a perpendicular drawn to the ecliptic, in an angle of twenty-three degrees and a half, and the city of London being situated in the fifty-second degree of north latitude, which is nigher the north pole than to the equinoctial, it will not pass through near so great a space of the dark side of the Earth, or that side opposite to the Sun, as it will through the illuminated part of the Earth, or part next to the Sun. The nights, in this season of the year, only continuing while London describes that part of its tract, from *c* to *b*, and from *b* to *c*, on the other side: but as soon as it arrives at *b* it will be daybreak, and the day continues while London describes that part of its track from *b* to *a*, and from *a* to *b* on the other side of the globe; and when it arrives at *a*, the Sun will be on the meridian, or it will be then noon-day. The length of the day, therefore, will be to that of the night, as the distance *c b* to that of *b a*.

But when the Earth arrives at B, which is on September

D d 2

the

the axis, the axis of the Earth always keeping in the position, the days will be equal to the nights; for the mirrored parts of the Earth will extend exactly from north to the south poles; and the more, in the passage of the Earth from A to B, the length of the days will gradually decrease in the northern latitudes, but increase in the southern latitudes.

Fig. 1. Mercury is the smallest of all the primary planets nearest the Sun; performing his revolution in less space of time than any of the rest, round the Sun, and with a very rapid motion. This occasioned the Greeks to give him his present name and character, calling it, as before observed, the messenger of the gods.

The mean distance of this planet from the Sun, compared with that of the Earth from the Sun, is as 387 to 100,000; therefore, his distance from the Sun is about 37,000 miles, or little more than one third of the Earth's distance from the Sun. Hence, the diameter of the Sun, seen from Mercury, will appear near three times as large as when seen from the Earth; consequently, that planet receives seven times as much heat and light from the Sun, as the Earth does.—This is a degree of heat sufficient to water gold.

The diameter of Mercury is not one third of that of the Earth, or about 2600 miles; therefore, its surface is but one ninth, and its solidity one twenty-seventh of that of the Earth.

The orbit of Mercury is inclined to the plane of the Earth's orbit, in an angle of 6 degrees 54 minutes. He performs one entire revolution round the Sun, in a space of 87 days 23 hours 15½ minutes; therefore, the years and winters in that planet cannot be more than 87 each. His greatest elongation from the Sun, that is, the greatest distance that he is seen by us to depart from the Sun, is 28 degrees 20 minutes: the eccentricity of his orbit is one fifth of his proportional mean distance from the

which is far greater than that of any of the other planets ; and the pace with which he moves in his orbit is at the amazing rate of about 95,000 miles in an hour.

The place of his aphelion is 14 degrees 13 minutes of Sagittary ; the place of his ascending node is 15 degrees 47 minutes of Taurus, and consequently that of the descending node, 15 degrees and 47 minutes of Scorpio.

The length of his days, or rotation on his own axis, inclination of his axis to his orbit, gravity on his surface, density and quantity of matter, are all unknown.

Mercury is observed to appear with various phases, like the Moon ; varying according to his various positions with regard to the Earth and Sun ; therefore he never appears with a full face towards us, except when he is too near the Sun to be distinctly seen ; for his enlightened face is always towards the Sun. From these observations it is plain, that Mercury, like all the other planets, is a dark opaque body, having no light of his own, but what he receives from the Sun ; for, if he had, he would always appear completely round.

The best time to make an observation on this planet, is when it is seen on the Sun's disk, called its transit ; at which time it passes before the Sun, and appears like a little round black spot on the Sun's surface, eclipsing a small part thereof, and is only visible through a telescope. And as these transits can happen only when the Earth and Mercury are both in the same node of Mercury, (that is, on the 6th of November, and the 4th of May ;) and at which time Mercury must also be in an inferior conjunction, it will follow, that at these times, when Mercury is in the inferior conjunction with the Sun, he will appear to pass over the Sun's disk.

But in all the other parts of his orbit, he will never make a transit over the Sun, though he may be in an inferior conjunction, because he goes either above or below the face of the Sun. Gassendi, in November 1631, first took an observation of this kind ; and Mr. Whiston has given a list of several

several transits of Mercury: viz.—November 12th, at 3 hours 44 minutes, afternoon, in 1782; May 4th, at 6 hours 57 minutes, forenoon, 1786; November 5th, 3 hours 55 minutes, afternoon, 1789; and the 7th, a hours 34 minutes, afternoon, 1799.

Venus, the brightest, and, to appearance, the largest of all the planets, is the next inferior planet, and is distinguished from all the rest by her brightness, and white appearance; her light is so considerable, that in a dusky place she will often cause an object to project a sensible shadow; and her brightness is so great, as often to render her visible in the day-time.

As *Venus* moves round the Sun in an orbit within the Earth's orbit, like Mercury, she can never be seen in opposition to the Sun, though she departs farther from him than Mercury does; her greatest distance from the Sun being about 47 degrees 48 minutes, as seen from the Earth.

When she is on the west of the Sun, which happens from her inferior to her superior conjunction with him, she rises in the morning before the Sun, and is then called the *morning star*: when she is to the east of the Sun, which is from her superior to her inferior conjunction, she sets after the Sun in the evening, and is then called the *evening star*.

The diameter of *Venus* is nearly equal to that of the Earth, being about 7687 miles; her apparent mean diameter, seen from the Earth, is 58 seconds; but her apparent diameter, seen from the Sun, or her horizontal parallax, is only 30 seconds: her distance from the Sun 68,000,000 miles; her eccentricity $\frac{1}{175}$ of her distance, or near 476,000 miles. The inclination of her orbit to the plane of the ecliptic is 3 degrees 23 minutes; the points of their intersections, or nodes, are in 14 degrees of Gemini and Sagittary. The place of her aphelion is 9 degrees 38 minutes of Aquarius. Her axis is inclined to her orbit in an angle of 75 degrees: her periodical course round the Sun is performed in 224 days 16 hours 49 minutes. The diurnal rotation round her axis is
not

not certainly known : Cassini makes it 23 hours, but others make it much more.

When Venus is observed through a good telescope, she is perceived to have various phases and changes, like those of the Moon ; her illuminated parts being constantly turned towards the Sun.

Dr. Herschel made many observations on this planet, between the years 1777 and 1793. The results of his observations are, that this planet has a revolution about its own axis, but the periodical time of which he was not able to ascertain ; that the position of its axis cannot certainly be known ; that the planet's atmosphere is very considerable ; the surface of the planet he also found to be diversified with hills and vallies, and other inequalities. But the atmosphere of the planet appeared so dense, or some other obstruction in the region of the planet prevented him, as he says, from having a particular view of the mountains, though assisted by the best instruments.

The transits of Venus happen but seldom. One of these transits was seen in England, in the year 1639, by Mr. Horrox and Mr. Crabtree : two more were seen in the present century ; the one, June the 6th, 1761 ; and the other in June 1769. There will not happen another till the year 1874.

In all other respects Venus has the same appearance to us regularly every eight years ; that is, her conjunctions, elongations, times of rising and setting, being very nearly the same, and on the same days.

Some astronomers have discovered, or imagined they have discovered, a satellite belonging to Venus : of this number was Cassini, who, with a telescope of 34 feet, in the years 1672 and 1686, thought he saw a satellite move round this planet, at the distance of about three fifths of Venus's diameters. And Mr. Short, in 1740, with a reflector of 16 $\frac{1}{2}$ inches focus, perceived a small star near Venus ; and with another telescope of the same focus, and magnifying to the

both power, he found its distance from Venus about 10 minutes; and with a glass magnifying to the 240th power, he observed the different phases of this satellite; and its diameter appeared to be near one third of that of Venus. And several other acute observers have imagined they discovered the same thing.

The *Earth*, the next primary planet in order from the Sun, is our habitation. It performs its revolution round the Sun, at the distance of 95,000,000 miles. Like all the other primary planets, it has both a diurnal and annual motion:—its diurnal motion is that by which it turns round its own axis in the space of 24 hours nearly, from west to east, and thereby causing the continual succession of day and night. Its annual motion is that by which it is carried round the Sun in its own orbit, and between the orbits of Venus and Mars, having the orbits of Venus and Mercury within its own orbit, or between it and the Sun, which is in the centre; and those of Mars, Jupiter, Saturn, the Georgian, &c. without, or above it, which are therefore called the superior planets, and Mercury and Venus the inferior ones. This annual motion is accomplished in the space of a year, or 365 days 6 hours, or rather 365 days, 5 hours, 48 minutes, 48 seconds,—this is called the tropical year. But the time the Earth takes to perform its annual revolution, from any fixed star to the same again, as seen from the Sun, is 365 days, 6 hours, 9 minutes, 17 seconds, which is the sidereal year. The figure of the Earth's orbit, as that of all the other planets, is elliptical; the eccentricity of the orbit, or distance of the Sun in the focus from the centre of the orbit, is about one sixtieth part of the mean distance of the Earth from the centre. The Earth, as well as all the other planets, performs his annual revolution according to the natural order of the signs.

By the diurnal rotation of the Earth on its axis the same appearance is produced as if it were fixed, and the sun and stars

stars moved round them every 24 hours. For their turning round from west to east, causes the Sun, and all the heavenly bodies, to appear to move the contrary way, or from east to west, as is imagined to be the case by the vulgar and illiterate. Thus, when, by the rotation of the Earth, the observer is brought to that part where he sees the Sun or a star just rising above the horizon, in the east, they are said to be rising; and as the Earth continues to move, other stars will appear to rise, and advance westward; and when, by the motion of the Earth, the observer is brought directly under the Sun or star, they are then said to be on the meridian; after which, by a continuance of the same motion, the observer is brought to the eastern side of the Earth, when the Sun or star will appear to set on the western side thereof.

The diameter of the Earth is $7957\frac{1}{2}$ miles; though some make it 7964 miles.

The Earth, throughout its annual orbit, always keeps its axis parallel to itself in every part of its orbit; thereby occasioning all the varieties of different seasons of the year, and the different length of the days and nights, as seen in *fig. 3*; the Sun enlightening more of the north polar parts, at one season of the year, and more of the southern parts at the opposite season of the year; thus producing the different degrees of heat in the different seasons; for in the summer season the heat of the Sun is increased by two causes, viz.—1. By the Sun's rays being more vertical to those situated in the north parts of the globe; and, consequently, the heat is not diminished in passing through so great a portion of the atmosphere, as when the rays come more oblique. And, 2dly, having the light, and consequently the heat of the Sun for a longer space of time in the summer than in the winter. All which will be evident from an inspection of the figure.

What is here asserted, concerning the Earth, holds good

with regard to all the other planets, as each of them revolves about an axis, which is not perpendicular to the plane of its orbit, but inclined thereto in a greater or less angle; and which axis is always parallel to itself in every part of the planet's orbit.

The figure of the Earth, like that of all the other planets, is that of an oblate spheroid, which Sir Isaac Newton demonstrated to arise from the rotation of the Earth about its axis: and by the observation and experiments of later astronomers, the polar diameter is to the equatorial diameter, as 178 to 179, as affirmed by that great genius.

The absolute gravity or density of the whole mass of the Earth is to that of the water as 9 is to 2, and to common stone, as 9 is to 5. Thus, we discover the very considerable mean density of the Earth, which is almost double to that of common stone; from whence it may be presumed, that the internal parts of the Earth contain great quantities of metals.

Having the density of the Earth, its quantity of matter is easily found; being always equal to the product of its density multiplied by its magnitude.

The Earth is, moreover, every where surrounded by an atmosphere, which is that large quantity of fluid matter extending over the whole surface of the Earth, consisting of air, aqueous and other vapours, electric fluid, &c. which surround the Earth to a considerable height, and partake of all its motions, both annual and diurnal.

The atmosphere serves for innumerable purposes, and is even essential to both animal and vegetable life; it is thin, insinuating itself into all the vacuities of bodies, which causes those mutations of generation, corruption, dissolution, &c.

This atmosphere, like all other matter, has the properties of weight and pressure, the quantity of which is now pretty well known, and is found by the barometer to be equal to an equal column of quicksilver, of 30 inches high; therefore, because a cubical inch of quicksilver is found to weigh
near

near half a pound avoirdupois, a column of quicksilver of 30 inches in height, whose base is one square inch, will weigh near 15 pounds; from whence it follows, that the weight of the atmosphere on every square inch of surface on the face of the Earth is also 15 pounds. Thus it appears, that the pressure upon the human body must be very considerable; for, as every square inch of surface sustains a pressure of 15 pounds, every square foot will sustain 144 times as much, or 2160 pounds.

The atmosphere has been proved, by late experiments, to become more rare, and of less density, the farther we remove from the Earth, and that in the following proportion:—At the height of three miles and a half from the Earth, the density of the atmosphere is nearly two times rarer than at the surface of the Earth: at the height of seven miles, four times rarer; at the height of 14 miles, 16 times rarer; at the height of 21 miles, 64 times rarer; at the height of 28 miles, 256 times rarer: and at the height of 35 miles, 1024 times rarer, &c.

Mars is the first of the four superior planets, and placed immediately next above the Earth, including the orbits of the Earth, Venus, and Mercury, within that of his.

The mean distance of Mars from the Sun is 1523 of those parts of which the distance of the Earth from the Sun is 1000, or upwards of 144 000,000 miles: his eccentricity 142 of those parts of which the Earth is distant from the Sun 1000: his mean diameter is 4189 miles. The length of his year, or the period of his completing one revolution round the Sun, is 686 $\frac{2}{3}$ days. His revolution round his own axis is performed in 24 hours 39 minutes and 22 seconds. His mean diameter, seen from the Sun, is 11 seconds. The inclination of his axis to his orbit is nothing, his axis being perpendicular. The inclination of his orbit to the ecliptic is 1 degree 51 minutes; the place of his aphelion 2 degrees 6 minutes 15 seconds of Virgo: his ascending node is 17 degrees 59 minutes of Taurus.

Dr. Herschel has made many observations on the rotation of this planet about its axis, from which he inferred that the mean diurnal rotation was between 24 hours 39 minutes 5 seconds, and 24 hours 39 minutes 22 seconds. He also observed several small remarkably bright spots, near both the poles, which had a sort of motion. He also concludes that the inclination of his axis to the ecliptic is 59 degrees 32 minutes: and the node of the axis to be in 17 degrees 47 minutes of Pisces. The obliquity of the planet's ecliptic 28 degrees 42 minutes, and the point Aries on Mars' ecliptic to answer to our 19 degrees 28 minutes of Sagittary.

The figure of Mars is that of an oblate spheroid, like the Earth, having his equatorial diameter to the polar one, as 1355 to 1272, or nearly as 16 to 15.

This planet also has a considerable, but moderate atmosphere; so that its inhabitants probably are, in their nature, similar to the inhabitants of this earth.

Mars always appears with a ruddy, disturbed light, occasioned by the nature of his atmosphere.

When he is in opposition to the Sun, that is, when the Earth comes between the Sun and him, he is nearly five times nearer to us than when in conjunction with the Sun, or when the Sun is between him and us; and consequently, in the former case, he appears near five times more large and bright than in the latter case.

As he receives his light from the Sun, like the other planets, he must necessarily have an increase and decrease apparently, like the Moon; he may also be observed sometimes almost bisected, when in the quadratures; but he is never seen cornicular, as the inferior planets are; which shows that his orbit includes that of the Earth, and that he shines with a borrowed light.

Between the orbits of Mars and Jupiter there have lately been discovered two new planets: viz. one named the *Pallas*, discovered March 23, 1802: its eccentricity is greater than that of Mercury; the inclination of its orbit to the ecliptic

33 degrees 39 minutes; its periodical revolution four years five months, and real diameter 95 miles. And the *Ceres de Ferdinand*, discovered by Mr. Piazzi, at Palermo, in Sicily, on January 1st, 1801: it is not apparently larger than a fixed star of the eighth magnitude; its elements, according to M. Gauss, are as follow,—inclination of its orbit to the ecliptic 10 degrees 36 minutes 57 seconds, ascending node $2^{\circ} 21'$, aphelion $10^{\circ} 26' 27'' 38''$, eccentricity of the orbit .082 parts of its mean distance, periodical revolution 1652 days 5 hours nearly; and real diameter, according to Dr. Herschel, 162 miles.

Jupiter is the next superior planet, and the largest of all the planets in the solar system; in brightness, he is next to the planet Venus: his orbit is situated between those of Mars and Saturn. His diameter is above 11 times that of the Earth; consequently his magnitude exceeds the magnitude of the Earth above 1300 times. His annual revolution round the Sun is performed in 4332 days 8 hours 51 minutes 30 seconds, or nearly 12 years. His diurnal revolution about his own axis, he performs in the short space of 9 hours 56 minutes, by which motion every part on his equator is carried round at the rate of 26,000 miles in an hour, being about 25 times faster than the equatorial parts of the Earth move.

The axis of Jupiter is nearly perpendicular to the plane of his orbit; therefore he has no sensible change of seasons, except very near the poles. This is wisely ordered by Providence. For if his axis made any considerable angle with the perpendicular of his orbit, just so many degrees as it was inclined thereto would be near six years in darkness round each of their poles, in their turn.

The orbit of Jupiter is inclined to the ecliptic in an angle of 1 degree 19 minutes 15 seconds: the place of his aphelion is 10 degrees 57 minutes 30 seconds of Libra: the place of his ascending node is 8 degrees 50 minutes of Cancer, and of his descending node 8 degrees 50 minutes of Capricorn.

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The eccentricity of his orbit is one twentieth part of his mean distance from the Sun.

There are several faint shining substances which surround Jupiter, of different dimensions, called his zones or belts, which are constantly changing their size and situations, and are, therefore, generally believed to be clouds. They have sometimes appeared of different breadths, at other times of the same breadth. Large spots have also been observed in these belts; and whenever a belt vanishes, as is often the case, the spots contiguous to it have also vanished. These belts have sometimes been interrupted and broken, and the broken ends of such belts have often been observed to revolve round the planet in the same time with the spots. The spots have also assumed different appearances; some of them changing their shape from a circular to an oblong form, others uniting together in one, and sometimes one large one dividing into two or three.

The difference between the equatorial and polar diameters of Jupiter is upwards of 6000 miles, the former being to the latter as 13 to 12.

Jupiter is attended by four moons, or satellites, some of them larger than our earth, which perform their respective revolutions round Jupiter in different periods of time; so that there is scarcely any part of this great planet but what is enlightened by one or more of these moons during the whole night. The periods, distances in semidiameters of Jupiter, and the angles of the orbits of these moons, seen from the Earth, are as follow:

Moons.	Periods round Jupiter.			Distances in Semidiameters of Jupiter.	Angles of the Orbits.
	D.	H.	M.		
1	1	18	27	$5\frac{1}{2}$	3' 55"
2	3	13	13	$9\frac{1}{8}$	6 14
3	7	3	42	$14\frac{1}{2}$	9 58
4	16	18	32	$25\frac{1}{4}$	17 30

The three nearest moons of Jupiter fall within his shadow, and are eclipsed once in every revolution; but the orbit of the fourth satellite is so much inclined, that it passes the shadow of Jupiter without falling into it two years in every six.

Saturn is the next primary planet, and the outermost from the Sun, except the Georgian. He shines with a feeble light, on account of his great distance from the Sun, which is also apparently increased by his great distance from us. This planet has attracted the most attention of all the primary planets, on account of his wonderful ring. This ring, or rather a double ring, one within the other, surrounds the body of Saturn, at a distance equal to the diameter of the planet. Beyond the ring, seven moons perform their respective revolutions round Saturn. The rings and the moons are all in the same plane, and are all dark dense bodies, and therefore cast their shadows upon each other.

The phenomena of the rings have engaged the attention of all the astronomers, since their discovery; some contending it was one entire ring, others dividing it into two or more; but the observations of Dr. Herschel have been more satisfactory on this head than those of his predecessors: he divides them into two rings, one within the other: their dimensions and spaces, he states in the following proportion to each other:—

	<i>Miles.</i>
Inside diameter of smaller ring	146,345
Outside diameter of ditto	184,393
Inner diameter of larger ring	190,248
Outside diameter of ditto	204,883
Breadth of the inner ring	20,000
Breadth of the outer ring	7,200
Breadth of the vacant space	2,839

This

This ring revolves in its own plane in 10 hours 38 minutes 15 seconds.

From the above statement it appears, that the outside diameter of the larger ring is almost 26 times the diameter of the Earth. This ring is inclined to the plane of the ecliptic in an angle of 30 degrees. When we see the ring most open, its shadow upon the planet is broadest; and from that time the shadow grows narrower as the ring appears to do to us, until, by the annual motion of Saturn, the Sun comes to the plane of the ring, or even with its edge; which, being then directed towards us, becomes invisible.

Saturn is found to have certain zones or belts, somewhat like those of Jupiter. Dr. Herschel has discovered and demonstrated that Saturn has a dense atmosphere; that he has a revolution about his axis; that his axis is perpendicular to the plane of his rings; that his figure, like that of the other planets, is that of an oblate spheroid, the polar diameter being to the equatorial as 10 to 11; that his rings have a revolution in their own plane, their axis being the same as that of Saturn.

The annual period of Saturn about the Sun is near 30 years, or 10,761 days 14 hours 36 minutes 45 seconds; his diameter is about 79,042 miles, being near ten times that of the Earth; his distance from the Sun is about $9\frac{1}{2}$ times that of the Earth.

From his great distance from the Sun, some have imagined that the portion of light and heat derived from the Sun is not sufficient for animal life. But that they have a greater portion of light, and consequently heat, than is at first imagined, is evident from the brightness of this planet and its satellites, in the night-time. Also, as the Sun's light to us is 45,000 times as great as that of the full moon, the Sun will afford 500 times as much light to Saturn, as
the

the full moon does to us ; and 1600 times as much to Jupiter. Thus, these two planets, without any moons to enlighten them, would receive more light from the Sun than might be at first imagined ; their number of satellites, the rings of Saturn, and the nature of their atmospheres, may also have a considerable effect in increasing their light and heat. For we find that, in our earth, the different degrees of heat do not entirely depend on the rays of the Sun. The inhabitants also of those planets are, no doubt, adapted to their situations.

Saturn has seven satellites, or moons, performing their revolutions round him in their respective periods, as follows :

Satellites.	Periods.				Distances in Semidiamet. of Saturn.	Distances in Miles.	Diameters of the Orbits.
	D.	H.	M.	S.			
1	1	21	18	27	$4\frac{1}{2}$	170,000	1' 27"
2	2	17	41	22	$5\frac{1}{2}$	217,000	1 52
3	4	12	25	12	8	303,000	2 36
4	15	22	41	13	18	704,000	6 18
5	79	7	48	0	54	2,050,000	17 4
6	1	8	53	9	$3\frac{5}{8}$	120,000	1 14
7	0	22	40	46	$2\frac{5}{8}$	91,000	0 57

The four first of these satellites describe ellipses, like those of the rings, and are also in the same plane. Their inclination to the ecliptic is from 30 to 31 degrees. The fifth satellite describes an orbit which is inclined from 17 to 18 degrees to the orbit of Saturn ; the plane of the orbit of this satellite lies between the plane of the ecliptic and the planes of the orbits of the other satellites. Dr. Herschel discovered that this satellite turns once round its own axis in the time that it makes one revolution about the planet Saturn ; in which respect it resembles our moon.

The *Georgium Sidus*, or *Herschel*, is the most distant of all the primary planets from the Sun ; its apparent diameter is

about four seconds. It is but seldom he can be seen by the naked eye ; but, having his situation, he may be plainly seen with a good telescope, in a clear night.

This planet is twice the distance from the Sun that Saturn is, and is nearly 83 years performing his annual course. It is 90 times as large as the Earth. The degree of cold in this planet is supposed to be extreme, as it is computed that the light of the Sun is not above the 300th part of what we enjoy on earth.

This planet is attended by six satellites, which perform their revolutions round him as follows :

Satellites.	Periods.			Distances in Semi-diameters of the Georgian.
	D.	H.	M.	
1	5	21	25	12 $\frac{1}{2}$
2	8	17	1	16 $\frac{1}{2}$
3	10	23	4	19
4	13	11	5	22
5	38	1	49	44
6	107	16	40	88

The orbits of these satellites are nearly perpendicular to the plane of the ecliptic ; and by the best observations that have been made, it is probable that their magnitudes are equal, if not greater, than that of Jupiter. The motion of these six satellites is retrograde, or contrary to the order of the signs.

These are the primary planets which constitute the solar system, each of them moving, in his own proper orbit, round the Sun, as the common centre. The Sun, therefore, can hardly be considered as a planet, though reckoned as such by the ancient astronomers ; but it may rather be ranked as a fixed star. He has, in every respect, the same properties, being a fixed luminous body, imparting light and

and heat to all the planets, both primary and secondary, found in his system. The reason he appears brighter and larger than any of the fixed stars, is his nearness to the Earth, in comparison with the great distance of the former. For a spectator, placed as near to any fixed star as we are to the Earth, would see that star, in every respect, as large and as bright as the Sun appears to us; and an observer, as far distant from the Sun as we are from the nearest fixed star, would see the Sun as small as the star appears to us, and would reckon it as one of the stars.

Though the Sun is said to be a fixed body, yet he has a revolution round his own axis, which he performs in the course of 27 days 12 hours and 20 minutes, which is found, by an observation of the several spots, to be seen on the Sun's disk, which pass from the western edge of his disk to the eastern edge thereof, in the space of less than 14 days. And these spots are found to perform one entire revolution round the Sun in the space of 27 days 12 hours and 20 minutes; therefore we reasonably suppose, that this is the Sun's proper motion from west to east, like that of all the planets.

Philosophers have been greatly divided in opinion concerning the matter of the Sun; some contending it was a ball of fire, from the property of the Sun's rays acting like fire, when collected by concave mirrors or convex lenses: others, as Boerhaave, maintain the contrary, the particulars of which arguments I have not room to insert; but the following properties of the Sun are demonstrated by Sir Isaac Newton:—
 1. That the density of the Sun's heat and light is seven times as great on the planet Mercury as it is with us.—
 2. That the quantity of matter in the Sun is to that in Jupiter nearly as 1100 is to 1; and that the distance of that planet from the Sun is in the same ratio to the Sun's semi-diameter.—
 3. That the quantity of matter in the Sun is to that in Saturn as 2360 to 1; and the distance of Saturn from

the Sun is in a ratio but little less than that of the Sun's semidiameter. And hence the common centre of gravity of the Sun and Jupiter is nearly in the superficies of the Sun; and that of the Sun and Saturn a little within it.—4. Hence the common centre of gravity of all the planets cannot be more than the length of the solar diameter distant from the centre of the Sun. This common centre of gravity is always at rest; and though the Sun, by the various positions of the planets, may be moved every way; yet it cannot recede far from this common centre.—5. The axis of the Sun is inclined to the ecliptic in an angle of 87 degrees 30 minutes nearly. The Sun's apparent diameter being sensibly larger in December than in June, the Sun must be proportionably nearer to the Earth in winter than in summer. In winter, the Earth will be in the perihelion; in the summer, in the aphelion. This is demonstrated by the Earth's motion being quicker in December than in June, by about a fifteenth part: for the Earth and every planet describe equal areas in equal times; thus, when it moves swifter, it must be nearer the Sun. From this we find that we have about eight days more in real time from the Sun's vernal equinox to the autumnal, than from the autumnal to the vernal.—6. The Sun's diameter is equal to 100 diameters of the Earth; therefore, the body of the Sun is 1,000,000 times greater than that of the Earth.—7. The apparent mean diameter of the Sun is 32 minutes 18 seconds. The Sun's horizontal parallax is now fixed at eight seconds, five twentieths of a second.—8. If 360 degrees (the whole ecliptic) be divided by the quantity of time in the solar year, it will give 59 minutes 8 seconds, &c. which is the mean quantity of the Sun's diurnal motion. And if these 59 minutes 8 seconds be divided by 24, the number of hours in a day, the quotient is 2 minutes 28 seconds, which is the Sun's motion in one hour; and which, divided by 60, will give his motion in one minute, &c. By this method,

method, the tables of the Sun's mean motion are constructed.

Though the planets above described perform their periods round the Sun, or rather round the centre of gravity, yet many of the planets seen from the Earth will appear to move in a contrary motion to the order of the signs; particularly the inferior planets; and sometimes they may appear stationary, or not to move at all, for several nights together. But these appearances are nothing but optical deceptions, arising partly from the motions of the planets, and partly from the motion of the Earth on which we are placed; for we always judge a planet to be in that part of the ecliptic which is on the opposite side of the planet to us; this is called its *Geocentric Longitude*: but the part of the ecliptic in which the planet is seen by an observer, supposed to be placed in the Sun, is called the *Heliocentric Longitude*. And the longitude of any planet or star is an arch of the ecliptic, counted from the beginning of Aries to the place where the ecliptic is cut by a circle perpendicular to the ecliptic, and passing through the star or planet.

SECT. II.

OF THE SECONDARY PLANETS.

THE secondary planets, or satellites, are certain planets which perform a revolution round any other planet, as the Moon does round the Earth. They are called satellites, because

asteroids that are always found attending their primary planet, and making the tour about the Sun together with them.

There are but four primary planets that are certainly known to have satellites, viz. the Earth, Jupiter, Saturn, and the Georgium sidus. Though some have imagined they have discovered satellites attending some of the other planets, as hath been hinted in the last section: but these observations have not been sufficiently confirmed.

The Earth is attended by one satellite, called the Moon, and marked γ . She performs her revolution round the Earth in an elliptic orbit, the mean eccentricity of which is one eighteenth part nearly of her mean distance from the Earth, or about 25,000 miles. Her mean distance from the Earth being 60, or diameters of the Earth; or about 240,000 miles.

The mean time of one revolution of the Moon about the Earth, or from one New Moon to another, when the overtimes the Sun is 12, is 29 days 12 hours 44 minutes 3 seconds 11 thirds. But the mean time in which she moves once round her whole orbit is 27 days 7 hours 43 minutes 8 seconds, which is at the rate of about 2200 miles in an hour. For the Moon has completed one revolution about the Earth, before she comes again in conjunction with the Sun; because while the Moon is performing her revolution, the Earth has advanced about a 13th part of the orbit for year.

The Moon turns once round her own axis exactly in the time that she goes round the Earth. This is the reason the face of the Moon is always turned towards the Earth; and day and night in the Moon, taken together, are just as long as a lunar month.

The diameter of the Moon is to that of the Earth, as 20 to 7; therefore it is equal to 2180 miles. The surface of the Moon is to that of the Earth as 3 is to 40, or as 1 to 13 $\frac{1}{3}$ nearly; therefore, the Earth reflects 13 times

as much light upon the Moon, as she does upon the Earth, when she is at her full. The solid content of the Moon is to that of the Earth as 3 is to 146: the density of the Moon's body is to that of the Earth as 5 is to 4; therefore, her quantity of matter is to that of the Earth as 1 is to 39 nearly. The force of gravity on her surface is to that on the Earth as 100 is to 293. The axis of the Moon is almost perpendicular to the plane of the ecliptic; therefore, she has little or no difference of seasons. The mean apparent diameter of the Moon is 31 minutes $16\frac{1}{2}$ seconds.

The various phases and appearances of the Moon have puzzled all the astronomers of antiquity. Her wanings and increasings, her various positions with regard to the Earth, and her frequent eclipses, were matters of constant admiration. The Moon being a dark spherical body, and shining only with the borrowed light of the Sun, can only have one half of her body illuminated at the same time, the opposite half remaining in its native darkness; therefore, as the Moon performs a revolution round the Earth, she will sometimes turn the whole of her illuminated face towards the Earth; at which time she appears perfectly round, and is a *full moon*: at other times only a certain portion of her illuminated face will be turned towards the Earth; she will then appear either horned, half round, or gibbous, according to the quantity of her illuminated part which is seen by us.

To illustrate this, let ABCDEFGH represent the orbit of the Moon, (*fig. 9, plate 17.*) Now, when the Moon is at A, in conjunction with the Sun, her dark side will be turned towards the Earth, and therefore she will be invisible, as at *a*, which is then called the New Moon. When she arrives at B, or has run through one eighth part of her orbit, one quarter of her illuminated face will be turned towards the Earth; she will then appear horned, as at *b*. When she arrives at C, one half of her illuminated face

face is turned towards the Earth, as at *c*, when she is said to be in her quadrature. When she arrives at *D*, which is called her second octant, three parts of her illuminated face will be turned towards the Earth, and she will appear gibbous, as at *d*. When she arrives at *E*, the whole of her illuminated face is turned towards the Earth, and she appears quite round, as at *e*, when she is said to be a full Moon. As she proceeds through the other half of her orbit, she decreases again from *e* to *a*, and nearly in the same ratio, as she increased, in the former half of her orbit. And the Earth has all the same appearances to an observer in the Moon, as the Moon has to us, but in a contrary order: viz. the Earth being at the full to them, when the Moon changes to us, and *vice versa*; as is evident from a view of the figure.

The motions of the Moon are all very irregular; the only equable motion she has, is the rotation on her own axis in the space of a month, being the time in which she moves round the Earth; which is the reason that she always exposes the same face towards the Earth.

The orbit of the Moon is very changeable, and does not long preserve the same figure; for though the orbit of the Moon be an ellipse, having the Earth in one of her foci thereof; yet the eccentricity is sometimes greater than at other times.

The plane of the Moon's orbit is inclined to that of the ecliptic, in an angle of five degrees.

The face of the Moon has the appearance, when viewed through a telescope, of being diversified with hills and valleys; this is also proved to be the case, from the edge or border of the Moon appearing jagged, especially about the line which separates the illuminated part of the Moon from the dark side thereof. The spots also of the Moon, which are taken for mountains, are found to cast a triangular shadow in the direction opposite to the Sun; and those
parts

parts which are taken for vallies or cavities are always dark on that side next the Sun, and illuminated on the opposite side, which is agreeable to experience. Sometimes the tops of the mountains are seen illuminated by the Sun, while their bases are in the dark side of the Moon; and by these means we have a good method of taking the height of the lunar mountains.

Thus, let ED (*fig. 14*) be the Moon's diameter, EC the line dividing the dark from the illuminated part of the Moon; and A the top of a hill in the dark part, just beginning to be illuminated: with a telescope, take the proportion of AE to the diameter ED , then there are given the two sides AE , EC , of the right-angled triangle AEC ; the squares of the two sides of the right-angled triangle being added together give the square of the hypotenuse AC , from the square root of which, subtracting BC , the radius, there remains AB , the height of the mountain.

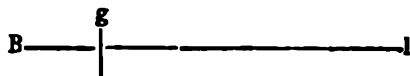
From late observations, Dr. Herschel has discovered that very few of the lunar mountains exceed half a mile in perpendicular height. The same gentleman has also observed three volcanoes in the Moon, which he thus describes: "I perceived (April 19th, 10 hours 36 minutes, sidereal time) three volcanoes in different parts of the dark part of the New Moon: two of them are either already nearly extinct, or otherwise in a state of going to break out; which, perhaps, may be decided next lunation: the third shews an actual eruption of fire, or luminous matter; its light is much brighter than the nucleus of the comet which Mr. Mechain discovered at Paris the 10th of this month." The following night he discovered it burn more violently; and by measuring, he found the shining or burning matter to be more than three miles in diameter. The actual fire or eruption of a volcano exactly resembled a small piece of burning charcoal, when it is covered by a very thin coat of white ashes; and it had a degree of brightness,

about as strong as that with which a coal would be seen to glow in faint daylight.

These are the chief phenomena observable in the Moon. All the other satellites are of a similar nature to this; but from their great distance from the Earth, we are unable to be so particular in our description of them.

Though it be asserted, that the Moon and the other secondary planets revolve round the primary planets as their centres, and the primary planets revolve round the Sun for their centre, yet it must be remembered, that this assertion is not the real mathematical truth. For the primary planets do not regard the Sun as their exact centre; but each primary planet, and the Sun, revolve round their common centre of gravity. Which common centre of gravity is that point where the two bodies, or the Sun and planet, will equiperpetuate each other. Thus, the centre of gravity in a common balance beam, or steel-yard, is the point of suspension.

To discover the common centre of gravity of two bodies, is to find that point, whose distance from the greater body is less than its distance from the least body, in the same proportion as the gravity of the less body is less than that of the greater; and in two bodies of equal gravities, their common centre of gravity is equally distant from their two centres.



Thus, if B be a body four times as great in magnitude as the body I, and both be supposed to be connected by an inflexible wire BI, the common centre of gravity of the two bodies will be at the point g, which is four times nearer to B than to I; or as B g is to g I, so is I to B.

Therefore, the common centre of gravity of the Earth and Sun is nearer to the centre of the Sun than to that
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of the Earth, by as great a ratio as the quantity of matter in the Sun exceeds that in the Earth, which centre of gravity is in the body of the Sun. The common centre of gravity of Jupiter and the Sun is also within the body of the Sun, though very near its superficies.

The Sun is not acted upon by our Earth only, so as to turn round the common centre of gravity of that and the Earth, without having regard to the other planets; but there is a common centre of gravity between the Sun and each of the primary planets; and each of these planets has its effect in causing the Sun to turn round their centres of gravity.

As the centre of gravity of Jupiter and the Sun is the farthest distant from the Sun's centre, owing to the great size of Jupiter, and its distance from the Sun, and as this centre is within the body of the Sun, it follows that the Sun is never removed above one of its own diameters out of its place.

Each of the secondary planets, and its primary planet also, turn round their common centre of gravity.

The figures 1, 2, 3, 4, 5, 6, 7, and 8, shew the true proportions of the planets Mercury, Venus, Mars, the Earth, Jupiter, Saturn, Georgium Sidus, and the Moon.

Jupiter has four satellites; the times of whose periods and distances have been noticed in the last Section. Their periods were found from their conjunction with Jupiter, after the same manner as the periods of the primary planets were discovered. Their distances from the body of Jupiter are measured by a micrometer, and computed in semidiameters of Jupiter, and then reduced into miles.

The satellites of Jupiter are of very great use in astronomy; for, by an observation of the eclipses of these satellites, we derive three great advantages:—1st, in determining the distance of Jupiter from the Earth; 2dly, we find the progressive motion of light: for by these eclipses, it is evident that light does not come to us from Jupiter

in an instant; for if the motion of light were instantaneous, it is evident we should see the commencement of the eclipses of the satellites at the same moment they really happen, whatever distance they might be from us; but, on the contrary, if light have a progressive motion, it is evident, the farther we are from a planet, the later we should be in seeing the beginning of its eclipse; and so it is found to happen: the satellites of Jupiter appear to be eclipsed later than the true computed time, and always proportionably later, as the Earth is removed farther from the planet. When the Earth and Jupiter are nearest to each other, that is, when they are both in conjunction, on the same side of the Sun, then the eclipses of Jupiter's satellites are seen to happen sooner than when the Sun is directly between Jupiter and the Earth; in which last case, the distance of Jupiter from the Earth is greater than it is in the former case, by the whole diameter of the Earth's annual orbit, or by double the Earth's distance from the Sun: in this last case, we cannot observe an eclipse of Jupiter's satellites, till near a quarter of an hour after the time we could have discovered it in the former case, that is, when Jupiter was at his least distance from the Earth. From hence it follows, that the light is near a quarter of an hour in passing through a space equal to the diameter of the Earth's orbit, or near eight minutes in passing from the Sun to the Earth; which is at the rate of about twelve million of miles in a minute.

But the third and greatest advantage derived from the observation of these eclipses, is the discovery of the longitude of different places on the Earth: for having the difference of time between two observations of the same eclipse, taken in two different places, we have the difference of longitude between the two places. For example, suppose there be two observers of an eclipse, the one at London, the other at Barbadoes, the eclipse will appear at the same moment of *real time* to each person; but being under different meridians, the hour of the day will be different at each place; thus, if it be 12

o'clock at noon at London, it will be 8 o'clock in the morning at Barbadoes, by which the observers find the difference of longitude between the two places to be sixty degrees, or four hours in time.

The planet Saturn has seven satellites, the sixth and seventh of which were discovered by Dr. Herschel, in the years 1787 and 1788. Their periods, distances, &c. have been described in the last Section.

The Georgian planet, or Herschel, is also found to have six satellites revolving round him, like those of Jupiter and Saturn. These satellites were discovered by Dr. Herschel, for which see the last Section.

These are the only primary planets which we are certain are attended by satellites. Some astronomers have imagined they discovered a satellite belonging to Venus; but the many repeated observations which have been made by others to observe it, and without effect, leave us room to suspect they were deceived.

S E C T. III.

OF THE FIXED STARS.

THE fixed stars, generally denominated *stars* by way of eminence, are those heavenly bodies which usually keep the same distance with regard to each other. All the heavenly bodies, except the primary and secondary planets, and comets, are of this class.

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The distances of the fixed stars are so great, that we have no distance in the planetary system with which we can compare them; for the diameter of the Earth's orbit, which is nearly 190,000,000 miles, bears no sensible proportion to the distance of the nearest fixed star from the Earth.

The distances of the fixed stars have been the subject of investigation to several astronomers. Various methods have been devised for the purpose of discovering the distances of these heavenly bodies, but without success, on account of their almost infinite distance; the most accurate observations only give us a distant approximation; but by the best observations, however, we can safely conclude that the nearest fixed star is upwards of forty thousand diameters of the Earth's orbit distant from us, or eighty thousand times farther distant from us than the Sun is.

The magnitudes of the fixed stars appear very different in different stars, owing, in some sort, to their real magnitudes, which are different, but principally owing to their different distances from us.

The stars are generally divided, according to their apparent magnitudes, into six, and by some into eight classes. The first class, called stars of the first magnitude, are those that appear largest: stars of the second magnitude appear somewhat less; and thus every following class comprehends those stars next in size to the former class: the stars of the sixth magnitude containing the smallest stars visible to the naked eye. All others that cannot be perceived, but by the help of telescopes, are called telescopic stars. It is not to be inferred from hence that Nature has divided the stars into those classes; for there are almost as many classes as there are stars, so great is their variety of magnitude and brightness!

The number of stars is also very great, and appears to be almost infinite; but astronomers have deduced all that are visible to the naked eye into catalogues. Mr. Flamsteed reduced 3000 stars into a catalogue, which contains all that are visible to the naked eye at any time of the year, and a
great

great number that are only visible through a telescope. The number of stars that are visible at one time, in the clearest heaven, seldom exceeds 1000; their appearing so much more numerous arises from their twinkling, and from viewing them confusedly, without reducing them into any order. But a good telescope will, nevertheless, bring great numbers to our view: and the more the magnifying power of the telescope is increased, the greater will be the number of stars discovered; till the number becomes so great as to baffle our computation.

From the great distance of the stars, we are at a loss to discover many of their properties; but from their phenomena we can with certainty deduce the following theorems concerning them:—1. That they are much greater than our earth; for, if that were not the case, they could not be visible at such a distance.—2. They are farther distant than the most distant of the planets; for we often find them hidden behind the body of the planets.—And, 3. They shine with their own natural light; for though they be much farther from the Sun than Saturn is, and appear much smaller to us, yet they shine much brighter than that planet. And it is known, that the more the telescope magnifies, the less is the angle under which the star is seen; because the telescope destroys all the adventitious rays. Thus a telescope magnifying 200 times will shew a star less in magnitude than it appears to the naked eye, insomuch, that it will appear to be only an indivisible point.

From hence we conclude, that the fixed stars are so many Suns; and that, in all probability, they are not much smaller than our Sun, but perhaps larger.

Therefore, it is generally believed that every star is the centre of a system, and has planets revolving round it in the same manner as the Earth and the other primary planets revolve round the Sun; for our Sun, together with the orbits of all the planets, would be almost invisible from the nearest fixed star.

To imagine that the stars are formed only to afford us a faint light, would be absurd, as we have incomparably more light from the Moon than from all the fixed stars taken together.

The fixed stars have two apparent motions; one called the first, common, or diurnal motion; the other called the second or proper motion. The former of these motions arises from the Earth's motion round its axis; by which the stars appear to be carried round the Earth, from east to west, in the space of 24 hours. The latter is that motion by which they appear to go backwards from west to east round the poles of the ecliptic with a very slow motion, describing only one degree of a circle in the space of 71½ years. This apparent motion is owing to the precession of the equinoxes; or, in other words, the axis of the Earth is directed to different parts of the heavens every year, describing a circle, one degree of which it describes in 71½ years.

The *Zodiac* is an imaginary ring or zone in the heavens, in the space of which all the primary planets revolve in their orbits: its breadth is made different by different astronomers, but is from eight to ten degrees on each side the ecliptic; and is divided into twelve parts, called the Twelve Signs of the Zodiac, and each sign is subdivided into 30 degrees; the degrees are each again divided into minutes, seconds, &c. But as the stars have a motion from west to east, these constellations, or signs of the Zodiac, do not now correspond to their proper signs, for the vernal equinox formerly happened when the Sun was in the first degree of Aries, and the Earth in the opposite degree of the Zodiac, or first degree of Libra; whereas now, the Sun has advanced a whole sign from that point at the Vernal Equinox.

The twelve signs of the Zodiac are distinguished by the following names and characters, viz. ♈ *Aries*, ♉ *Taurus*, ♊ *Gemini*, ♋ *Cancer*, ♌ *Leo*, ♍ *Virgo*, ♎ *Libra*, ♏ *Scorpio*,
♐

pis, † *Sagittarius*, † *Capricornus*, = *Aquarius*, † *Pisces*: or, according to the English names, the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Balance, the Scorpion, the Archer, the Goat, the Water-bearer, and the Fishes.

Besides these constellations in the Zodiac, the stars in every other part of the heavens are reduced into constellations of some certain figures, to which it is supposed each set of stars bears some resemblance. In the northern hemisphere are 21 constellations, of which the following are the names:—the Little Bear, Great Bear, the Dragon, Cepheus, Bootes, the Northern Crown, Hercules, the Harp, the Swan, Cassiopeia, Perseus, Auriga, Serpentry, the Serpent, the Arrow, the Eagle, the Dolphin, the Horse, Pegasus, Andromeda, and the Triangle. In the southern hemisphere are 15 constellations, viz. the Whale, Orion, the Eridanus, the Hare, the Great Dog, the Little Dog, the Ship, the Hydra, the Cup, the Raven, the Centaur, the Wolf, the Altar, the Southern Crown, and the Southern Fish.

This division was introduced by Ptolemy, and to these Bayer added 12 more, about the southern pole, viz. the Peacock, the Tucan, the Crane, the Phoenix, the Dorado, the Flying Fish, the Hydra, the Chamelion, the Bee, the Bird of Paradise, the Triangle, and the Indian.

To these, Mr. Royer has added 11 other constellations, viz. the Giraffe, the River Jordan, the River Tigris, the Sceptre, and the Flower-de-Luce, being on the north. The following six are on the south part, viz. the Dove, the Unicorn, the Cross, the Great Cloud, the Little Cloud, and the Rhomboid.

Hevelius also added the following new constellations, composed of some unformed stars, viz. the Unicorn, the Camelopardalis, the Sextant of Urania, the Dogs, the Little Lion, the Lynx, the Fox and Goose, Sobieski's Crown, the Lizard, the Little Triangle, and the Cerberus, to which Gregory has added the Ring, and the Armilla.

It must, however, be remarked, that some of these constellations introduced by Hevelius answer to some of those of Royer; as the Camelopardalis to the Giraffe, the Doge to the River Jordan, and the Fox to the River Tigris. The foregoing is the number of the constellations as they stand at present; but an attempt has lately been made by Dr. Hill, to introduce fourteen new ones; they are, however, not yet adopted by mathematicians.

Besides the stars in the foregoing constellations, there are a great number of stars not included in any constellation, and therefore called unformed stars.

The Galaxy, or Milky Way, is that long, white, luminous tract which seems to encompass the heavens, and is easily seen in a clear night, when the Moon is not up. It is of a considerable breadth, and in some parts double. Its luminous appearance is owing to the great number of small stars with which it is every where bespangled, and which by a good telescope may be plainly discovered.

Of Comets.

A comet is a wandering body, appearing suddenly, and as suddenly disappearing; and moves in its own proper orbit, like a planet.

It is usually furnished with a long train of light, called its tail, which is always opposite to the Sun. Comets are divided into three kinds: viz *bearded*, *tailed*, and *bairy* comets; which division arises from the different situation of the comet. Thus, when the comet is eastward of the Sun, and moves from him, it is said to be a bearded comet, because the light precedes it in the manner of a beard: when the comet is westward of the Sun, and sets after him, it is said to be a tailed comet, because the train of light follows it in the manner of a tail: and when the Sun and comet are in opposition to each other, the Earth being between them,

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the train of the comet is hid behind its body; except the extremities of the train, which being broader than the body of the comet, appear, as it were, round the edges of it like a border of hair, from which it is called a hairy comet.

The comets make a part of the solar system, and move in elliptical orbits, having the Sun in one of their foci. and describe areas proportional to the times of their motions, like the planets. The reason why they sometimes appear visible, and sometimes not, is the great eccentricity of their orbits, which is very considerable, for when they are in that part of the orbit most remote from the Sun, they are much beyond the orbit of Jupiter; and in their perihelion they frequently descend within the orbit of Mars, and sometimes within those of the inferior planets.

SECT. IV.

OF ECLIPSES.

An eclipse is the privation of the light of one of the luminaries by the interposition of some opaque body, either between the luminary and the eye, or between it and the Sun.

The *duration* of an eclipse is the time of its continuance.

The *immersion*, or incidence of an eclipse, is the moment when the eclipse begins; or when part of the luminary first begins to be obscured.

The *emersion*, or expurgation of an eclipse, is the time

when the eclipsed luminary begins to reappear, or emerge out of the shadow.

The *quantity*, of an eclipse, is the part of the luminary eclipsed. To determine this quantity, the diameter of the eclipsed body is divided into 12 equal parts, called digits; and the eclipse is said to be of so many digits as are contained in that part of the diameter which is eclipsed.

Eclipses are either those of the Sun, the Moon, or of some of the satellites, and are either *total*, *partial*, *annular*, *central*, &c.

A *total* eclipse, is when the whole body of the luminary is darkened.

A *partial* eclipse, is when only a part of the luminary is eclipsed.

A *central* eclipse, is when the centres of the two luminaries and the Earth come in a straight line, and is always total.

An *annular* eclipse, is when the whole body is eclipsed, except a ring or annulus, which appears round the border or edge.

An eclipse of the Moon, is a privation of the light of the Moon, and occasioned by the body of the Earth being directly between the Sun and the Moon, and so intercepting the Sun's rays, that they cannot arrive at the Moon; consequently the Moon passes through a part of the conical shadow of the Earth, as seen in *fig. 12, plate 17*, where *DEC* represents the Earth, and *DGFC* the conical shadow thereof, in which is the Moon in an eclipse. The dotted spaces *DGr*, and *FCr*, shew those parts of the shadow, called the *penumbra*, in which the Moon is deprived only of part of the Sun's light.

An eclipse of the Sun, is an obscuration of the Sun's body, occasioned by the Moon's coming between the Earth and the Sun, and thus intercepting the light of the Sun from us, on which account some have considered it an eclipse of the Earth.

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The solar eclipse is represented *fig. 11*, where m represents the Moon, CD the Earth, and $rmso$ the Moon's conical shadow, travelling over that part of the Earth $C\phi D$, and causing a complete eclipse of the Sun to all the inhabitants who reside in the tract CD . The spaces Cro and Dso include the penumbra, and all the inhabitants within those spaces will perceive a faint shadow of the eclipse.

Hence, an eclipse of the Moon can happen only at the time of the full Moon, or when she is opposite to the Sun; and an eclipse of the Sun will take place only at the time of a new Moon, or when the Moon is between the Sun and Earth.

From hence some may imagine that there may be two eclipses, viz. one of the Sun and another of the Moon, in every lunation, which would really be the case, if the Moon moved in the same plane with the ecliptic; but the orbit of the Moon not being in the plane of the ecliptic, but inclined thereto in an angle of 5 degrees 35 minutes, and passing through the plane of the ecliptic, it must necessarily follow, that an eclipse can only take place when the Moon is near that part of its orbit which passes through the plane of the ecliptic. These two opposite points where the Moon's orbit intersects the ecliptic, are called its Nodes. That point where the Moon ascends from the south to the north side of the ecliptic, is called the *ascending node*, or dragon's head, and marked Ω ; and the opposite point, where the Moon descends from the north to the south side of the ecliptic, is called the *descending node*, or dragon's tail, and marked \oslash ; and a line drawn from one node to the other, is called the line of the nodes. Thus, if (*fig. 13*) $abcd$ be the orbit of the Moon, and eg the ecliptic, the points ac , where the orbit cuts the ecliptic, are the two nodes, and the dotted line ac the line of the nodes. From a view of the figure, it is plain, when the full or new Moon happens when the Moon is at the points b or d , there
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can be no eclipse, the shadow of the Moon or Earth falling either above or below the other luminary; but when the full or new Moon is at the points *a* or *c*, or within 17 degrees of these points, there will be an eclipse of one of the luminaries.

In order to calculate an eclipse, it is necessary to know how to take the parallax of the Sun, or any heavenly body; as also to take the parallactic angle.

The parallactic angle, called also the *parallax*, is the angle EST (*Fig. 1, plate 18*), made at the centre of a star, or other bodies, by two lines, one drawn from the centre of the Earth T , and the other from its surface E ; or, which is the same thing, it is the difference of the two angles CBA and BTA .

Parallax is an arch of the heavens intercepted between the true and apparent place of any star, or heavenly body.

The true place of a star, S , is that point of the heavens, B , where it would be seen by an observer placed in the centre of the Earth T : and the apparent place of the same star, is the point C in the heavens, where it would appear to an observer on the surface of the Earth, at E . This difference of the two places of the same star is the parallax, sometimes called, for distinction sake, the parallax of altitude; and is an angle formed by two visual rays, the one drawn from the centre, and the other from the circumference, of the Earth, and traversing the body of the star; the measure of it being an arch of a great circle, intercepted between the points of the true and apparent places, B and C .

The parallax BC is also the difference between the true distance of the star from the zenith A , and the apparent distance AC . Hence the parallax diminishes the altitude of a star, or increases its distance from the zenith.

The parallax is greatest in the horizon, which is therefore called the horizontal parallax, as $BF T$. From the horizon the parallax decreases all the way to the zenith A , where the true and apparent places of the star coincide.

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The *parallax of the annual orbit of the Earth*, is the angle under which the semidiameter of the Earth's orbit is seen.

To find the parallax of a celestial body, observe when the body is in the same vertical line with a fixed star which is near it; and while it is in that position, measure its apparent distance from the star; then observe when the star and body are at equal altitudes from the horizon, and there measure their distances again, and the difference of these distances will be the parallax.

The Astronomy of Eclipses.

To calculate a lunar eclipse it is necessary first, to find the length of the Earth's conical shadow, which may be found by finding the distance between the Earth and Sun, and the proportion of their diameters. Thus, suppose the semiaxis of the Earth's orbit to be 95,000,000 miles, and the eccentricity of the orbit 1,377,000, which, added together, make 96,377,000 miles, or 24,194 semidiameters of the Earth; and the Sun's semidiameter being to that of the Earth as 112 to 1; then, as AD is to BE, so is DB to EC (*fig. 2*), that is, as 111 is to 1, so is 24194 to 218 semidiameters of the Earth, equal to EC, the length of the Earth's shadow.

To find the apparent semidiameter of the Earth's shadow, in the place where the Moon passes through it, add together the parallaxes of the Sun and Moon, and from the sum subtract the apparent semidiameter of the Sun, and the remainder will be the apparent semidiameter of the shadow at the place where the Moon passes through it.

Note. The Sun's parallax may very well be omitted in this calculation; and the apparent semidiameter of the shadow increased by adding one whole minute.

It is also necessary to have the true distances of the Moon from the node at the mean opposition; also the true time of the opposition, with the true place of the Sun and Moon reduced

reduced to the ecliptic, and the Moon's true latitude at the time of the true opposition; likewise the angles of the Moon's way with the ecliptic, and the true horary motions of the Sun and Moon; from which every particular concerning the eclipse may be computed by common arithmetic and trigonometry.

The method of constructing an eclipse of the Moon is as follows:—Let *EW* (*fig. 3*) represent a part of the ecliptic, *C* the centre of the transverse section of the Earth's shadow. Draw the line *CN* perpendicular to the ecliptic, and towards the north, if the Moon have north latitude; but if she have south latitude, draw a line *CS*. Make the angle *NCD* equal to 5 degrees 35 minutes, which is the angle the Moon's orbit makes with the ecliptic. Bisect this angle by the right line *CF*, and in this line the true equal time of opposition between the Sun and Moon falls by the tables.

Take the Moon's latitude at the true time of full Moon from a scale of equal parts, which is supposed to represent minutes of a degree, and set this distance from *C* to *G* on the line *CF*. Through the point *G* draw a line *HI* at right angles to *CD*, which line represents a portion of the Moon's orbit. Then *L* is the point where the Moon's centre is at the middle of the eclipse; *G* the place of her centre at the tabular time of her being full; and *K* the point of her centre at the instant of her ecliptic opposition; *I* is the Moon's centre at the moment of her immersion, and *H* her centre at the end of the eclipse, or emerſion.

From the same scale take the Moon's ſemidiameter, and deſcribe three circles on the points *IGH*, which represent the Moon in the beginning, middle, and end of the eclipse.

Then, to find the length of time or the duration of the eclipse, meaſure the line *IH* on the ſame ſcale, as ſay, as the Moon's horary motion from the Sun is to *HI*, ſo is one hour, or 60 minutes, to the whole duration of the eclipse.

From the above figure the eclipses may be computed. For, first, in the right-angled triangle CGL , right-angled at L , there is given the hypotenuse CG , which is the Moon's latitude at the time of the full Moon; also we have the angle GCL equal to the half of 5 degrees 35 minutes, wherefore the legs CL and GL may be found. Secondly, in the right-angled triangle CLH , or CLI , are given the legs CL , and CH or CI , which latter is the sum of the semidiameter of the Moon and the Earth's shadow; therefore LH or LI may be found, which is equal to half the difference of the motions of the Sun and Moon during the eclipse. Thirdly, as the difference of the horary motions of the luminaries is to one hour, or sixty minutes, so is HL to the semiduration of the eclipse; and so is GL to the difference between the opposition and middle of the eclipse. This last, therefore, taken from the time of full Moon, gives the time of the middle of the eclipse; from which, subtracting the time in LI , before found, gives the beginning of the eclipse; and, added to the same, gives the end of the eclipse.

Lastly, from CO , the semidiameter of the Earth's shadow, subtract CL , and the remainder LO , added to LP , gives OP , the quantity eclipsed.

A solar eclipse, or an eclipse of the Sun, would be discovered in the same manner as a lunar eclipse, if the Moon had no sensible parallax; but as the Moon has a parallax, the method is somewhat different.

1. Add the apparent semidiameters of both Sun and Moon together, both in the aphelion and perihelion, which gives 33 minutes 6 seconds for the greatest sum, and 30 minutes 31 seconds for the least sum.

2. As the parallax diminishes the northern latitude and increases the southern, let the greatest parallax in the latitude be added to the former sums, and also subtracted from them: thus may be had, in each case, the true latitude, beyond which there can be no eclipse. Having this latitude, the

Moon's distance from the nodes is found in the same manner as for the lunar eclipse.

To find the digits eclipsed, add the apparent semidiameters of the luminaries together, from which subtract the Moon's apparent latitude; the remainder is the part of the diameter eclipsed. Then say, as the semidiameter of the Sun is to the scruples eclipsed, so are six digits reduced into scruples (that is, 360 scruples or minutes) to the digits eclipsed.

To determine the duration of a solar eclipse, find the horary motion of the Moon from the Sun for an hour before the conjunction, and for one hour after it. Then say, as the former horary motion is to the seconds in an hour, so are the scruples of half the duration to the time of immersion; and as the latter horary motion is to the same number of seconds, so are the scruples of half the duration, to the time of emerſion. Then adding the times of immersion and emerſion together, the ſum is the whole duration.

To find the beginning, middle, and end of a solar eclipse, find the arch GL from the Moon's latitude, for the time of conjunction. Then say, as the horary motion of the Moon from the Sun before the conjunction is to one hour, so is the distance of the greatest darkness to the interval of time between the greatest darkness and the conjunction. Subtract this interval in the first and third quarter of the anomaly from the time of the conjunction, but in the other quarters add it to the same, and the result is the time of the greatest darkness. Lastly, from the time of the greatest darkness, subtract the time of incidence, to which is to be added the time of emerſion; the difference in the first case will be the beginning, and the ſum in the latter case will be the end of the eclipse.

To calculate eclipses of the Sun, it is necessary, first, to find the mean new Moon, and from thence the true one, with the place of the luminaries for the apparent time of
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the true new Moon.—2. For the apparent time of the true new Moon, compute the apparent time of the new Moon observed.—3. For the apparent time of the new Moon seen, compute the latitude seen.—4. Thence determine the number of digits eclipsed.—5. Find the times of the greatest darkness, emergence, and immersion.—6. And from thence determine the beginning and ending of the eclipse.

From this it is evident that the trouble in the calculation of eclipses arises from the parallaxes of longitude and latitude, without which the calculation of solar eclipses would be the same as those of lunar ones.

S E C T. V.

OF TIME.

TIME is a mode of duration marked by certain periods, measures, and motions; and the chief method we have of measuring time is by the revolution of the two luminaries, the Sun and Moon, but particularly that of the Sun.

Mr. Locke observes, that the idea we have of time is acquired by considering any part of infinite duration, as set out by periodical measures. The idea of any particular time, or length of time, as a day, an hour, &c. is acquired by observing certain appearances of some bodies, moving

with a regular motion, and at regular and seeming equidistant periods. Now, by being able to repeat these lengths or measures of time as often as we please, we can imagine duration where nothing really endures or exists, and thus we imagine to-morrow, or next year, &c.

Time is also the duration of a thing which has both a beginning and an end, and in this sense it is distinguished from eternity.

Thus, time is the duration of some motion; for without some regular and uniform motion we should have no methods to compute time, or distinguish it from eternity.

Time may be divided into *absolute* and *relative*.

Absolute time is time considered in itself, and without any relation to motion.

Relative or apparent time is the sensible measure of any duration, by means of motion, as, by the motions of the luminaries, the hands of a clock, watch, &c.

Relative time is subdivided into *astronomical* and *civil*.

Astronomical time is that which is measured by the motions of the heavenly bodies.

Civil time is formed for civil purposes, and distinguished into years, months, days, hours, &c.

The year, in the full extent of the word, is a system of several months, or a space of time measured by the revolution of some celestial body in its orbit. Thus, the time in which the fixed stars make one revolution is called the great year; and the times in which Jupiter, Saturn, &c. complete their revolutions, and return to the same point again, are respectively called the years of Jupiter, Saturn, &c. For a year originally denoted a revolution, and is not limited to that of the Sun; therefore we find some ancient nations at different times called the revolutions of the Moon, or the space of a month, a year; which occasions such strange accounts in the chronology of some very ancient nations, as in those of the Egyptians, Babylonians, &c.

The

The solar year, called also *year* by way of eminence, is that space of time in which the Sun moves through the twelve signs of the Zodiac. This year, by the best observations, is found to contain 365 days 5 hours 48 minutes 48 seconds; but the quantity, according to the authors of the Gregorian calendar, is 365 days 5 hours 49 minutes. In the civil account, this year is said to contain only 365 days; and one day is added to every fourth year, to make up for the odd hours, which is therefore called *leap year*.

The Julian year, so named from Julius Cæsar, who established it, consists of 365 days 6 hours, which exceeds the true solar year by upwards of 11 minutes, which excess amounts to a whole day in near 131 years. And one day is added to the end of February every fourth year, which is composed of the odd six hours every year. This year is, therefore, called *bissextile*, or *leap year*.

The Gregorian year, introduced by Pope Gregory XIII. in 1582, is the Julian year corrected by this rule, viz. That instead of every hundredth year being a leap year, as it would be in the Julian calendar, in this way, only one hundredth year out of four is a leap year, the other three being common years. By this omission of three days in every 400 years, the civil year would nearly keep pace with the solar year for time to come.

Yet this year is not quite perfect, for, as in four centuries the Julian year gains 3 days 2 hours 40 minutes, and as there are only three days omitted in the Gregorian account, there is still an excess of 2 hours 40 minutes in 400 years, which amounts to a whole day in 3600 years.

In the year 1752 this style was adopted in England, and the eleven days were thrown out after the 2d of September, by accounting the 3d the 14th of that month. This was called the *New Style*, in distinction from the former, which was called the *Old*.

The

The solar year is either astronomical or civil. The astronomical solar year is that which is determined precisely by astronomical observations, and is of two kinds, viz. *tropical* and *fidereal* or *astral*.

The tropical or natural year is the time the Sun takes to pass through the twelve signs of the Zodiac, and is the only proper natural solar year, because the seasons always fall in the same months.—The fidereal or astral year is the space of time the Sun takes in passing from any fixed star till his return to the same again, and is 20 minutes 29 seconds longer than the true solar year.

The lunar year is the space of twelve lunar months, and is either astronomical or civil.

The lunar astronomical year consists of twelve lunar synodical months; and is, therefore, 354 days 8 hours 48 minutes 38 seconds, being 10 days 21 hours 10 seconds shorter than the solar year.

The lunar civil year is either common or embolismic. The common lunar year consists of twelve lunar civil months, and contains 354 days. The embolismic lunar year consists of thirteen lunar civil months, and contains 384 days.

The civil or legal year, in England, formerly began on the 25th of March, or the day of the Annunciation of the Virgin Mary: but the historical year begun on the 1st of January. The part of the year between these two terms was usually expressed thus, 1735-6, or 173 $\frac{1}{2}$. But according to the new style, the civil year now begins on the 1st of January.

The ancient Roman year, as first settled by the Romans, contained only ten months, and in all 304 days.

The Egyptian year, called also the year of Nabonassar, from the epoch of that name, contains only 365 days, divided into twelve months of 30 days each, with five intercalary days added at the end. Thus the Egyptian year loses a whole day of the Julian year every four years, and after the

the space of 1460 years, it begins with the Julian year, which length of time is called the Sothic Period.

The ancient Greek year consisted of twelve months, which at first were divided into 30 days each, but afterwards each month contained 29 and 30 days alternately; and this year was computed from the first appearance of the new Moon, with the addition of an embolismic month of 30 days, every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year, in order to keep the new and full Moons to the same seasons of the year.

The ancient Jewish year consisted of twelve months, containing 29 and 30 days alternately. To which were added eleven or twelve days to make it agree with the solar year.

The Syrian year was the same in quantity as the Julian year, but commenced in the beginning of October, according to the Julian year.

The Persian year contained twelve months, of 30 days each, with five intercalary days added.

The Arabic, Mahometan, or Turkish year, called also the year of the Hegira, consists of 354 days 8 hours 48 minutes, divided into twelve months, containing 29 and 30 days alternately; though sometimes it contains 13 months; and intercalary days also added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21st, 24th, 26th, and 29th year. The months commence with the first appearance of the new Moon.

The year is divided into twelve parts, called months, from the Moon, by whose motions it was regulated: the month is properly the time in which the Moon passes through the Zodiac, and is of several kinds: as,

1. The *illuminative month*, which is the interval between the appearance of one new Moon and that of the next; and always varies in quantity. This month is used by the Turks and Arabs.

2. The *lunar periodical month*, or the exact time in which the Moon runs through the Zodiac, and consists of 27 days 7 hours 43 minutes and 8 seconds.

3. The

3. The *lunar synodical month*, called a lunation, is the time between two new Moons, as seen from the Earth, and consists of 29 days 12 hours 44 minutes 3 seconds and 11 thirds.

4. The *solar month* is the time the Sun runs through one sign of the ecliptic, and consists at a mean rate of 30 days 10 hours 29 minutes 5 seconds.

5. The *civil* or *common month* is an interval of a certain number of whole days—such are the calendar months.

6. The *civil lunar month* consists alternately of 29 and 30 days. Thus two of these months are equal to two astronomical months, and the new Moon will be kept to the first day of such civil months for a long time together. This month was in most common use till the time of Julius Cæsar.

7. The *civil solar month*, which consisted alternately of 30 and 31 days, excepting one month which had 29 days, introduced by Julius Cæsar. But under Augustus, the sixth month, till then called Sextilis, received the name of Augustus, from thence called August; and one day more was added to it, which was taken from February. This is the regular civil month in use in England.

A week is a space of time containing seven days, and originated from the division of the lunar month into four parts.

The division of the month into weeks was used by the Syrians, Egyptians, and most of the Oriental nations. The Roman week consisted of nine days, and the ancient Greeks used decades, or a system of ten days.

But the Jews used the week of seven days. The days of the week they denominated, the first, second, third, fourth, fifth; and the sixth day they called the preparation of the Sabbath, the Sabbath being the seventh day: and this division is still observed by the Christians, Arabs, Persians, Ethiopians, &c.

The

The ancient heathens denominated the days of the week from the seven planets, calling each day after that planet which they supposed governed the first hour of the day. Thus, the first day was called *Dies Solis*, or Sunday, from the Sun; the second, *Dies Lunæ*, or Monday, from the Moon, &c.

But our Saxon ancestors, before their conversion to Christianity, named the days of the week from the Sun and Moon, and also from some of their deified heroes, to whom they were peculiarly consecrated, which names we still retain. Thus, Sunday was dedicated to the Sun; Monday to the Moon; Tuesday to *Tuisco*, or Mars; Wednesday to Woden, or Mercury; Thursday to Thor, the Thunderer, or Jupiter; Friday to *Friga*, or *Friya*, the wife of Thor, or Venus; and Saturday to *Seater*, or Saturn. And the days of the week are often expressed by modern astronomers by the characters of the planets, as, \odot for Sunday, and ♄ for Monday, &c.

A *day* is that space of time which arises from the appearance or disappearance of the Sun, and is either natural or artificial.

The *natural* day is the portion of time in which the Sun apparently performs one revolution round the Earth; that is, the time in which the Earth makes a rotation on its own axis.

The *artificial* day is the time from Sun rising to Sun setting.

The *natural* day is either astronomical or civil.

The *astronomical* day begins at noon, or when the Sun's centre is on the meridian, and contains 24 hours to the following noon.

The *civil* day is the time allotted for the space of a day in civil purposes, and includes one entire rotation of the Earth on its axis. This day begins at different times in different nations: at Sun rising among the ancient Babylonians, Persians, Syrians, and most other Eastern

nations, and the present inhabitants of the Balearic Islands, the Greeks, &c. It began at Sun setting among the ancient Athenians and Jews; it is also used by the Austrians, Bohemians, Marcomanni, Silesians, modern Italians, and Chinese. With all modern astronomers, and the ancient Umbri and Arabians, the day is begun at noon: and at midnight among the ancient Egyptians, Romans, and with the modern English, French, Dutch, Germans, Spaniards, and Portuguese.

An *hour* is the twenty-fourth part, but sometimes only the twelfth part of a day.

There are various kinds of hours, as, 1. Equal hours, which are the twenty-fourth part of a natural day. They are called equinoctial hours, because they are measured on the equinoctial; and astronomical, because used by astronomers. 2. Babylonish hours, of which there are 24 equal ones in the day, and reckoned from Sun rising. 3. European hours, used in civil computations, and are reckoned from midnight; 12 hours from thence till noon, and 12 more from noon to midnight. 4. Jewish, or planetary, or ancient hours, which are the twelfth parts of the artificial day, and the same parts of the artificial night. They are called ancient or Jewish hours, because used by the ancients, and still used by the Jews: they are called planetary hours, because ancient astrologers pretended that a new planet presided over every hour. 5. Italian hours, of which there are 24 equal ones to a day, reckoning from sunset.

The hour is divided into 60 minutes; and each minute into 60 seconds; each second into 60 thirds, &c.

As time, for the purposes of chronology, is calculated by years, it is necessary to have some certain fixed point of time from which calculations can be made with certainty, which fixed point of time is called an *epoch*, or epoch.

Different nations use different epochs or æras; the Christians chiefly use that of the nativity of Jesus Christ; the Mahometans, that of the Hegira, or flight of Mahomet;

the Jews, that of the creation of the world, or that of the Deluge; the ancient Greeks, that of the Olympiads; the Romans, that of the building of Rome; the ancient Persians and Assyrians, that of Nabonassar, &c.

The doctrine and use of epochs is of great importance in chronology. And to find what year of one epoch corresponds with that of another, a period of years has been invented, which commenced before all the epochs, and is a common standard of them all, and called the Julian period. To this period all the epochs are reduced; that is, the year of this period when each epoch commences is determined. Thus, adding the given year of one epoch to the year of the period corresponding with its beginning, and from the sum subtracting the year of the same period corresponding to the other epoch, the remainder is the year of that other epoch.

A TABLE

OF THE

Years of the most remarkable Epochs.

N. B. The years before Christ, are those before the reputed year of his birth, and not reckoned back from the first year of his age, as is usually done.				Julian Period	Years of the World.	Years before Christ.
Creation of the World	—	—	—	706	0	4007
The Deluge, or Noah's Flood	—	—	—	2362	1656	2351
Affyrian Monarchy, founded by Nimrod	—	—	—	2537	1831	2176
Kingdom of Athens, founded by Cecrops	—	—	—	3157	2451	1556
Entrance of the Israelites into Canaan	—	—	—	3262	2556	1451
The Destruction of Troy	—	—	—	3529	2823	1184
Solomon's Temple founded	—	—	—	3701	2995	1012
The Arconautic Expedition	—	—	—	3776	3070	937
Lycurgus formed his Laws	—	—	—	3829	3123	884
Arbaces, first King of the Medes	—	—	—	3838	3132	875
Olympiads of the Greeks began	—	—	—	3938	3232	775
The building of Rome	—	—	—	3961	3255	752
Era of Nabonassar	—	—	—	3967	3261	746
First Babylonish Captivity, by Nebuchadnezzar	—	—	—	4107	3401	606
The Second Babylonish Captivity, and Birth of Cyrus	—	—	—	4114	3408	599
Solomon's Temple destroyed	—	—	—	4125	3419	588
Cyrus began to reign in Babylon	—	—	—	4177	3471	536
Peloponnesian War began	—	—	—	4282	3576	431
Alexander the Great died	—	—	—	4390	3684	323
Captivity of 100,000 Jews, by Ptolemy	—	—	—	4393	3687	320
Archimedes killed at Syracuse	—	—	—	4506	3800	207
Julius Cæsar invaded Britain	—	—	—	4659	3953	54
He corrected the Calendar	—	—	—	4667	3961	46
The true Year of Christ's Birth	—	—	—	4709	4003	4

<i>The Christian Æra.</i>			<i>Julian Period</i>	<i>Years of the World.</i>	<i>Years since Christ.</i>
Dionysian, or vulgar Æra of Christ's Birth	—	—	4713	4007	0
Christ crucified, Friday, April 3d	—	—	4746	4040	33
Jerusalem destroyed	—	—	4783	4077	70
Adrian's Wall built in Britain	—	—	4833	4127	120
Dioclesian Epoch of Martyrs	—	—	4997	4291	284
The Council of Nice	—	—	5038	4332	325
Constantine the Great died	—	—	5050	4344	337
The Saxons invited into Britain	—	—	5158	4452	445
Hegira, or Flight of Mahomet	—	—	5335	4629	622
The Death of Mahomet	—	—	5343	4637	630
The Persian Yefdegird	—	—	5344	4638	631
The Moon, and all the Primary Planets, seen in the					
Sign Libra, from the Earth	—	—	5899	5193	1186
Art of Printing discovered	—	—	6153	5447	1440
The Reformation begun by Martin Luther	—	—	6230	5524	1517
The Calendar corrected by Pope Gregory	—	—	6295	5589	1582
Sir Isaac Newton born, December 25th	—	—	6355	5649	1642
Oliver Cromwell died	—	—	6371	5665	1658
Sir I. Newton made President of the Royal Society	—	—	6416	5710	1703
Died March 20th	—	—	6440	5734	1727
The New Planet discovered by Dr. Herschel	—	—	6494	5788	1781
The Ceres de Ferdinand discovered by M. Piazzi	—	—	6514	5808	1801

S E C T. VI.

OF ASTRONOMICAL PROBLEMS, AND THE USE OF
THE GLOBES.

THE celestial globe differs from the terrestrial one in having the images of the constellations and figures of the stars upon it, instead of the several parts of the Earth. The meridian circle drawn through the poles and the point Cancer represents the solstitial colure; and that meridian, drawn through the point Aries, represents the equinoctial colure.

P R O B L E M I.

To exhibit a true Representation of the Face of the Heavens for any given Time and Place.

Rectify the globe for the latitude of the place (as taught in Geography), placing the north pole of the globe towards the north pole of the world. Having found the Sun's place in the ecliptic, and brought it to the meridian, set the index to 12 o'clock at noon; then turn the globe on its axis till the index points to the given hour. In this position the globe exactly represents the face of the heavens as it appears at that time; every constellation and star in the heavens corresponding in situation to those on the globe.

P R O B L E M II.

To find the Declination and right Ascension of any Star.

Bring the star to the brazen meridian; and the number of degrees on the meridian between the equator and the star is its declination; and the degree of the equator, cut by the meridian, is the right ascension of the star. Thus the right
ascension



ascension of any star is an arch of the equator, intercepted between the first degree of Aries, and that point where the meridian or circle passing through the star cuts the equator.

PROBLEM III.

To find the Latitude and Longitude of any Star.

Bring the solstitial colure to the meridian, and then fix the quadrant of altitude over the pole of the ecliptic, in the same hemisphere with the star, and bring its graduated edge to the star; then the degree on the quadrant, cut by the star, is its latitude, counted from the ecliptic; and the degree of the ecliptic, cut by the quadrant, is the star's longitude.

PROBLEM IV.

To find the Place of any Star or heavenly Body, having its Declination and right Ascension.

Find the point of right ascension on the equinoctial by Problem II. and bring it to the meridian; then count the degrees of declination upon the meridian from the equinoctial, and there make a mark upon the globe, which will be the place of the star, &c.

PROBLEM V.

To find the Place of a Star, Planet, Comet, &c. having the Latitude and Longitude.

Bring the pole of the ecliptic to the meridian, and there fix the quadrant of altitude, which turn round till its edge cut the given longitude on the ecliptic; then count the given latitude from the ecliptic upon the quadrant of altitude, and there make a mark upon the globe, which will be the place of the star, planet, &c. The place of any star, planet,

net,

net, &c. being found by this or the foregoing Problem, its rising, setting, or any other circumstance concerning it, may be found by the proper Problems, as those of the Sun are found.

PROBLEM VI.

To find the rising, setting, and culminating of a Star, or any celestial Body, and consequently its Continuance above the Horizon for any Place and Day; also its oblique Ascension and Descension, with its eastern and western Amplitude and Azimuth.

Adjust the globe to the state of the heavens at 12 o'clock at noon on the given day; bring the star, &c. to the eastern side of the horizon, which will give its eastern amplitude and azimuth, and the time of rising; as for the Sun. Again, turn the globe till the same star comes to the western side of the horizon; so will the western amplitude and azimuth, with the time of setting, be found. Then the time of rising subtracted from that of setting, leaves the continuance of the star above the horizon; which, subtracted from 24 hours, leaves the time it is below the horizon. Lastly, bring the star to the meridian, and the hour to which the index then points is the time of its culminating, or southing.

PROBLEM VII.

To find the Altitude of a Star, &c. for any given Hour.

Adjust the globe to the positions of the heavens, and turn it till the index point to the given hour; then fix the quadrant of altitude at 90 degrees from the horizon, and turn it to the place of the star: then the degrees of the quadrant intercepted between the horizon and the star will be the altitude sought.

PROBLEM

PROBLEM VIII.

Having the Altitude of a Star by Night, or the Altitude of the Sun by Day, to find the Hour of the Day or Night.

Rectify the globe as in the foregoing Problem, and turn the globe and quadrant till that degree of the ecliptic where the Sun is, or the star itself, cut the quadrant in the given degree of altitude; then the index will point to the hour required.

PROBLEM IX.

Having the Azimuth of a Star, or the Sun, to find the Time of the Night or Day.

Rectify the globe as before, and bring the quadrant to the given azimuth in the horizon; then turn the globe till the star or Sun come to the quadrant, and the index will then shew the hour of the night or day.

Mars.	Pallas.	Ceres de Ferdinand.	Jupiter.	Saturn.	Herschel, or Georgian
7° 24'	*	*	11° 51'	6° 29'	3° 44'
<i>b. m.</i> 23 30 $\frac{3}{4}$	<i>d.</i> 1594	<i>d. b.</i> 1652 5	<i>d. b. m.</i> 4332 8 51 $\frac{1}{2}$	<i>d. b. m.</i> 10761 14 36 $\frac{3}{4}$	<i>d. b.</i> 30445 18
<i>m. s.</i> 39 22	*	*	<i>b. m.</i> 9 56	*	*
° 51'	33° 39'	10° 36' 57"	1° 19 $\frac{1}{2}$ '	2° 30 $\frac{1}{2}$ '	46' 26"
17° 59'	*	2° 21°	3° 8° 50'	3° 21° 48 $\frac{1}{4}$ '	3° 13° 1'
2° 6 $\frac{1}{4}$ '	*	*	6° 10° 57 $\frac{1}{2}$ '	9° 0° 45 $\frac{1}{2}$ '	11° 23° 23'
25''	*	*	46''	20''	4''
4189	95	162	89170	79042	35109
52369	*	*	520098	953937	1903421
1262	*	*	123778	227028	453000
millions	*	*	490 millions	900 millions	1800 millions
14218	*	*	25277	53163	4759
43	*	*	3.7	1.1	0.276
$\frac{7}{12}$	*	*	1 $\frac{2}{3}$	1000	90
.7	*	*	.23	.02	*

AN EXPLANATION

OF THE

Principal Terms used in Astronomy.

Eras. certain periods of time from whence chronologers and astronomers begin their computations.

Altitude, the height of the Sun, Moon, or stars above the horizon, and it is always reckoned upon a vertical circle.

Azimuth, an arc of the horizon contained between the east or west point of the heavens, and the centre of the Sun, or a star, at the time of its rising or setting.

Anomaly (true), the distance of a planet in signs, degrees, &c. from that point of its orbit which is the farthest from the Sun.

Anomaly (mean), is that which would take place, if the planet moved uniformly in the circumference of a circle.

Antecedentia, the motion of any heavenly body when it is contrary to the order of the signs, as, through Aries, Taurus, Gemini, &c. towards Pisces, &c.

Aphelion, that point in the orbit of a planet which is the farthest distant from the Sun.

Apoceen, that point in the orbit of a planet, in which it is at its greatest distance from the Earth.

Apsees, the two points in the orbit of a planet, which are its greatest and least distance from the Sun; the line joining these points is called the line of the apsees.

Armillary

Armillary sphere, an instrument having the principal circles which are usually drawn upon the artificial globe.

Ascensional difference, an arc of the equinoctial contained between that point of it which rises with the Sun, Moon, or star, and that point which comes to the meridian with them; or it is the time the Sun rises or sets before or after six o'clock.

Atmosphere, that collection of vapours, or body of air, that surrounds the Earth.

Axis, of the Earth or any planet, is an imaginary line passing through the centre, from one pole to the other.

Azimuths, great circles passing through the zenith and nadir, and they are perpendicular to the horizon. The azimuth of any celestial object is an arc of the horizon, contained between the east or west point of the heavens, and a vertical circle passing through the centre of that object.

Bisextile, or leap year, every fourth year, so called because the Romans reckoned the sixth day of the calends of March, in this year, twice over.

Cardinal points, the East, West, North, and South points of the compass.

Cardinal points of the ecliptic, the first points of the signs Aries, Cancer, Libra, and Capricorn.

Centrifugal force, that force by which any body, revolving in a circular orbit, endeavours to fly off from the centre of motion in a right line, or tangent to the circle.

Centripetal force, that force which attracts any heavenly body towards the centre of its orbit, and which, together with the centrifugal force, preserves the body in the proper path of its orbit.

Colures, two great circles or meridians, one of which passes through the solstitial points Cancer and Capricorn, and is called the solstitial colure; the other passes through the equinoctial points Aries and Libra, and is called the equinoctial colure.

Conjunction,

Conjunction, is when two stars or planets, seen from the Sun or Earth, appear in the same point of the heavens.

Constellation, several stars lying near each other, which astronomers, for the sake of remembering with more ease, supposed to be circumscribed by the outlines of some animal, or other figure.

Cosmical rising or setting of a star, is when they rise with the Sun in the morning, or set with him in the evening.

Consequentia, the motion of the planets according to the order of the signs, as from Aries towards Taurus, &c.

Culminating of the Sun or a star, is when they come to the meridian of any place.

Cycle of the Moon, a period of 19 years, in which time the changes and eclipses of the Moon are nearly the same, and happen at the same time.

Day, that portion of time in which the Earth performs an entire revolution upon its axis; and is either natural, artificial, or astronomical.

Declination of any celestial body, is its distance north or south from the equator, reckoned in degrees, minutes, &c. upon a circle perpendicular to the equator.

Degree, the 360th part of a circle.

Direct, a planet is said to be direct when it moves according to the order of the signs.

Disc of the Sun, or Moon, is its round face, which, on account of its distance from us, appears flat, like a plane surface.

Digit, is the twelfth part of the Sun or Moon's diameter.

Eccentricity, the distance between the centre of an ellipse and either of its foci.

Ecliptic, that great circle in which the Sun appears to move.

Elevation of the pole, is an arc of the meridian contained between the pole and the horizon, and is always equal to the

the distance of the zenith from the equator, that is, the latitude of the place.

Elongation, the angular distance of a planet from the Sun, as it appears to a spectator upon the Earth.

Ellipsis, a figure formed by cutting a cone obliquely. The orbits of all the planets are of this form.

Emersion, the time when any planet that is eclipsed begins to recover its light again.

Epoth, the Moon's age at the end of the year, or the difference between the solar and lunar year.

Equations, certain quantities by which are estimated the inequalities in the motion of a planet: the Moon being subject to many irregularities, has a great number of equations.

Equation of Time, the difference between equal and apparent time, or that shewn by a clock and a sun-dial.

Equinoxes, the two points where the ecliptic cuts the equator.

Galaxy, or the Milky Way, a large irregular zone in the heavens, illuminated with a great number of stars.

Geocentric place of a planet, is that part where it is seen from the Earth.

Heliacal rising of a star, is when it appears above the horizon, before the Sun in the morning: and heliacal setting of a star, is when it is not seen after the Sun in the evening.

Heliocentric place of a planet, is that part in which the planet is seen from the sun.

Hemisphere, the half of a globe or sphere, and is either celestial or terrestrial.

Horizon, is the circle which separates the visible from the invisible hemisphere, and is either sensible or rational. The former passing over the surface of the Earth, and the latter through the centre.

Hour circles, are great circles passing through the poles of the world.

Immerision,

Immersion, the moment when an eclipse begins on a planet.

Inclination, the angle which the orbit of one planet makes with that of another.

Latitude of a star or planet, is its distance from the ecliptic, reckoned in degrees, minutes, &c. upon the arc of a great circle.

Longitude of a star or planet, is its distance from the first point of Aries, in degrees, minutes, &c. upon the ecliptic.

Luminaries, the Sun and Moon, so called by way of eminence.

Lunation, the time between one new Moon and the next.

Magnitudes, the different classes of the stars, of which there are usually reckoned six or eight.

Mean motion of a planet, is that which would take place if it moved in a perfect circle, and an equal space every day.

Meridian, that great circle which passes through the poles and the zenith of any place.

Minute, the 60th part of an hour in time, or the same part of a degree of space.

Nadir, that point in the heavens directly under our feet.

Nodes, the two points where the orbit of a planet intersects the ecliptic.

Northern signs of the ecliptic, are those six on the north of the equinoctial, viz. Aries, Taurus, Gemini, Cancer, Leo, and Virgo.

Nucleus, the head of a comet, or the central part of a planet.

Oblique ascension of the Sun, or a star, is an arc of the equinoctial contained between the first degree of Aries, and that point of it which rises with the Sun or star.

Oblique

Oblique Sphere, is that position of the globe when either pole is above the horizon less than 90 degrees.

Opposition, when two stars or planets are 180 degrees distant from each other.

Orbit, the path a planet describes in its course round the Sun.

Orbis magnus, the orbit of the Earth.

Parallax, the difference between the places of any celestial body, as seen from the centre, and from the surface of the Earth.

Parallax of the Earth's annual orbit, is the angle at any planet which is subtended by the distance between the Sun and Earth.

Parallels of latitude, are small circles of the sphere, parallel to the equator.

Perigee, that point of a planet's orbit in which it is nearest the Earth.

Perihelion, that point of a planet's orbit nearest the Sun.

Pole star, a star of the second magnitude in the tail of the Greater Bear, so called from being situated near the North Pole of the world.

Poles of the world, the two points at the extremities of the Earth's axis.

Precession of the equinoxes, a slow motion of these two points, whereby they are found to go backwards about 50 seconds in a year.

Quadrant, the fourth part of a circle; also an instrument for measuring angles.

Retrograde, is that motion by which some of the planets seem to go backwards, or contrary to the order of the signs.

Right ascension, is that degree of the equator which comes to the meridian with any celestial body, reckoning from the first point of Aries.

Satellites, the secondary planets.

the body which communicates the motion is the power, and that which receives the motion is the weight.

9. *Friction* is the resistance which any machine suffers by the parts rubbing against each other.

In the practice of mechanics, though all bodies are rough in some degree, and all engines imperfect; yet it is necessary to consider all planes as perfectly even, all bodies perfectly smooth, and all bodies and machines to move without friction or resistance, all lines straight and inflexible, all cords very pliable, &c.

S E C T. I.

ON THE SIX MECHANICAL POWERS.

THE whole principles of relative motion in mechanics depend upon this one single rule:—*That the whole force of a moving body is the result of its quantity of matter multiplied by the velocity of its motion.* Thus, when the product arising from the multiplication of the particular quantities of matter in any two bodies by their respective velocities are equal, the entire forces are so too. For example:—suppose a body, A, which weighs 40 pounds, to move at the rate of two miles in a minute; and another body B, which weighs only four pounds, to move 20 miles in a minute: the entire forces with which these two bodies will strike against any other would be equal to each other, and therefore it would require equal powers to stop them; for 40 multiplied by 2, gives 80, the force of the body A: and 80 is

is also the product of 4, multiplied by 20, the force of the body B. Thus, the heavier any body is, the greater is the power required, either to move or stop it. And again, the swifter it moves, the greater is its force; therefore, when two bodies are suspended on any machine, so as to act contrary to each other, if the machine be put in motion, and the perpendicular ascent of one body, multiplied into its weight, be equal to the perpendicular descent of the other body, multiplied into its weight; those bodies, how unequal soever in their weights, will balance one another in all situations; for as the whole ascent of one is performed in the same time with the whole descent of the other, their respective velocities must be directly as the spaces through which they move; and the excess of weight in one body is compensated by the excess of velocity in the other. Upon this principle the power of any machine may be easily computed; for it is only finding how much swifter the power moves than the weight does (that is, how much farther in the same time), and just so much power is gained by the engine.

A *lever* is a bar, either of iron or of wood, one part of which is supported by a prop, as its centre of motion. And the velocity of every part or point in the lever is directly as its distance from the prop.

There are four kinds of levers:—1. The *common lever*, where the prop is placed between the weight and power, but much nearer the weight than the power. 2. Where the prop is at one end of the lever, the power at the other end, and the weight between them. 3. Where the prop is at one end, the weight at the other end, and the power applied between them. 4. The *bent lever*, which differs from a lever of the first sort only in being bent. Levers of the first and second kind are often used in mechanical engines; but the third kind are seldom used, as no power can be gained by them.

When the power is at the same distance from the prop as the
the

the weight is, and the power and weight are both alike, the machine will remain in equilibrium, and no power can be gained. This is the principle upon which the common balance is formed. Let CD (*fig. 14, plate 19*) be a beam or lever, E the middle point, or centre of motion, which may be considered as the prop; A B two weights hanging at the ends C and D ; then, when the machine is suspended at the point E , and put in motion, the points C and D , being equidistant from E , will describe equal arches, therefore their velocities will be equal; and if the bodies A and B be also equal, then the motion of A will be equal to that of B , as the velocities and quantities of matter are equal; and consequently, if the machine be at rest, neither of the weights can move the other, but they will remain in equilibrium.

The use of the balance or common pair of scales, is to compare the weights of different bodies; for any body, whose weight is required, put into one scale, will be balanced by a body of the same weight, put into the other scale.

In order to have a pair of scales perfect, they should have the following properties:—1. The points of suspension of the scales, and the centre of motion in the beam C , E , D , must be in a right line. 2. The arms CE and DE , must be of equal length. 3. The centre of gravity must be in the centre of motion E . 4. There should be as little friction as possible. 5. The scales must be in equilibrium when empty.

If the centre of gravity of the beam be above the centre of motion, and one end of the balance be put lower than the other, that end will continually descend, till it be stopped at the handle H ; but if the centre of gravity of the beam be below the centre of motion, the balance will preserve an equilibrium.

Hence, to examine a pair of scales, let the weights in the two scales be in equilibrium, then change the weights to the contrary scales, and if they remain in equilibrium, the balance is true, otherwise it is false.

Let

Let ABC (*fig. 1, plate 19*) represent a lever of the first kind, supported by the prop D; the parts AB and BC, on each side of the prop D, are called the arms of the lever; the end A of the shorter arm AB is applied to the weight to be raised, and the power is applied to the end C, of the other arm BC. The principal use of this lever is to loosen large stones which are fixed in the ground, or to raise great weights to a small height, in order to place rollers under them, or ropes for raising them higher by other machines.

In this lever, the shorter arm AB should be as much thicker than the longer arm BC, as will be sufficient to balance it on the prop D. Thus, if P represent a power whose weight is equal to one ounce, and W a weight of twelve ounces, and if the power be twelve times farther from the prop than the weight is, they will exactly counterpoise each other; and a small addition applied to the power P will raise the weight W; and the velocity with which the power descends will be to the velocity with which the weight rises, as 12 is to 1; that is, directly as their distances from the prop; and consequently, as the spaces through which they move. Thus, it is evident, that if a man, by his natural strength, could lift an hundred weight, he will, by a lever of this sort, be able to raise twelve hundred weight. If the weight be less, or the power greater, than in the foregoing case, the prop may be placed so much farther from the weight, and then it can be raised to a proportionably greater height by the same addition of force: but if the weight be greater, or the power less, the prop should be placed so much nearer the weight. For, universally, if the gravity of the weight, multiplied by its distance from the prop, be equal to the gravity of the power, multiplied by its distance from the prop, the power and weight will exactly balance each other. Thus, if the weight W be twelve ounces, and its distance from
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the prop 1 inch; the product of 12 multiplied by 1 is 12; and if the power P be 1 ounce, and its distance from the prop 12 inches, the product of these two is also 12; therefore they counterpoise each other. If a power equal to 2 ounces be applied at 6 inches distance from the prop, it will also balance the weight W , for 6 multiplied by 2 is 12. And if the power be 3 ounces, and placed at 4 inches distance from the prop, it would also balance the weight W , for 3 times 4 is 12. And the like in any other proportion. A poker stirring a fire is a lever of this kind; the bar upon which it rests is the prop, the hand applied to the end of it is the power, and the incumbent coals on the other end the weight. Several sorts of instruments are formed of two levers of this kind, as, scissars, snuffers, pincers, &c.; the prop, or centre of motion, is the pin which holds them together.

The *statera*, or Roman steelyard, is a lever of this kind, and is used to find the weight of any body by one single weight, placed at different distances from the prop. $G X$ (fig. 13) is a steelyard, suspended by the hook O , from the centre of motion D ; the shorter arm DG is of such a weight as exactly to counterpoise the longer arm DX ; if this longer arm be divided into as many equal parts as it will contain, and each part equal to OD , the single weight P will weigh any body as heavy as itself, or as many times heavier as there are divisions in the arm DX . Thus, if the weight P be one pound, and placed at the first division 1, in the arm DX , it will balance one pound in the scale W ; if it be removed to the second division at 2, it will balance two pounds in the same scale; if to the third, three pounds, &c. And if each of these integral divisions could be divided into as many equal parts, as a pound contains ounces, then the weight P , placed at any of these subdivisions, would shew the odd ounces, over and above the number of pounds in the body in the scale.

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The second kind of levers have the weight between the prop and the power (*fig. 2*). In this, as well as the former, the advantage gained, is as the distance of the power from the prop. to the distance of the weight from the prop; and the rules for computing the force of this lever are the same with those of the former. Thus, if W be a weight of six ounces, hanging at the distance of one inch from the prop G , and P a power or weight of one ounce, hanging at the end B , six inches distant from the prop (by the cord CD running over the fixed pulley E), the power will just support the weight: and a small addition to the power will raise the weight one inch for every six inches that the power descends. Thus the power acts with the same force upon the weight, as it would do, if the weight were at the same distance from the prop, and on the other side thereof, in which case it would be a lever of the first sort.

Two men carrying a burden upon a stick exhibit a specimen of a lever of this kind; and the portion of weight borne by each man, is in proportion to his distance from the weight. In yoking two horses of an unequal strength to draw any load, the point of action is placed as much nearer to the stronger horse than to the weaker, as the strength of the former exceeds that of the latter.

Of this kind of levers are oars, rudders of ships, doors turning upon hinges, cutting knives fixed at the point, &c.

The third kind of lever has the power applied between the weight and the prop, in which, in order that the power may counterpoise the weight, the gravity of the power must exceed that of the weight as, much as the distance of the weight from the prop exceeds the distance of the power from the prop. Thus, if E (*fig. 3*) be the prop of the lever AB , and W a weight of one pound, which is placed three times as far from the prop as the power P , which acts at F by the cord C going over the pulley D ; the power P must be three pounds, to counterpoise the weight of W of one pound;

weight will balance each other. Again, let AB be a wheel, ED the axle, and the circumference of the wheel eight times as great as that of the axle; then a power P of one pound hanging by the cord I , which goes round the wheel, will balance the weight W of eight pounds hanging by the rope K , which goes round the axle; and a small addition to the power will cause it to descend and raise the weight; but the weight will rise with only an eighth part of the velocity wherewith the power descends; and consequently will move through only an eighth part of an equal space in the same time. If the wheel be pulled round by the handles SS , the power will be increased in proportion to their length.

In this mechanical power the radius of the wheel and the opposite radius of the axle may be considered as the longer and shorter arms of a lever of the first kind, the centre of the axle being the prop.

Sometimes the wheel, or the axle, is indented, or cut into teeth, which have another wheel working in them, as in jacks, clocks, mill-work, &c. by which means they give a much greater mechanical force. To compute the power of a combination of wheels, multiply the radii of all the axles continually together, as also the radii of all the wheels; then, as the former product is to the latter, so is a given power applied to the circumference of a wheel to the weight it can sustain. For example: in a combination of five wheels and axles, to find the weight a man can sustain or raise, whose force is equal to 150 pounds, the radii of the wheels being 30 inches, and the radii of the axles three inches. Here 3 multiplied four times into itself, produces 243; and 30 multiplied four times into itself, produces 24,300,000; therefore, as 243 is to 24,300,000, so is 150 to 15,000,000 pounds, the weight he can sustain, which is more than 6696 tons, or above 100,000 times as great a weight as he could sustain by his own natural force.

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gravity of the power is to that of the weight. Thus if *P* descend eight inches, *A* will ascend four inches, *B* two inches, *C* one inch, and *W* half an inch.

A, B, C, D, (*fig. 11*,) are four pulleys, two of which, *A* and *B*, are in a fixed block *X*; the two others, *C* and *D*, in a moveable block. Here the weight *W* is raised by pulling the cord at *P*, which goes successively over the four pulleys, and is fastened at the end to the fixed block at *f*. The purchase of this machine is seen to be as 4 to 1, for *P* is sustained by the single cord; but *W* by four folds of the cord, viz.—*s, f, u, k*, so that if *P* be one pound, *W* will be four.

The velocity of the power to that of the weight is also as in the former case, as the gravity of the weight to that of the power, or as 4 to 1; for when *P* descends four inches, the parts of the cord at *k* will ascend four inches towards *e*, and all the other parts of the cord will equally follow each other; and as there are four folds in the cord, viz. *s, f, u, k*, they will each of them be shortened one inch, and *C* or *W* will be so much raised.

In the same manner the purchase of any combination of pulleys may be determined; for the momenta of the weight and power will always be equal; as in the other mechanical powers.

The fourth mechanical power is the *inclined plane*. In this machine the advantage gained is as great as its length exceeds its perpendicular height. Let *AB* (*fig. 5*) be a plane parallel to the horizon, and *CD* a plane inclined to it; if the length *CD* be three times as great as the perpendicular height *GF*, the cylinder *E* will be supported on the plane *CD*, by a power equal to a third part of the weight of the cylinder; or, it may be rolled up the plane with a third part of the power, which would be sufficient to draw it up the side of an upright wall. If the plane were four times as long as the perpendicular height, it would require only the
fourth

The power gained by the wedge is in proportion to the length of the slant side to half the thickness of the back. Thus, if the back of the wedge be two inches thick, and the side 20 inches long, any weight performing on the back will balance 20 times as much acting against the sides. To use a wedge to the greatest advantage, it should be forced, not by pressure, but by percussion, as by the blow of a hammer or mallet; by which means a wedge may be driven in below any weight, and so made to lift it up, as the largest ships, &c.

The wedge has a very great mechanical force, and effects what would be impossible by the lever, wheel and axle, or pulley; for the force of the blow shakes all the adjacent parts, and thereby makes them separate more easily; so that not only wood, but even rocks, can be split by it.

To the wedge may be referred the axe or hatchet, the chisel, the spade and shovel, knives of all kinds; as also the bodkin and needle, and all sorts of instruments which, beginning from an edge or point, become gradually thicker.

The sixth, and last mechanical power, is the *screw*, which is not properly a simple machine, because it cannot be used without a lever to turn it, called the winch or handle. It is a compound engine of very great force, and is a kind of perpendicular or endless inclined plane, still farther assisted by the power of the handle or lever: and the gain of power is in proportion of the circumference described by the power to the distance between one thread and the next in the screw.

Thus, let C be a wheel (*fig. 8*), having a screw *a b*, on its axis, working in the teeth of the wheel D, which suppose to be 48 in number. Then it is evident, that for one revolution of the wheel C, screw *a b*, and winch A, the wheel D, will be moved one tooth by the screw; and therefore in 48 revolutions of the winch A, the wheel D will be turned once round.

fourth part of the weight, on copper a fifth part, and on brass a sixth part of the weight: iron or steel running in brass has the least friction of any. And metals of the same sort have more friction than different sorts; and in general the friction increases in the same proportion with the weight, but is greater with a greater velocity.

The friction in pulleys is now almost reduced to nothing, by the contrivance of Mr. Garnett, in his patent friction rollers, which produce a great saving of labour and expense; as well as wear of the materials, both when applied to pulleys and the axles of wheel carriages. By this contrivance, there is a hollow space left between the nave and axle, or centre and pin-box, which is filled up by solid equal rollers, nearly touching each other, and furnished with axles, each of which is inserted into a circular ring at each end, by which their relative distances are preserved; and they are kept parallel by means of wires fastened to the rings between the rollers, and to which the wires are rivetted.

It is a general property in all the mechanic powers, that when the weight and power balance each other, if they be put in motion, the power and weight will be to each other reciprocally as the velocities of their motion; or the power is to the weight as the velocity of the weight is to the velocity of the power; so that their two momentas are equal: viz.—The product of the power, multiplied by its velocity, is equal to the product of the weight multiplied by its velocity. And hence the general rule: viz. *That what is gained in power is lost in time.* For the weight moves as much slower as the power is less.

general the effect of any power or force is as the product of that force multiplied by the distance of its direction from the centre of motion ; or the product of the power, and its velocity when in motion, for the velocity is proportional to the distance from that centre.

The second general problem, in Mechanics, is of the greatest importance, though it has been little attended to by mechanical writers, viz.—To determine the proportion between the power and weight, so that when the power prevails, and the machine is in motion, the greatest effect possible may be produced by it in a given time. When the power is only a little greater than what is sufficient to sustain the weight, the motion is usually too slow ; and though a greater weight be raised in this case, it is not sufficient to compensate for the loss of time. And when the power is much greater than what is sufficient to sustain the weight, the weight is raised in less time ; but it often happens, that this is not sufficient to compensate for the loss which arises from the load being reduced ; therefore, the only general rule that can be given is, to find when the product of the weight, multiplied by its velocity, is the greatest ; for this product measures the effect of the machine in a given time, which is always greater in proportion as both the weight and velocity are greater.

In the construction of compound machines, where it is necessary to alter the direction of the motion, recourse must be had to what is called *bevel gear*, the principle of which is as follows :—

Let A and B (*fig. 15*) be two cones revolving on their centres a c and a b ; if their bases be equal, they will each perform their revolution in the same time ; and any two points in each cone equally distant from the centre, as d 1, d 2, d 3, &c. will revolve in the same time as f 1, f 2, f 3, &c. respectively. But if one cone be twice the diameter of the other, as the cone a d e (*fig. 20*), which is twice the diameter of the cone f a d, then as they turn upon their

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centres,

and also where the equality of the motion is not regarded, the universal joint may be used (*fig. 22*) instead of the bevel gear. This joint may be constructed by a cross, as shewn in the figure; or with four pins fastened at right angles upon the circumference of a hoop, or solid ball. This is of great use in some machines, where the tumbling shafts are continued to a great distance from the moving power, as it is in cotton mills. The shafts, by applying this joint, may also be cut to any length, which is a great advantage where there is much resistance.

CHAP. XVI.

OF ELECTRICITY.

SECT. I.

THE PRACTICAL PART OF ELECTRICITY.

THE earth, air, and all terrestrial bodies are supposed to contain a certain quantity of an elastic subtle fluid, called by philosophers, the *electric fluid*; and when any body possesses

But the friction of an electric body against a rubber separates these fluids, and causes the vitreous electricity of the rubber to pass to the electric, then to the prime conductor of the machine, while the resinous electricity of the conductor and electric is communicated to the rubber; thus the quality of the electric fluid possessed by the conductor and the rubber is changed, while the quantity remains the same in each. In this separated state the two electric fluids will exert their respective powers; and any number of bodies charged with either of these may repel each other, attract those bodies that have less of each particular fluid than themselves, and be still attracted more by bodies that are either only destitute of it, or loaded with the contrary. In this theory the electric spark makes a double current; one fluid passing to the electrified conductor from any substance presented to it, while the same quantity of the other fluid passes from it; and when each body receives its natural quantity of both fluids, the balance of the two powers is restored, and both bodies are unelectrified.

The other theory, and that which is commonly received, is that distinguished by the name of *positive* and *negative electricity*, suggested by Dr. Watson, and demonstrated by Dr. Franklin; in which it is supposed, that all bodies possess a certain share of one and the same fluid, which is extremely subtle and elastic, by which the particles of it are strongly attracted, as they are repelled by one another. When bodies possess their natural share of this fluid, they are said to be in an unelectrified state; but when the equilibrium is destroyed, and they have an additional quantity from other bodies, or when they lose part of their natural share by the communication to other bodies, they then become electrified, and exhibit electrical appearances; which are generally the same in both cases. In the former case they are said to be electrified positively, or *plus*; and in the latter case negatively, or *minus*. It is also supposed, that electrics always contain

natural share. Here the glass is said to be *excited*, because the friction seems to excite the electric power which was in the glass.

In the same manner the friction of the glass globe against the rubber in the electrical machine makes the electrical fluid which was in the rubber pass to the glass, from whence it is conveyed to the prime conductor, the points of which are presented to every part of the globe in succession, as it is turned in the machine; and as the friction is continued, there will be a constant supply of electric fluid to the prime conductor (though other bodies be presented to it), and keep discharging all the while in visible sparks. The hand, in the former of these cases, and the rubber, in the latter, part with their natural share of electric fluid to the glass against which they were rubbed, but receive an immediate supply from the conducting substances to which they are connected; and these are again supplied by the general mass of fluid that is in the earth.

Again, if a stick of sealing-wax, a piece of sulphur, or a tube of rough glass, be drawn through the hand, the electric fluid belonging to them will pass from them to the hand, and being surrounded by the air, which is a non-conductor, they remain exhausted, and are ready to take sparks of electric fire from any bodies presented to them. The sulphur, sealing-wax, &c. in this case are said to be excited, as well as the glass, which was overloaded with fluid, though the state they are in be the reverse of one another. It is impossible to distinguish by the eye the course of the electric matter, its velocity is so great.

There are a variety of inventions for the construction of the electrical machine, but the most simple is that represented in figure 2, which, by reason of its simplicity, is not liable to be put out of order, as it has neither wheel nor string, though both might be attached thereto, if required. It may also be fixed firm on a table, and easily taken off: the globe may also be taken out with the greatest ease, in

Instead of the above composition, some use a mixture made of four parts of Venice turpentine, one part of resin, and one of bees-wax; which is prepared and used in the same manner as the former.

There are also other methods for making amalgam: as,
1. By four parts of spelter and six parts of mercury; 2. also by adding six ounces of quicksilver to one pound of molten tin, which, when cold and reduced into power, is to be mixed with seven ounces of sulphur, and six ounces of sal ammoniac; the whole is sublimated in a mattrafs.

The parts of the machine which are insulated should be varnished over with a varnish made of highly rectified spirits of wine and sealing-wax; as also the glass pillars, in order to keep off the moisture they would imbibe from the damp air.

It is necessary for the young practitioner to attend to the following rules, in the performing of his experiments, as it will often happen, that though he be in possession of very good instruments, yet, through some inadvertencies, his experiments will not succeed according to his expectation, for want of a sufficient practice in the art.

1. The electrical machine, coated jars, and every part of the electric apparatus, should be kept clean, and free from dust and moisture.

2. In clear weather, when the air is dry, and particularly in frosty weather, the machine will always work well: but in hot weather, and damp weather (except it be brought in a warm room, and the apparatus made thoroughly dry), it will not work so well.

3. The cylinder should always be wiped clean with a soft dry linen cloth that is warm, and then with a clean hot flannel, before the machine be used; then applying a little amalgam, turn the winch of the machine, and the electric fluid will come like a wind from the cylinder to the knuckle, and some sparks and cracking will soon follow. This indicates that the machine is in good order. But if these appearances

themselves upon the lower plate P, which is supported on the pedestal GH; after which they will be attracted and repelled again as before, and fly from one plate to the other with great rapidity, if the electrification be strong. It is usual to cut the pieces of paper into the figures of men and women, when they exhibit a kind of dance, which affords some entertainment to the beholders.

The electrical bells are often used in concert with the above experiment, and depend on the same principle. There are four bells *a, b, c, d*, which hang from the ends of two brass rods (*fig. 5*), communicating with the prime conductor, and with another bell *e*, fixed on a pedestal A, reaching to the ground. Between the four bells hang four brass balls, suspended by silken strings; each of these balls hang between the centre bell *e* and one of the outermost bells. The outermost bells being connected with the prime conductor by brass chains, are electrified, and attract the brass balls which hang between them and the centre bell; and the attraction being strong, each ball strikes its outer bell with some violence, and makes it ring: being then loaded with electricity, it is immediately repelled, and flies to unload itself by striking upon the centre bell, which is insulated by the glass pillar B upon the pedestal A, and from which pillar the electric matter passes to the floor, by means of the brass pedestal A. The balls are then again attracted by the outermost bells, as before; and thus the ringing may be continued as long as necessary.

When a person is to be electrified, and stands upon a stool with glass feet, or baked wood (*fig. 7*), having the chain in his hand, fastened to the prime conductor, he is then said to be insulated, and may be considered as part of the prime conductor; for every part of his body will exhibit the same appearance as the prime conductor. For if the finger of any person, standing upon the floor, be presented to him, a spark of fire will be seen to issue from him, and both he and the person who receives it will feel a painful sensation; and

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a snapping

electric substances, the point projecting towards the clouds, will draw the electric matter from them, and become sensibly charged with electricity, as if it had been connected with the prime conductor of an electrical machine. It will attract all light bodies, and sparks of electric matter may be drawn from it; and in short, it will exhibit every appearance of common electricity; and on the other hand, by common electricity may be produced, in miniature, all the known effects of lightning.

This discovery was effected by Dr. Franklin, by raising a kite, called the *electrical kite*; and formed of a large thin silk handkerchief, extended and fastened at the four corners to two slender strips of cedar, and accommodated with a tail, loop, and string, so as to rise in the air like a common paper kite. To the top of the upright stick of the cross was fixed a sharp-pointed wire, extending a foot upwards, above the wood; and to that end of the twine which is next to the hand, was tied a silk riband. At the junction of the twine and silk was suspended a key, from which, when the kite is raised during a thunder-storm, a phial may be charged and electric fire collected, as by means of an electrical machine. From which it appears, that points have a remarkable property, both of throwing off, and receiving the electric fluid; from whence has risen that useful invention of applying metallic conductors to houses or buildings, in order to preserve them from the dreadful effects of lightning, as will be hereafter shewn.

2. *The dancing Balls.*—Present the point of a wire, which is fixed on the prime conductor, to the inside surface of a glass tumbler, grasping it on the outside with the hands. The glass then will soon become charged with electric matter; its inside surface acquiring the electricity from the point of the wire, and its outside surface losing its natural quantity of electric fluid through the hands, and which, in this case, serves as a coating to the glass. Then put a few pith-balls upon the table, and cover them with this glass tumbler, and they will immediately jump up by the sides of the glass (*fig. 9*), and continue in motion for some time, being attracted, and repelled, by the electric fluid upon the surface of the glass, which they gradually conduct to the table, the outside of the glass acquiring the electric fluid from the surrounding air.

3. *Electrics become Conductors when made hot.*—Tie the silk string G (*fig. 10*) to a crooked glass tube A E F B, by which it may be held; fill the middle part of this tube E F with resin, sealing-wax, or any other electric substance; then fix the two wires A E, B F, in the sealing-wax, &c. Hold the tube over a clear fire to melt the resin or sealing-wax within it; at the same time connecting one of the wires A or B with the outside of a charged jar, and touching the other wire with the knob of the jar. Then endeavour to make the discharge through the resin, wax, &c. and it will be observed, that while the resin is cold, no shock can be transmitted through it, but as it melts it becomes a conductor; and when perfectly melted, the shocks pass through it freely: by which it may be seen, that glass and other electrics become conductors, when they are made very hot.

4. *The Thunder House.*—A (*fig. 11*) represents the side of a house, being a board about three quarters of an inch thick. This is fixed perpendicularly upon the bottom board B, upon which is also fixed, in a hole in the same board, the glass pillar C D, about eight inches distant from the board A. In the board A is a small square hole I L M K, about a

experiment, the ball G represents a thunder cloud, which being arrived sufficiently near the top of the house A, the electricity strikes it; and as the house is not secured with a proper conductor, the explosion will break part of it, by knocking out the piece of wood I M.

Again, let the piece of wood I M be so situated that the wire L K may stand in the direction I M; in this case, the conductor H O is not discontinued; and repeating the experiment as before, it will be seen, that the explosion will have no effect upon the house, as the piece of wood L M will remain in the hole unmoved; which shews the usefulness of a metallic conductor. The instrument used in this experiment is called the thunder house, as it shews the effect thunder has upon a house, both secured and unsecured.

5. *The electrical Battery*—is the most formidable and entertaining part of electricity; and is formed of a number of glass-coated jars connected together, so that their whole force may be united. And if a battery of no great power is required, as containing about eight or nine square feet of coated glass, common pint or half pint phials will answer the purpose very well; but when a large battery is required, it is necessary to have cylindrical glass jars, of about fifteen inches high, and four or five inches in diameter.

The best method of coating these jars, is to coat them with tin foil on both sides, which may be fixed upon the glass with paste, made of wheat flour; but in coating the inside of phials or jars, whose mouths are not large enough to admit the tin foil, brass filings are used, mixed with gum-water or bees-wax, &c. And the coatings should not come within two inches of the mouth of the jar, otherwise the jar may discharge of itself. Some kind of glass is not capable of holding any charge: the jars or phials should therefore be examined, before any experiment be performed.

A very good battery may be formed of twelve jars, coated on both sides with tin foil, containing in the whole about twelve square feet of coated glass. In the middle of each

of cork B (*fig. 14*) by a silk thread; in the cork a few short threads are drawn, to represent its legs. It is to be hung in the midway, between the knob E of the wire D E, which is connected with the jar A D, and the knob A: then the jar being charged, by connecting its knob A with the prime conductor, the spider will be attracted by the knob A, and then repelled by it to the knob E, where it discharges its electricity; and is then again attracted by the knob A, and again repelled to the knob E; and will continue this motion till it has completely discharged the jar.

7. *To represent any luminous Figures.*—The spiral tube is composed of two glass tubes C D (*fig. 18*), one within the other, the ends being closed with two brass caps A and B. On the outside of the innermost tube is stuck a spiral row of small round pieces of tin foil, about a twelfth of an inch distant from each other; then holding it by one end, and presenting the other end to the prime conductor, small sparks will appear between all the pieces of tin foil, and in the dark it will have the appearance of a spiral line of fire. If, instead of the spiral tube, the tin foil be stuck upon a flat plate of glass A B C D (*fig. 19*), it may be so formed as to represent any other figures, letters, flowers, &c.

8. *To prove that Electricity prefers a short Passage through the Air to a long one through good Conductors.*—A B D (*fig. 16*) is a wire about ten feet long, at the ends of which is fixed a piece of glass G, to keep the ends A B at a proper distance, and to let them slide within half an inch of each other, if required; then connect the chains belonging to the sliding wires with the hook of the battery, and the discharging rod, and send the charge of the battery through them. On making the explosion, a spark will be seen between A and B; which proves that the electric fluid chooses a short passage through the air, rather than a long one through good conductors; for very little of the electric fluid will pass through the bent wire A D B.

9. *The*

tube. This instrument is fastened to the pillar CD, by a brass ring C. When the air within the tube AB is rarefied, it will press upon the water at the bottom of the tube, and so cause it to rise in the small tube; and the rise and fall of the water shew the rarefaction of the air in the glass tube AB, which has no communication with the external air.

If the knobs GI of the two wires be brought into contact with each other, and the ring E or F be connected with one side of the charged jar, and the other ring with the other side, and a shock be made to pass through the wires, the water in the small tube will not be at all moved; which shews that the passage of the electric fluid, through conductors sufficiently large, occasions no rarefaction of the air. But if the knobs GI be placed a little distant from each other, and the shock sent through the wires as before, the spark between the two knobs will considerably rarefy the air, and the water will be suddenly forced up the small tube quite to the top.

12. *To shew the Course of the electric Fluid in the Discharge of a Jar, and to make it visible by the Star and Pencil.*—For this purpose, the jar must be charged; then taking the discharging rod (fig. 4) without its knobs, present one point within an inch of the knob A (fig. 15), and the other point at an equal distance from the outside coating of the jar. By these means the jar will be discharged silently; and if its inside be electrified positively, the point C of the discharging rod will be illuminated with a star, because it receives the electric fluid; and the point B with a pencil, because it gives out the electric fluid to the outside of the jar; and if the jar be electrified negatively on the inside, the pencil will appear upon the point C, and the star upon the point B.

13. *The universal Discharger.*—This is an instrument of very extensive use, and is composed of the following parts: A is a flat board, about fifteen inches long, four broad, and one

always be made very dry before the fire, when it is used, taking care that it be not heated.

If the jar or battery be charged with positive electricity, and it be required to know the exact time that it becomes discharged, while you are attempting to charge it negatively, observe the moment the index comes to the perpendicular station, and at that moment there will not be the least spark left in the jar. If the operation be continued, the index will again advance along the semicircle; and thus shew the exact quantity of negative electricity which the jar has acquired.

SECT. III.

OF MEDICAL ELECTRICITY.

ELECTRICITY was no sooner brought to any degree of perfection, but it was applied to medical purposes. For by late observations, it has been found to possess the invariable properties of increasing the sensible perspiration, quickening the circulation of the blood, and promoting all the glandular secretions. And among all the variety of cases in which it has been used, there are none in which it has been found prejudicial, except those of pregnancy, and venereal disease; and there are a number of cases, in which it has been applied with considerable success. In most disorders where it has been used with perseverance, it has given at least a temporary and partial relief; and in some cases it has effected a total cure. Of which, numerous instances may be seen in

an inch, therefore the electrometer may be made very small. Sometimes there is a scale on the wire C E, which serves to set the balls B C to any given distance from each other, with more certainty. When this instrument is used, the jar is so placed, that the ball B may touch the conductor. Then, suppose the ball C to be set at one tenth of an inch distance from the ball B, and the electrometer E be connected to the outside coating of the jar at I by a chain E I. In this case, if the electrical machine be put in motion, the jar will be charged; and when the charge is so high, that the electric fluid accumulated within the jar, can pass from the ball B to C, which is here supposed to be one tenth of an inch; the discharge will take place, the spark will appear between the balls, and the shock will pass through the chain E I from E to I; for the part F D being of glass, and generally covered with sealing-wax, is impervious to the electric fluid; therefore, the electric fluid has no way to pass from the inside to the outside of the jar, but from the ball B to the ball C, and along the wire C E, and from thence along the chain E I.

When the electrical shock is to be administered to any part of the human body (as, for example, to the arm), instead of the chain, which must now be taken away, two small pliable wires E L, I L, are to be fastened, one to the ring E, and the other to a hook I, of the stand H I, which communicates with the outside coating of the jar (if the jar have not the stand H I, the extremity of the wire I may be put in contact with the outside coating of the jar in any other convenient manner); the other end of the said wires is fastened to the brass wires L L of the directors K L. Each director consists of a knobbed brass wire L, connected to a glass handle K, by means of a brass cap. Then the operator, holding the directors by the extremities of the glass handles, brings their balls into contact with the extremities of that part of the patient's body through which the shock is to be sent. Then it is evident, from a view of the figure,

the cause is sometimes a contraction of the muscles; in which case, electricity has often proved an effectual remedy: but the loss of motion is sometimes occasioned by a relaxation, as well as contraction; as, when the hand is bent inwardly, and the patient has no power to straighten it. In these cases it is often difficult to discover the real cause; but the surest method is, to electrify both those muscles which are contracted, and also their antagonists; for no injury can attend electrifying a sound muscle. In rheumatic disorders, the electric fluid should be drawn from the parts afflicted; or the sparks may be drawn from the conductor.

The operation should be continued for four or five minutes, and may be repeated once or twice a-day. When the shocks are strong, their greatest number at one operation seldom exceeds a dozen or fourteen, except they be given to different parts of the body.

The *electrophorus* (*fig. 2*) consists of two plates, from six to 18 inches diameter, in general, and sometimes much larger. The upper plate is generally made of brass; but a tin plate will serve the purpose, having a wire turned upon its edge, in the common manner; on the centre of this plate is fixed the socket O, in which the glass handle I is fixed, which is nine or ten inches long. When the electrophorus is to be of a large diameter, a thin board, covered with tin foil, and suspended by silken strings, will answer exceedingly well.

The lower plate (only the edge of which could be shewn in the figure) may be made either of galls, sealing-wax, or the following composition, viz.—resin four parts, pitch three parts, shell-lac three parts, Venice turpentine two parts, melted together over a gentle fire; then poured and spread upon a thin linen cloth, about a quarter of an inch thick; the linen cloth is then stretched upon a hoop, and made as tight as possible.

To charge a jar with this machine, rub the coated side of the under plate with a piece of fine new flannel, or a hare, or cat's

balls is always contrary to that with which they are charged, for they do not receive any electricity from the wires of the jar or battery; for all bodies placed within the influence of electrified bodies are affected with the contrary electricity.

But to discover the kind of electricity, when the charge is very small, instead of the pith-balls, a piece of downy feather should be used, suspended by a single silken thread, as it comes from the worm, or at least by a very few of those threads, to render it as light as possible. If any electrified body be presented to this, the feather will be repelled by it, if it be of the same kind with its own, and attracted by it, if the electricity be contrary to it. For this light body, when once electrified, either positively or negatively, will retain its virtue a long time, with very little loss.

Notwithstanding electricity might be rendered so generally useful in the application of it to medical purposes, yet it is frequently found to be ineffectual, where it might be expected to prove the most salutary, which is very often the effect of ignorance in the operator; for many persons, particularly in the metropolis, undertake to administer electricity, who are entirely destitute of any medical knowledge, and, consequently, of the cause and situation of disorders; hence, the failure of it is owing, generally, to an error in the application.

The Technical Terms used by Writers on Electricity.

Battery, electrical, a number of jars combined together, to be all charged and discharged at the same time, (*fig. 12, plate 20*).

Charging, throwing an additional quantity of electric fluid upon one side of a plate of glass, or a jar, while the other side is exhausted in the same proportion. All electric substances may be charged as well as glass.

Circuit, those conducting substances used to connect the

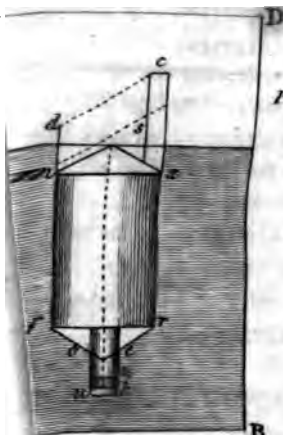


Fig. 11.



Fig. 15.

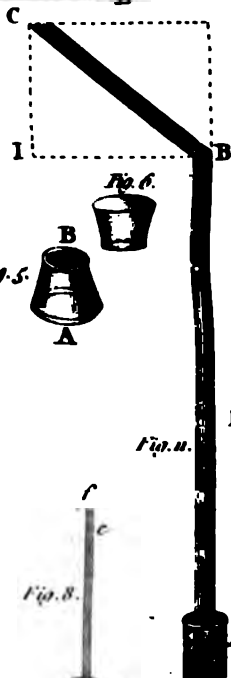


Fig. 12.

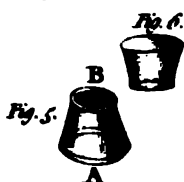


Fig. 5.



Fig. 6.

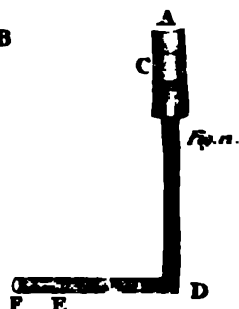


Fig. 13.



Fig. 8.

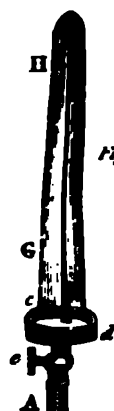


Fig. 9.

Point, the appearance of the electric fluid issuing from the point of a body, electrified positively.

Star, the appearance of the electric fluid issuing from the point of a body, electrified negatively.

Shock, electric, the convulsion given to the animal muscles by the discharge of a jar or battery.

Wire of a jar, &c. the wire or metal rod which touches the inside coating of a jar, &c.

CHAP. XVII. OF PNEUMATICS.

SECT. I.

OF THE PROPERTIES OF AIR.

PNEUMATICS is that part of natural philosophy which treats of the weight, pressure, and elasticity of the air, with the effects arising from them.

The air is that thin, transparent, fluid body which surrounds the whole earth to a considerable height; and which, together with the clouds and vapours that float in it, is called the atmosphere. That the air is a fluid, is evident from the following properties, which it possesses in common with all other fluids: viz —1. It yields to the least force impressed on it.—2. Its parts are easily moved among one another.—3. It presses according to its perpendicular height.—4. And its pressure is every way equal.

S a s

But

an elastic nature, and capable of being compressed into a less space, for the lowermost parts of the atmosphere, being pressed with the weight of all that is above them, must consequently be rendered more dense and compact at the earth's surface than at any height above it. And that air, towards the upper part of the atmosphere being less pressed, is consequently less dense and compact than that near the earth; for the density of the air is always as the force that compresses it. The following table given by Dr. Cotes, shews that the rarity of the air at a distance from the earth's surface increases in a geometrical proportion, while its height from the earth increases in an arithmetical proportion :

At the altitude of	Miles above the surface of the earth the air } is				times thinner and lighter than at the earth's surface.
	7				4
14	—	—	—	—	16
21	—	—	—	—	64
28	—	—	—	—	256
35	—	—	—	—	1024
42	—	—	—	—	4096
49	—	—	—	—	16284
56	—	—	—	—	65536
63	—	—	—	—	262144
70	—	—	—	—	1048576
77	—	—	—	—	4194304
84	—	—	—	—	16777216
91	—	—	—	—	67108864
98	—	—	—	—	268435456
105	—	—	—	—	1073741824
112	—	—	—	—	4294967296
119	—	—	—	—	17179869184
126	—	—	—	—	68719476736
133	—	—	—	—	274877906944
140	—	—	—	—	1099511627776

From this table it appears, that at the height of seven miles from the earth, the air is four times rarer and lighter than at the earth's surface; at the height of 14 miles it is 16 times rarer and lighter; at 21 miles, 64 times rarer, &c. From hence it may be proved, that a cubical inch of such air as we breathe, near the earth's surface, would be so much more

weight on each square foot, gives 12,043,468,800,000,000,000 pounds, for the pressure of the whole atmosphere.

All common air is impregnated with a certain kind of what is called *vivifying spirit*, which is essential to preserve animal life; and in a gallon of air there is enough of it for one man during the space of a minute, but not much longer. This spirit is also in that air which is in water, as appears by the fish dying, when they are excluded from fresh air, as in a pond that is frozen over.

This spirit in air is lost by passing through the lungs of any animal, and is the reason why an animal dies so soon when deprived of fresh air. The little eggs of insects, also, when stopped up in a glass, and excluded from the air, do not produce their young, though they be assisted by warmth. The seeds also of plants, though mixed in good earth, will not grow if they be deprived of air.

The vivifying quality is also destroyed by the air passing through fire, particularly charcoal fire, or the flame of sulphur.

Air may also become vitiated, by being closely confined in any place for a considerable time, or by being mixed with malignant steams: and lastly, by the corruption of the vivifying spirit; as in the holds of ships, in oil cisterns, wine cellars which have been shut some time, or brewers vats. In any of them, the air may be so much vitiated as to be immediate death to any animal that enters them.

When the air has lost its vivifying spirit, it is called *damp*, because it abounds with humid and moist vapours, and because it deadens fire, extinguishes flame, and destroys life. The effects of these damps are sufficiently known to those who work in mines.

When part of the vivifying spirit of air in any country begins to putrefy, the inhabitants of that country will be subject to an epidemical disease, which will rage till the putrefaction is over. And as the putrefying spirit occasions the disease, so if the diseased body contributes towards the
putrefaction

wards that part which is most heated by the sun. But the course of the sun being from east to west, with respect to the earth, the common course of the air which supplies the place of the rarefied air must be in the same direction, viz. from east to west; but on the north side its course will be directed a little towards the north, and on the south side as much towards the south.

This would be the general course of the wind about the equator, if it were not affected by other causes, which change its direction: as, 1. By exhalations that arise out of the earth, at different times and different places, occasioned by subterraneous fires, volcanoes, &c.; 2. by a sudden inundation of rain, which causes a contraction of the air; 3. by the violent heat of some burning sands, which cause an extraordinary rarefaction of the contiguous air; 4. by high mountains, which alter the direction of the wind; 5. by the declination of the sun towards the north or south, thereby causing a greater heat in the air on the same side of the equator.

These are the principal causes which create such a great variety and uncertainty in the winds in most countries distant from the equator; as, 1. The variations of the winds in the different parts of Europe; 2. the monsoons which are found in the Indian seas; 3. those winds which always blow from west to east, on the western coast of America, and on the coast of Guinea; and the sea breezes, which, in hot countries, blow from sea to land in the daytime; and the land breezes, which blow towards the sea in the night; and all those other irregularities in the wind, as storms, whirlwinds, hurricanes, &c.

sphere, before the fermentation takes place ; but when it is more dense, it will hover near the surface of the earth, where, when an explosion takes place, the fire is visible, and often dangerous ; the explosion also has a violent force ; and the heat being great, will rarefy and drive away all the adjacent air, kill men and cattle, split trees, rocks, &c.

Lightning differs from all other fires ; for it has often been known to pass through wood, leather, cloths, and other substances, without hurting them ; at the same time melting iron, steel, silver, gold, and other hard bodies. It has melted or burnt asunder a sword, without hurting the scabbard ; and melted money in a man's pocket, without hurting his clothes. So fine are the particles of this fire, that they pass through soft, loose bodies, without injuring them, and spend their force upon those that are more dense.

Any steel instruments, as knives, forks, &c. that have been struck with lightning, have a strong magnetic virtue, which they retain many years. The lightning striking the mariner's compass has often turned it quite round, and made it stand the contrary way, that is, with the north pole towards the south.

Those explosions which sometimes happen in mines, and called fire damp, are of the same nature with lightning, and occasioned by sulphurous and nitrous vapour rising from the mine, which, mixing with the air, take fire from the lights used in the mine. This fire, when once kindled, continues to run from one part of the mine to another, as the combustible matter happens to lie ; and as the elasticity of the air is increased by the heat, the air in the mine will swell considerably ; and, for want of room, will at length explode, with a degree of force equal to the violence of the fire, the quantity of effluvia, and density of the vapours. This is sometimes so strong as to blow up the mine ; at other times it is so weak, that when it has taken fire it may be easily blown out.

The air-pump, with all its apparatus, is shewn (*fig. 1*, *plate 22*), where LL is the plate, on which is placed a wet leather, and the large glass receiver M, placed upon the leather, so that the hole *i* in the plate may come within the glass. Then, by turning the handle F (*fig. 2*), the air will be pumped out of the receiver, which will be held down to the plate by the force of the external air or atmosphere. For, as the handle F is turned backwards, or towards D, it raises the piston *de* in the barrel BK, by means of the wheel E, and rack work D and C; and as the piston is leathered so tight, as to fit the barrel exactly, no air can get between the piston and barrel, and therefore all the air above *d*, in the barrel, is lifted up towards B, and a vacuum is made in the barrel from *b* to *e*; upon which part of the air in the receiver M (*fig. 1*), by its spring, rushes through the hole *i* in the brass plate LL, through the pipe GCG, which communicates with both barrels, by means of the hollow trunk IHK (*fig. 2*), and pushing up the valve *b*, enters into the vacant place *be* of the barrel BK. For wherever the resistance or pressure of the air is diminished, the air will run to that place, if it can find a passage. Then, if the handle F be turned the contrary way, the piston *de* will be lowered in the barrel; and as the air which came last into the barrel cannot be pushed back through the valve *b*, it will ascend through a hole in the piston, and make its escape through a valve at *d*; and by that valve be prevented from returning into the barrel, below the piston. At the next raising of the piston, a vacuum is again made in the barrel, between *b* and *e*, as before, when more of the air that is left in the receiver M (*fig. 1*) escapes by its spring into the barrel BK, through the valve *b*. What is here explained concerning the barrel BK must be understood with regard to the other barrel AI. And as the handle F is turned backwards and forwards, it raises and depresses the pistons in each barrel alternately, raising one, while it depresses the other. And as there is
vacuum

the quicksilver in the vessel: and if the receiver could be perfectly exhausted of air, the quicksilver would stand as high in the tube, as it does at that time in the barometer; for the quicksilver in both cases is supported by the same power, viz. the weight of the atmosphere on the quicksilver in the open vessel.

Every turn of the handle F exhausts a portion of air from the receiver, and consequently raises the quicksilver in the tube; and the ascent of the quicksilver is always proportionable to the quantity of air exhausted; and the quantity of air remaining in the receiver is proportionable to the defect of the height of the quicksilver in the gauge, from its height in the barometer.

There are several experiments made with this air-pump, to shew the resistance, weight, and elasticity of the air.

The resistance of the air is measured by a small machine, having two mills, *a* and *b* (*fig. 3*), which are of equal weights, and each turning freely on its own axis, independent of each other. Each mill has four thin sails fixed to its axis; those of the mill *a* have their planes at right angles to its axis; and those of *b* have their planes parallel to it. Therefore, when these mills turn round in common air, the mill *a* has but little resistance from the air, because its sails cut the air with their thin edges; but the mill *b* is greatly resisted by the air, because it exposes the whole plane of its sails against the air. In each axle is a fine pin near the middle of the frame, which goes quite through the axle, and stands out a little on each side of it. Upon these pins the slider *d* is made to bear, to hinder the mills from going round, when the strong pin C is set on bend against the lower end of the pins.

Set this machine in motion, by drawing up the slider *d* to the pins on one side, and setting the spring C on bend at the opposite ends of the pins; then pushing down the slider *d*, the spring C, acting with equal force upon each mill, will
set

16 grains less than when it was full of air, when the quick-silver stands at $29\frac{1}{2}$ inches in the barometer.

If the receiver O (*fig. 1*), or M, be placed over the hole *i* in the pump plate, and the air be exhausted therefrom, this small receiver will be pressed down to the pump plate, by the weight of the atmosphere, which will be found to be equal to as many fifteen pounds as there are square inches in that part of the plate which the receiver covers; and which will hold down the receiver so fast, that it cannot be removed until the air be let into it, by turning the cock *k*, when it will be perfectly loose.

Place the small glass A B (*fig. 5*), which is open at both ends, over the hole *i*, on the pump plate L L (*fig. 1*); and having put your hand close upon the top of it, at B, exhaust the air out of the glass, and your hand will be pressed down upon the glass with a weight equal to as many times 15 pounds as the end of the glass B contains square inches, as before.

If a piece of wet bladder be tied over the end of the glass (*fig. 6*), and, when it is dry, the glass be exhausted of its air; the outer air will press upon the bladder, which will have a spherical concave figure, and will grow more concave as more air is pumped out of the glass, till, at length, it will break with a report as loud as that of a gun. If, instead of the bladder, a flat piece of glass be laid on the top of this receiver, and joined to it by a ring of wet leather between them, to exclude the air upon exhausting the air out of the receiver, the pressure of the outward air will soon break the flat piece of glass to pieces.

Let the two brass cups A and B (*fig. 7*) be joined together with a wet leather between them, having a hole in the middle of it; then fix the end of the pipe D into the hole *i* of the pump plate, and exhaust the air out of them, having turned the cock E, which permits the air to come through the pipe C D. Then turn the cock E again, to

keep out the air, and unscrew the pipe D from the pump plate, and screw on the handle F; then it will require a great force to pull these two cups asunder; for if the diameter of the cups be four inches, they will be pressed together, by the external air, with a force equal to 190 pounds. But if they be put under the large receiver M (*fig. 1*), and the air exhausted out of the receiver, they will fall asunder, having no external air to keep them together.

Place the vessel A (*fig. 8*) on the pump plate, having some quicksilver in it, and cover it with the receiver B, in which is inserted through the collar of leathers, in the brass neck C, the tube *d e* open at the lower end; then exhaust the air out of the receiver, and it will also be exhausted out of the tube. When the receiver is sufficiently exhausted, push down the tube, so as to immerse the lower end into the quicksilver. In this experiment, though the tube be exhausted of air, yet none of the quicksilver will rise in it, because there is no air in the receiver, to press upon its surface; but if the air be let into the receiver, by the cock *k* in the pump plate, the quicksilver will immediately rise in the tube, and stand nearly as high as it does at that time in the barometer.

This experiment shews, that the quicksilver is supported in the tube, merely by the pressure of the air on its surface, in the open vessel, in which the tube is immersed; and that the more dense and heavy the air is, the higher the quicksilver rises; and, on the contrary, the thinner and lighter the air is, the less it will rise. This is the reason why the quicksilver in the barometer falls before rain or snow, and rises before fair weather; for, in the former case, the air is too thin and light to bear up the vapours; and in the latter case, too dense and heavy to let them fall.

Note. In all experiments made with mercury, by the air-pump, there should be a short pipe screwed in the hole *i* of the pump plate, so as to rise about an inch above the plate, to prevent

prevent any quicksilver from getting into the air-pipe and barrels; for, should any get loose into the pipe or barrels, it spoils them, by loosening the solder, and corroding the brass.

To shew the elasticity or spring of the air, screw the pipe A (*fig. 9*) into the pump plate, and place the receiver G H, upon the plate *c d*, which is fixed to the pipe, and exhaust the air out of the receiver; then turn the cock *e*, to keep out the air, and unscrew the pipe from the pump, and screw it into the mouth of the copper vessel A (*fig. 10*), which is half filled with water. Then, upon opening the cock *e*, the elasticity of the air which is confined in the upper part of the copper vessel A, will force the water up through the pipe A B in a jet or fountain, into the exhausted receiver.

There are a great number of other experiments to be made with this useful machine, the air-pump, as:—1. To shew how necessary air is for the support of animal life; by putting any small animal under a receiver, and exhausting the air. 2. The different effects it has on different bodies; by increasing their gravity.—3. How long it will supply flame or fuel; by putting a lighted candle under the receiver.—4. The property of air in conveying sound; by putting a bell under the receiver, and striking it when the air is exhausted, &c.

2. *Of the Barometer.*

This is an instrument used for measuring the weight of the atmosphere, foretelling the changes of weather, and measuring the height of mountains, &c.

The common barometer is formed of a glass tube, hermetically sealed at one end, and filled with quicksilver, defecated, and purged of its air. The open end of the tube is then immersed in a vessel of quicksilver; and by the pressure of the atmosphere on the quicksilver, in the open vessel, the mercury in the tube will rise to the height of

twenty-nine inches and a half, when the weight of the atmosphere is at a mean rate. When the weight of the atmosphere is greater, then the mercury in the tube will rise higher; and when the weight of the atmosphere is less than its mean weight, the mercury in the tube will fall lower.

To construct the Barometer. Being provided with a glass tube of one third or one half of an inch wide (the wider the better), and about thirty-four inches long, being close at the top, or hermetically sealed, pour into it well purified quicksilver, with a small funnel, of either glass or paper, till it wants about half an inch of being full; then stopping it close with the finger, invert it slowly, and the air in the empty part will ascend gradually to the other end, and collect in its way any small air-bubbles, which will unavoidably get in, in filling the tube: then again invert it: and thus continue to invert it several times, turning the two ends alternately upwards, till all the air-bubbles are collected, and brought up to the open end of the tube; then the tube will appear like a fine polished steel rod, without a speck in it. Then pour in a little more quicksilver to fill the tube quite up, and stopping the open end of the tube with the finger, invert the tube, and immerse the finger and end of the tube, thus stopped, into a basin of purified quicksilver; withdraw the finger, and the mercury will descend in the tube to some place between 28 and 31 inches above the mercury in the open vessel, as these are the limits between which it always stands in this country, on the common surface of the earth. Measure from the surface of the quicksilver in the open vessel, to the height of twenty-eight inches, and also to the height of thirty-one inches, dividing the three inches between these two numbers, into inches and tenths of an inch, which are marked on a scale placed against the side of the tube; and the tenths subdivided into hundredth parts of an inch, by a sliding index, carrying a *vernier* or *nonius*. These three inches, between twenty-eight and thirty-one, divided thus,

thus, will answer all the purposes of a stationary or chamber barometer; but for experiment on altitude and depths, it is necessary to have the scales continued higher up, and a great deal lower.

Several precautions are necessary in filling the tube, and fitting up the barometer, as:—1. The bore of the tube should be pretty wide, to allow a free motion to the quicksilver.—2. The basin at bottom should also be pretty large, that the surface of the mercury in it may not sensibly rise or fall with that in the tube.—3. The bottom of the tube should be cut off rather obliquely, that, when it rests on the bottom of the basin, there may be a free passage for the quicksilver. And, lastly, It is best to boil the quicksilver in the tube, which will expel all the air from it, and render it very pure.

This instrument owes its invention to *Toricelli*, the disciple of Galileo, who lived about the beginning of the seventeenth century, and who having discovered that water could not be raised in a pump unless the sucker was within 33 feet of the surface of the well, desired Toricelli to investigate the cause of it. After some time, Toricelli discovered, that the pressure of the atmosphere was the cause of the ascent of the water in the pump; and that a column of water, 33 feet high, was the just counterpoise to a column of air of the same base, and which extended up to the top of the atmosphere: and this was the true reason why the water did not ascend any higher. He also discovered, that a column of quicksilver, about $2\frac{1}{2}$ feet high, would be a counterpoise to a column of water of the same base, and 33 feet in height; as quicksilver is nearly 14 or rather 13.6 times heavier than water. This supposition he soon verified, by filling a glass tube with quicksilver, and inverting the open end of it into a basin of the same, when the mercury descended till its height above that in the basin was above $2\frac{1}{2}$ feet, just as he expected.

It

It was not till after some time, that it was discovered, that the pressure of air was various at different times. This, however, was no sooner made known, than it was also observed, that the variations in the mercurial column were always succeeded by certain changes in the weather, as rain, wind, snow, &c. Hence, this instrument was soon used as the means of foretelling the change of the weather, and on this account obtained the name of *weather-glass*, as it did that of barometer, from its being the measure of the weight or pressure of the air.

The phenomena of the barometer are so various, that authors have not yet agreed upon the causes of them; nor is the use of it as a weather-glass yet perfectly ascertained; though daily observations lead us still nearer to precision. The most general rules for judging of the weather are those delivered by Mr. *Patrick*, which are esteemed the best of any, and are as follow :

1. The rising of the mercury prefaces, in general, fair weather: and its falling, foul weather, as rain, snow, high winds, and storms.

2. In very hot weather, the falling of the mercury indicates thunder.

3. In winter, the rising prefaces frost: and in frosty weather, if the mercury falls three or four divisions, there will certainly follow a thaw. But in a continued frost, if the mercury rises, it will certainly snow.

4. When foul weather happens soon after the falling of the mercury, expect but little of it; and, on the contrary, expect but little fair weather, when it proves fair shortly after the mercury has risen.

5. In foul weather, when the mercury rises much and high, and continues so for two or three days before the foul weather is over, then expect a continuance of fair weather to follow.

6. In

6. In fair weather, when the mercury falls much and low, and continues so for two or three days before the rain comes, then expect a great deal of wet, and probably high winds.

7. The unsettled motion of the mercury denotes uncertain and changeable weather.

8. You are not so strictly to observe the words engraved on the plate, as the mercury's rising and falling: though, in general, it will agree with them. For, if it stand at the words, *Much Rain*, then rises up to *Changeable*, it presages fair weather; though not to continue so long as if the mercury had risen high: and on the contrary, if the mercury stood at *Fair*, and falls to *Changeable*, it presages foul weather; though not so much of it as if it had sunk lower.

Upon these rules of Mr. *Patrick*, Mr. *Rowning* remarks, that it is not so much the absolute height of the mercury in the tube that indicates the weather, as its motion up and down; therefore, to pass a right judgment of what weather is to be expected, we ought to know whether the mercury is actually rising or falling; to which end the following rules should be observed:

9. If the surface of the mercury be convex (standing higher in the middle of the tube than at the sides), it is a sign that the mercury is then rising.

10. But, if the surface be concave (or hollow in the middle), it is then sinking.

11. If it be plain, or rather a little convex, the mercury is stationary; for, mercury being put into a glass tube, especially a small one, naturally has its surface a little convex; because the particles of mercury attract each other more forcibly than they are attracted by the glass.

12. Sometimes the mercury will stick to the side of the tube; therefore, when an observation is to be made with such a tube, the tube should be shaken a little; then, if the air be grown heavier, the mercury will rise about a twentieth of an inch higher: but if the air be lighter, it will sink as much:

method: and if it be the vernal barometer, dip it gently with the finger, which will give the mercury a free motion.

The foregoing rules may be added the following, taken from *Art and Mystery* ~~WINDS AND WEATHER~~.

13. In winter, spring, and summer, the sudden falling of the mercury, and that for a large space, denotes high winds and storms: but in summer, it denotes heavy showers, and winter thunder: and it always stands lowest of all for great storms, though not accompanied with rain; for wind and rain together, it falls more than for either of them alone. After a winter rain, the wind change into any part of the North, with a clear and dry sky, and the mercury rise, it is a certain sign of fair weather.

14. After very great storms of wind, when the mercury has been low, it commonly rises very fast. In settled, fair, and dry weather, expect but little rain, except the barometer sink much: for a small sinking then, only denotes a little wind, or a few drops of rain; and the mercury soon rises again to its former station. In a wet season, suppose in hay-time and harvest, the smallest sinking of the mercury must be noticed, for, when the constitution of the air is much inclined to showers, a little sinking then denotes more rain, as it never then stands very high; and if in such a season it rises suddenly, very fast and high, expect not fair weather more than a day or two, but rather that the mercury will fall again very soon, and rain immediately to follow: the slow gradual rising, and keeping on for two or three days, being most to be depended on for a week's fair weather; and the unsettled state of the quicksilver always denoting uncertain and changeable weather, especially when the mercury stands any where about the word *Changeable* on the scale.

15. The greatest heights of the mercury in this country are found upon easterly and north-easterly winds; and it may often rain or snow, the wind being in those points, and the barometer sink little or none, or may even be in the rising

rising state the effect of those winds counteracting. But the mercury sinks for wind as well as rain in all the other points of the compass; but rises as the wind shifts about to the north, or to the east, or between those points; but if the barometer should sink, with the wind in that quarter, expect it soon to change from thence; or else, should the fall of the mercury be much, a heavy rain is likely to ensue.

16. The barometer being lower, and continuing so longer than what can be accounted for, by immediate falls, or stormy weather, indicates the approach of very cold weather for the season; and also cold weather, though dry, is always accompanied by a low barometer, till near its termination.

17. Warm weather is always preceded, and mostly accompanied, by a high barometer; and the rising of the barometer, in the time of cold weather, is a sign of the approach of warmer weather: and also, if the wind be in any of the cold points, a sudden rise of the barometer indicates the approach of a southerly wind, which, in winter, generally brings rain.

The barometer is also found to sink in a certain ratio to its distance from the surface of the earth: it is, therefore, used for measuring any accessible heights. Various rules have been given by writers on the barometer, for applying it to this purpose, or computing the height ascended, from the fall of the mercury in the tube, the most accurate of which is that of Dr. Halley, now greatly improved by De Luc, by introducing into it the correction of the columns of the mercury and air, on account of heat, with other corrections and modifications. This rule is as follows, viz. $10,000 \times \logarithm$ of $\frac{M}{m}$ is the altitude in fathoms, in the mean temperature of 31 degrees; and for every degree of the thermometer above that, the result must be increased by so many times its 435th part, and diminished when below it. In this theorem M denotes the length of the column of mercury in the barometer tube at the bottom of the hill or eminence; and m

Vol. II.

X x

denotes

denotes the same at the top of the hill or eminence; and it is to be observed, that the result is always in fathoms of six English feet each.

As the scale of variation in the barometer is but small, being included within three inches, viz. from 28 to 31 inches, several contrivances have been devised for enlarging the scale, to render the small variations of the mercury more apparent; this has given rise to the invention of so many different kinds of barometers:—a few of the most improved are the following:

1. *The Diagonal Barometer.*

This is a method of enlarging the common scale of three inches perpendicular height, by extending it to any length, *BC* (*fig. 11*), in an oblique direction. This barometer was invented by Sir Samuel Moreland. The perpendicular height of the diagonal part *BC*, is equal to the scale of variation of three inches, or *CI*; and consequently, while the mercury in the common barometer rises the whole length of the scale, which is three inches, and equal to *CI*; in this barometer it will move from *B* to *C*: thus the scale is enlarged in this barometer, in the proportion of *BC* to *IC*. But it is found, that the diagonal part *BC*, cannot be bent from the perpendicular, more than in an angle of 45 degrees, which increases the scale only in the proportion of 7 to 5. This form is liable to some inconveniences, on account of the obliquity of the part *BC*, which makes the mercury frequently divide into several parts, and renders it necessary to fill the tube again.

2. *The Horizontal Rectangular Barometer.*

This barometer (*fig. 12*) was the invention of *J. Bernoulli*, and *Cassini*, and consists of a tube *ACDF*, sealed at the upper end *A*, and bent to a right angle at *D*; the end *F* being

F being open. The mercury, in this, stands in both legs from E to C. The scale of variation from D to F, is here made larger; and it is evident, in moving three inches from A to C, it will move as much more in the small leg D F, as the area of the tube at A C is greater than that of D F: wherefore the motion of the mercury at E must be very sensible. Though the end of the tube F be open, yet the mercury cannot run out, being opposed there by the pressure of the atmosphere. This instrument is founded on that theorem in Hydrostatics, that fluids of the same base press according to their perpendicular altitude, and not according to the quantity of their matter. So that the same pressure sustains the quicksilver, that fills the tube A D F, and the cistern C, as would support the mercury in the tube alone. This form is, however, liable to some inconveniences; for the attrition of the mercury against the side of the glass, and the quick motion of it in the part D F, is apt to break the mercury, and render its motions unequal: it is also apt to be thrown out at the open end F, by any sudden shock.

3. *Dr. Hook's Wheel Barometer (Fig. 13).*

This barometer tube has a large ball A at the top, and is bent up at the lower end, which is open, where an iron ball floats on the top of the mercury in the tube, and which ball is connected to another ball H, hanging freely over a pulley, and turning an index K about its centre. When the mercury rises in the part K, it raises the ball, and the other ball H descends, and turns the pulley with the index round a graduated circle from M towards N; and the contrary way when the quicksilver and the ball sink in the bent part of the tube. This scale is easily enlarged 10 or 12 fold, being increased in proportion to the axis of the pulley to the length of the index K. If this instrument could be constructed without any friction of the pulley and axis, it would answer

X x 2

extremely

extremely well; but the friction often obstructs the motion of the quicksilver.

4. Mr. Caswell's Baroscope, or Barometer.

This instrument is the most useful of any, for enlarging the scale of variation, and at the same time being the most exact. A B C D (fig. 14) is a bucket of water, in which is the baroscope, *x r k z f n*, which consists of a body *x r f m* and a tube, *e k z n*, which are both concave cylinders, made of tin, or rather glass, communicating with each other. The bottom of the tube, *k z*, has a leaden weight: to sink it, so that the top of the body of the baroscope, or barometer, may swim just even with the surface of the water, by adding the weight of a few grains to the top. When the instrument is forced with its mouth downwards, the water ascends into the tube to the height of *n*. A small concave cylinder or pipe is added to the top, to keep the instrument from sinking down to the bottom. *m d* is a wire, *m S* and *d e* are two threads, oblique to the surface of the water, which answer as diagonals; for while the instrument sinks, more or less, by an alteration in the gravity of the air, where the surface of the water cuts the thread, there is formed a small bubble, which ascends up the thread, while the mercury of the common baroscope ascends; and *vice versa*.

This instrument, as the author has shewn, marks the alteration in the air 1400 times more accurately than the common barometer. The bubble on the thread will seldom stand still a minute. A small blast of wind, which cannot be heard in a chamber, will make it sink sensibly; and even a cloud passing over it always makes it descend.

The common *barometer*, or *weather-glass*, is usually fixed up in a neat mahogany frame; and consists of the common tube barometer, with a thermometer by the side of it, and a hygrometer at the top.

The Air-gun.

This instrument is an ingenious pneumatical invention, for driving a bullet, with great violence, by means of condensed air, forced into an iron ball by a condenfor.

The condenfor (*fig. 15*) has at the end *a*, a male screw, on which the hollow ball *b* is screwed, in order to be filled with condensed air. In the inside of this ball there is a valve, to prevent the air from escaping (after it is injected into it), until it be forced open by a pin, *a* (*fig. 16*).

When the air is to be condensed into the ball, place your feet on the iron cross, *b b*, in order to hold down the piston rod *d c*; then lift up the barrel *e a* by the handles *i i*, until the piston *c* be brought below the hole *e*; the barrel *a c* and ball *b* will then be filled with air through the hole *e*. Then thrust down the barrel *a c*, until the piston *d c* reach the neck of the iron ball at *a*; then all the air between *e* and *a* will be forced up through the valve into the ball; and when the handles *i i* are again lifted up, the valve in the ball will close, and so keep in the air: thus by rapidly continuing the strokes up and down, the ball will presently be filled: then unscrew the ball from the condenfor, and screw it upon another male screw, which is connected with the barrel, and goes through the stock of the gun (*fig. 16*). A bullet then being deposited in the barrel of the gun, the hammer of the lock at *a* strikes against the pin, which opens the valve in the ball, and lets out as much air as will drive a musket-ball to a considerable distance.

There are several kinds of air-guns; but that here described, is the most improved and useful, as the gun need not be any larger than a small fowling-piece: and several balls, filled with condensed air, may be taken to any distance from home with very little trouble, and which will save the trouble of filling the same ball every time it is wanted. A ball of $3\frac{1}{2}$ inches diameter, may be made to contain twelve

penny-

penny-weights of air, which will discharge 12 or 15 bullets with considerable force.

There are some air-guns that have a smaller barrel contained within a larger one; and the space between the two barrels holds the condensed air. In this instrument there is a valve fixed at *a* (*fig.* 16), with a condensor fixed to the barrel at *a*, and continued through the butt end to *e*, where the piston rod may be left in. Here the whole gun serves instead of the handles *ii* (*fig.* 15), to condense the air into the barrel.

The magazine air-gun differs from the others, by having a serpentine barrel, which contains 10 or 12 bullets; these are brought into the barrel of the gun successively, by means of a lever; and they may be discharged as fast as if they were in separate guns.

CHAP. XVIII.

OF HYDROSTATICS.

Definitions.

1. **HYDROSTATICS** treat of the equilibrium of fluids; or the gravitation of fluid bodies remaining at rest. When this equilibrium is removed, and the fluid body set in motion, the effects it then produces are called *Hydraulics*.

2. A *Siphon* is a bent tube (*fig.* 5, plate 23).

3. A

3. A *Valve* is a kind of flap or cover, fixed to a pipe, or to the aperture of any body, and which, by opening, only one way, suffers water or any fluid body to pass, but not to return.

4. A *Piston* is a small cylinder, fixed to the end of a rod, and fitted to the bore of a pipe, and frequently contains a valve.

Axioms.

1. All fluids, except air, are incompressible, or incapable of being compressed into a less space.

2. In a vessel of water, or any other fluid body, the pressure of the upper parts on the lower is in proportion to the depth; and is the same at the same depth, whatever the diameter of the vessel may be.

3. The pressure of a fluid upwards, is equal to its pressure downwards, at any given depth.

4. The bottom and sides of a vessel are pressed by the fluid it contains, in proportion to its height, without any regard to the quantity.

5. If fluids of different gravities be contained in the same vessel, the heaviest will be at the bottom, the lightest at the top, and the others will be farther distant from the top in proportion to their specific gravities.

6. The direction of the pressure of a fluid against the sides of the vessel which contains it, is in lines perpendicular to the sides of such vessel.

7. A body that is heavier than an equal quantity of a fluid, will sink in that fluid; but, if it be lighter, it will swim at the top of the same fluid; and if it be of the same gravity, it will neither sink nor swim, but will remain suspended in any part of the fluid.

8. A solid immersed into a fluid, is pressed by that fluid on all sides, in proportion to the height of the fluid above the solid. And bodies very deeply immersed in any fluid, may be considered as equally pressed on all sides.

9. Every

SECT. I.

OF FLUIDITY.

A FLUID body, in Sir Isaac Newton's definition, is *a body yielding to any force impressed, and which has its parts very easily moved one among another.* This is the definition of a perfect fluid : if the fluid require some sensible force to move its parts, it is an imperfect fluid ; and the imperfection is in proportion to that force : such perhaps are all the fluids with which we are acquainted.

Fluids are either elastic, such as air ; or non-elastic, as water, mercury, &c. The latter are incompressible, and occupy the same space under all pressures or forces ; but the former dilate and expand themselves continually, by taking off the external pressure from them. The properties of the former fluids constitute the doctrine of Pneumatics, before treated of ; the latter contain the principles of Hydrostatics.

Fluidity differs from liquidity, or humidity ; the latter implying wetting or adhering. Thus, air, ether, mercury, and other melted metals, and even smoke and flame, are fluid bodies, though not liquid ones ; while water, beer, milk, &c. are both fluids and liquids.

The modern opinion concerning the original and constituent parts of fluids, is, that they are small, smooth, hard, globular particles ; consequently, each particle must be a solid globular body ; and considered singly, is no fluid ; but becomes a fluid, by being joined with other particles of the same or a similar kind.

That the particles of fluid bodies are very small, is evident, from their texture having never been discovered by the finest microscope : that they are smooth, appears from that freedom wherewith they glide over one another, when set in motion : that they are hard and impenetrable, is plain from their

being incapable of compression : and that they are spherical, is obvious, from their being so easily put in motion ; and from the interstices or vacancies, which are hereafter proved to subsist between them ; which could not be the case, unless they were spherical, and touched each other only in some single points of their surfaces. For, upon mixing salt with water, a certain quantity of the salt will be dissolved, without increasing the dimensions of the water ; which demonstrates the vacuities between the particles of the water. When a fluid becomes more buoyant, it is a proof that its specific gravity is increased, and consequently, many of its vacuities filled up ; and even then it may receive a certain quantity of other dissoluble bodies, the particles whereof are adapted to the remaining vacancies, without adding any thing to its bulk, though the absolute weight of the whole fluid be thereby increased. This is demonstrated by taking the weight of a phial of rain water with a nice balance : when the water is poured out, and some salt added to it, and the phial again filled with the water, it will be found to weigh more than when before the salt was put in, from the vacuities of the fresh water being filled with saline particles.

It has also been found by experiment, that the particles whereof fluids are composed, consist of spheres of different diameters, whose interstices may be successively filled with proper ingredients ; and where these interstices are smaller, the gravity of the fluid will be greater, and *vice versa*.

For example : if a barrel be filled with any large spherical bodies, as bullets many small shot may afterwards be placed in the interstices of these bullets ; the vacuities of the shot may then be filled with sea sand ; the interstices of which may again be filled with water ; which will also admit of a certain quantity of salt in the vacuities ; and thus the weight of the barrel may be greatly increased, without increasing the space occupied by these materials. This reasoning holds good in fluid bodies, as well as in those which are solid ; for river water will dissolve a certain quantity of salt ;

salt ; after which it will dissolve a certain quantity of sugar ; and after that a certain quantity of alum ; and then perhaps will receive other dissoluble bodies, without increasing the dimensions of the whole.

If fluids were not compounded of such primary particles, but made up of one homogeneous substance, equally dense, without consistence, there would be no difference in their specific gravities, and all fluids would be of the same weight, which is not the case.

That a fluid has vacuities, is evident from the following consideration, viz. if all space were absolutely full of matter, that matter must be either fluid or fixed. If it were fixed, no motion could possibly be therein ; as is evident from reason and experience ; it must therefore be fluid. But a fluid without vacuities would be denser, and consequently heavier, than all other fluids ; and if denser, all bodies will emerge, and swim at the top, by hydrostatical laws, and there would be no such thing as gravity. But as gravity exists, all space therefore cannot be filled, even with a fluid.

By the experiments of *Borcelli*, it has been demonstrated, that the constituent parts of all fluids, are not fluids themselves, but consistent bodies ; and that the elements of all bodies are perfectly firm and hard. The incompressibility of water, proved by the Florentine experiment, is a sufficient evidence that each primary particle of this fluid is a perfect impenetrable solid.

This famous experiment was first attempted by the ingenious Lord *Verulam*, who enclosed a quantity of water in a piece of lead, and found, that the water would sooner make its way through the pores of the lead, than be reduced to less compass, by any force that could be applied. This experiment was afterwards made at Florence, with a globe of silver ; which being filled with water, and well closed, was gently pressed, when a small quantity of water issued through the pores of the silver in the form of dew.

3. The specific gravities in bodies of the same absolute weight, are reciprocally as their bulks.

4. The specific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitudes.

5. The absolute gravities, or weights of bodies, are in the compound ratio of their specific gravities, and magnitudes or bulks.

6. The magnitudes of bodies are directly as their weights, and reciprocally as their specific gravities.

7. When a body is immersed in a fluid that is specifically lighter than the body, the body loses as much of its weight as is equal to the weight of a quantity of the fluid of the same bulk or magnitude.

Therefore, the specific gravities of two bodies are as the absolute gravities, under the same bulk. The specific gravity of the fluid will be to that of the body immersed in it, as the part of the weight lost by the solid is to the whole weight. And the specific gravities of two fluids are as the weights lost by the same solid immersed in them.

8. The specific gravity of any fluid or solid body is found as follows:—Suspend a globe of lead by a fine thread from one arm of a balance, and to the other arm fasten an equal weight. Immerse the globe of lead into the fluid, and observe what weight it will require then to balance it, and consequently what weight it has lost, which is proportional to the specific gravity of the fluid, compared with the whole weight of the solid. Thus the proportion of the specific gravity of one fluid to another, is determined, by immersing the globe successively in all the fluids, and observing the weight the globe has lost each time, which will be the proportions of the specific gravities of the fluids.

This operation also gives the specific gravity of the solid immersed, whether it be a globe, or of any other shape, if the gravity of the fluid be known. For the specific gravity of the fluid is to that of the solid, as the weight lost is to the whole weight.

The foregoing theorems have been sufficiently demonstrated by various authors, from the principles of mechanics: they are also exactly conformable to experience; as hath been sufficiently proved from several courses of philosophical experiments.

Various tables have been given, by different authors, of the specific gravities of many kinds of bodies. It will be sufficient in this place to give the specific gravities of some of the most useful bodies that have been determined with greater certainty and accuracy. The numbers in this table express the number of avoirdupois ounces, contained in a cubical foot of each body; that of common water being just 1000 ounces, or $62\frac{1}{2}$ pounds.

Table of specific Gravities.

I. Solids

Platina, pure . . .	23000	Crude antimony . .	4000
Fine gold . . .	19640	Diamond . . .	3517
Standard gold . .	18888	Granite . . .	3530
Lead . . .	11325	White lead . . .	3160
Fine silver . . .	11091	Island crystal . .	2720
Standard silver . .	10535	Marble . . .	2705
Copper . . .	9000	Pebble-stone . . .	2700
Copper halfpence .	8915	Rock crystal . .	2650
Gun-metal . . .	8784	Pearl . . .	2630
Fine brass . . .	8350	Green glass . . .	2600
Cast brass . . .	8000	Flint . . .	2570
Steel . . .	7850	Onyx stone . . .	2510
Iron . . .	7645	Common stone . .	2500
Pewter . . .	7471	Crystal . . .	2210
Cast iron . . .	7425	Clay . . .	2160
Tin . . .	7320	Oyster-shells . .	2092
Lapis calaminaris .	5000	Brick . . .	2000
Loadstone . . .	4930	Common earth . .	1984
Mean specific gravity of		Nitre . . .	1900
the whole earth . .	4500	Vitriol . . .	1880

Alabaster

For example, required the content of an, irregular block of common stone, weighing one hundred weight, or 1792 ounces: here, as 2500, the ounces in a cubic foot of common stone, is to 1792, so is 1728, the inches in a cubical foot, to 1238 $\frac{1}{2}$ cubical inches, the contents.

Again, what is the weight of a block of granite, the length whereof is 63 feet, and the breadth and thickness each 12 feet, being the dimensions of one of the stones of granite in the walls of Balbeck? Here the solid content of this stone is 9072 feet; therefore, as 1 is to 9072, so is 3500 ounces to 31,752,000 ounces, or 885 tons 18 cwt. 3 qrs. the weight of the stone.

SECT. II.

THE CONSTRUCTION AND USE OF THE HYDROSTATIC BALANCE, HYDROMETER, AND HYDRO- STATIC BELLOWS.

The Hydrostatic Balance.

THE hydrostatic balance is the most convenient instrument of any hitherto invented, for discovering the specific gravity of all substances, whether fluid or solid. It is constructed in various forms; but that which is most generally retained is the following:

VCG (*fig. 1, plate 23*) is the stand or pillar of the instrument, which is to be fixed in a table. From the top A, by two filken strings, hangs the horizontal bar BB, from which

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Z z

is

the other scale, hangs, by means of a horse-hair, a large glass bubble R.

To apply this instrument to use, let the weight L be taken away, and a wire $p\pi$ be suspended from the hook s ; and let the bubble R be taken away from the other scale, and a weight suspended in the room thereof. Suppose the weight to be sufficient to keep the parts belonging to the other scale in equilibrium: and that the middle point of the wire $p\pi$ is at the surface of the water in the vessel N. And, *Note*, the wire $p\pi$ is to be of such a size, that the length of one inch shall weigh four grains.

It is evident, that as brass is eight times heavier than water, for every inch the wire sinks in the water, it will become an eighth part lighter, which is half a grain; and heavier in the same proportion, for every inch it rises out of the water: thus, by sinking it two inches below the middle point, or raising it as much above it, the wire will become one grain lighter or heavier. Therefore, when the middle point of the wire is at the surface of the water, if the balance be in equilibrium, and the index T set to the middle point a of the graduated wire rs , and the distance on each side ar and as contain an hundred equal parts; then, if, in weighing bodies, the weight is required to the hundredth part of a grain, it may be easily found by the following method: let the body to be weighed be placed in the scale d , put the weight x in the scale e , and let this be so adjusted, as one grain more shall be too much, and one grain less too little. Then, by moving the balance gently up or down by the screw P, till the equilibrium be exactly shewn at O; if the index T be at the middle point a of the wire rs , it shews that the weight put in the scale e is just equal to the weight of the body.

The foregoing method discovers the absolute weight of the body; but to find the relative or specific weight, it must be weighed hydrostatically in water, as follows:—Instead of putting the body to be weighed into the scale d as before, let it hang by a horse-hair, as the weight R, supposing the vessel

tain wine, it will respectively sink to the marks *br*, *ra*, *pa*, and *no*. Instruments of this kind are sometimes called *Areometers*.—And, *Note*, that the globe *D* (*fig. 2*) should be made of copper; for ivory imbibes spirituous liquors, and consequently alters their gravity; and glass globes require too much attention.

This hydrometer was the invention of Mr. Clarke, and answers very well to discover the specific gravity of spirituous liquors, and to shew whether any spirit be above or below proof, and how much.

But the most perfect hydrometer is that represented in figure 4, which may be made to shew the specific gravity of fluids to the greatest degree of exactness. It consists of a large hollow ball *B*, with a smaller ball *b*, screwed to its bottom, and partly filled with mercury or small shot, in order to render it but little specifically lighter than water. In the larger ball at *C*, there is a short nick, into which is screwed the graduated brass wire *AC*, which has a small weight *A* at the top, to cause the instrument to descend in the fluid.

When this instrument is immersed in any fluid contained in a jar *LM*, the quantity of the fluid displaced by it will be equal in bulk to that part of the instrument which is under water, and equal in weight to the whole instrument. Therefore, if the weight of the whole instrument be 4000 grains, we can by these means compare the different dimensions of 4000 grains weight of several sorts of fluids; for if the weight *A* be sufficient to sink the instrument in rain water, till the middle point of the stem, marked 20, come to the surface of the water; and after that, if it be immersed in common spring water, and the surface of the water stand at one tenth of an inch below the middle point 20, it is evident that the same weight of each water differs only in bulk by the magnitude of one tenth of an inch in the stem of the instrument.

Then, suppose the stem of the instrument to be ten inches in length, and to weigh just 100 grains; every tenth
of

and that even to the thirty, forty, or fifty-thousandth part: and is infinitely superior to the common method used by excise officers and others, of shaking the spirits in a phial, and forming a judgment of the strength by the breaking of the bubbles.

Of the Syphon.

The syphon is a bent tube used to decant fluids from any standing vessel; and serves to perform some curious experiments. It depends upon the pressure of the air; and may be made in various forms.

If a small syphon, whose legs are of equal length, be filled with water, and the ends turned downward, the water will remain suspended in the syphon, as long as it is held exactly level; but when it is the least inclined to either leg, whereby in effect one leg is made shorter than the other, the water will run out by the longer leg; for the air being a fluid, whose density near the surface of the earth is to that of water as 1 to 850; and according to the nature of all fluid bodies pressing the surface of all things exposed to it every way equally, it must necessarily follow, that the weight of the atmosphere above, being kept off by the machine, and the air below bearing against and repressing the water, which endeavours to fall out of either leg with equal force, keeps it in suspense, and prevents its falling. But if the syphon be the least inclined to either leg, one of the legs is in effect shortened, and the other prolonged; by which advantage is given to the weightier fluid in the longer leg to preponderate, or overweigh the other part of the fluid in the other leg; and the water will all descend by the longer leg.

The least inclination of the syphon to either end will be sufficient to produce this effect, which may be proved by experiment, thus:—Hang a small syphon, whose legs are of equal length, upon the edge of a jar filled with water; then from the sloping of the jar, the external leg of the syphon

through the tubes, expelling all the air from them, while the weight of the atmosphere presses on the surface of the fluid in the cup.

This is sometimes called Tantalus's cup, and has a hollow figure, representing Tantalus, placed over the inner tube, of such a length, that when the fluid is got nearly up to the mouth of the figure, the syphon begins to act, and empty the cup.

This is the same in effect as if the two legs of the syphon were both in the vessel (*fig. 7*), when the water poured into the vessel will rise in the shorter leg of the syphon, to its own level; but when it has gained the bend of the syphon, it will begin to run off by the longer leg, and continue running till the vessel be emptied as low as the extremity of the shorter leg of the syphon.

The Hydrostatical Paradox.

Any quantity of fluid, however small, may be made to counterpoise, and sustain any weight, how large soever.—This is called the hydrostatical paradox: and depends upon the equal pressure of the parts of fluids every where at the same depth.

Let A B D G (*fig. 8*) represent a cylindrical vessel, to the inside of which is fitted a cover, which, by means of leather round the edge, will easily slide up and down in the vessel, without permitting any water to pass between its edge and the surface of the vessel. In the cover is fixed a small tube E F open at the top, and extending through the cover at the bottom. Then, the vessel being filled with water, and the cover put on, and loaded with a weight, suppose of a pound, it will be depressed, and the water will rise in the tube to E, and the weight will be sustained. If another pound be laid on the cover, the water will rise to F, and the weight also be sustained: and thus the water will rise higher in the tube in proportion to the weight that is laid on the cover. And though the weight of the water in the tube be but a few

upon the upper board of the bellows; as, suppose three weights, weighing 100 pounds each, and pour more water into the pipe, which, by running into the bellows, will raise up the board with all the weights upon it; and if the pipe be kept full until the weights are raised as high as the board can rise, the water will remain in the pipe, and support all the weights; though the water in the pipe weigh no more than a quarter of a pound, and the weights on the bellows 300 pounds.

The reason of this experiment appears evident from what has been said of the pressure of fluids, of equal heights, without any regard to their quantities. For if the tube be fixed in the upper board of the bellows, instead of the side, the water will rise in it to the same height as it did in the pipe, in the former case; and if as many tubes were fixed in the upper board as it would contain, the water would rise as high in each of them. The pressure of the fluid upwards is thus computed:—If one pipe be fixed in the upper board, and the pipe hold just one quarter of a pound of water, and if a person put his finger upon the hole of the pipe, when the fore-mentioned weights are placed upon the bellows, he will find his finger pressed upwards with a force equal to a quarter of a pound; and as the same pressure is equal upon equal parts of the board, each part whose area is equal to the area of the hole of the pipe, will be pressed upwards with an equal force, that is, with a force equal to that of a quarter of a pound; the sum of all which pressures against the under side of an oval board, 16 inches broad and 18 long, will amount to 300 pounds; and therefore, this quantity of weight will be raised up, and sustained by only one quarter of a pound of water in the pipe.

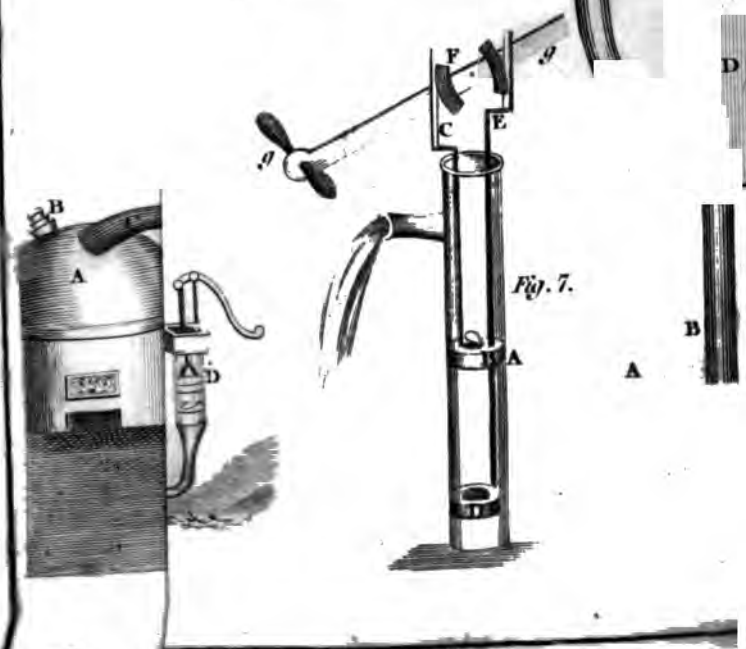
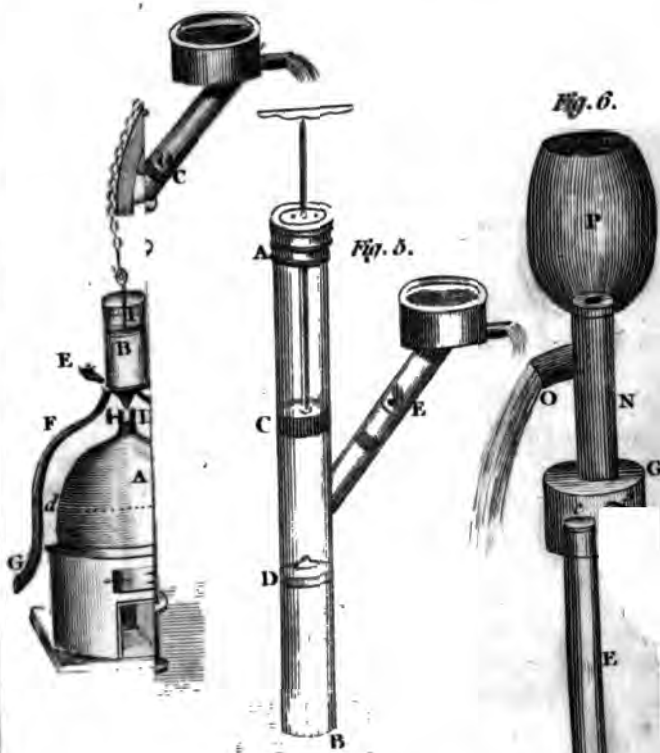
It is by this instrument that a man may raise himself upwards by his breath; for if he stand upon the upper board, and blow through the pipe, he will raise the upper board of the bellows, with himself upon it; and the smaller the bore of the pipe is, the more easily is the operation performed.

weight, which will then be too great for the pressure of the water round the tube upon the column of water below it.

Again, a piece of wood, however light, may be made to lie at the bottom of the water, by not suffering any water to get under it. Thus, having two pieces of wood, planed quite flat and smooth, so that no water may get between them, when they are put together; and cementing one of the pieces, as *ab*, to the bottom of the vessel *AB*, place the other piece upon it, and let it be held down by a stick, while the water is poured into the vessel; then, upon removing the stick, the upper piece of wood will not rise from the lower one, being pressed down both by its own weight, and the weight of all the water above it, while the contrary pressure of the water upwards is kept off by the wood placed beneath it; but if the top piece of wood be raised ever so little at any part of its edge, some water will get under it, which will be forced by all the weight of the water above, and will immediately press it upwards; and being lighter than its own bulk of water, it will float upon the surface of the water.

To prove that all fluids weigh just as much in their own elements as they do in open air, put as much shot in a phial as when corked, will make it sink in water; then let it be weighed, both in the air and in the water, and the weight in each case wrote down; then, as the phial hangs suspended in water, and counterpoised by another weight, pull out the cork, that the water may run into it, when it will descend and pull down that end of the beam. Next, put as much weight into the opposite scale as will restore the equilibrium; which additional weight will be found to answer exactly to the additional weight of the phial, when it is again weighed in the air with the water in it.

The velocity with which water spouts out of a hole, or through a tube in the side or bottom of a vessel, is as the square root of the depth or distance of the hole below the surface of the water. Therefore, in order to make double the



of the vessel, at equal distances above and below the pipe D, the perpendicular Cc and Ee , from these pipes to the semi-circle will be equal, and the jets F and H, which spout from them, will each go to the same horizontal distance NK; which is double the length of either the equal perpendiculars Cc and Ee .

CHAP. XIX.

HYDRAULICS.

SECT. I.

OF PUMP-WORK.

HYDRAULICS is that part of fluids which treats of the properties of fluids in motion, with a special attention to artificial water-works: and in this sense it stands opposed to hydrostatics, which concern fluids, as they remain at rest.

The laws of fluid bodies, as given in the last chapter, obtain also in this, and therefore need not be repeated.

The

Of the pump, there are simply three kinds, viz. the sucking, the forcing, and the lifting, pump. By the former, the water is raised by the general pressure of the atmosphere on the surface of the water in the well, and cannot be raised to a greater height than 33 feet, as before observed; though, in practice, it is seldom raised above 28 feet, because the air is not always dense enough to support a column of water of 33 feet. By the two latter, water may be raised to any height, having an adequate apparatus, and sufficient power.

Of the Sucking Pump.

The sucking pump is that in most common use, and consists of a tube or pipe, open at each end, having within a sliding piston, as large as the bore of the pipe, and which fits the pipe so exactly, as to admit no air to pass between it and the pipe. The pipe is called the barrel.

If the lower end of the barrel B be immersed in water (*fig. 1, plate 24*), and the piston D be raised, a vacuum will be made in the barrel, by lifting up the column of upper air from A to D, and thereby permitting the air in the lower part of the barrel to expand itself; and the atmosphere pressing upon the surface of the water in the well, will force it to follow the piston, and that even to the height of 33 feet, if the stroke could be of that continued length. But when the piston is let down again in the barrel, the water will fall with it, to prevent which, there is a valve fixed in some convenient part of the barrel, as at C, which valve consists of a wooden frame A (*fig. 2*), exactly fitted to the bore of the barrel, and a leather flap, B, lined with lead, in order to give it sufficient weight and strength. This valve opening with the upward motion of the water, and again closing when the piston is let down, serves to retain the water above, which flows through it:

A similar effect will be produced at the repetition of the stroke, till by degrees the water shall have reached the moving valve or bucket, and then the process will go on as before described. Thus, water, even by this machine, may be raised to any height whatever, provided the power be adequate to the weight, and the pipe strong enough to bear the fluid's natural pressure.

The proportion of the pressure of the water on the pipes in pump-work is according to the height of the water above the part considered: but the incumbent weight on the bucket of a pump, in action, is nearly proportional to that of a column of water raised; for though the weight of the atmosphere on the surface of the water, when the bucket rises, be really equal to the weight of 33 feet of water; yet this weight is exactly counterbalanced by the weight of the atmosphere, ever incumbent on the surface of the water thereby raised. Thus, all the advantage to be obtained by the hydraulic machines, is ranging matters into a convenient method of being performed; the performance itself depending entirely upon the moving power, with all the disadvantages of friction.

In this pump, if both the valves be placed towards the bottom of the pipe, the pump will work as easy, and require no greater power than if they were fixed 30 feet, or 33 feet, above the surface of the water.

It is generally found to be more advantageous in practice, to place both the valve and bucket pretty low in the barrel; for should a leak happen beneath the bucket, which is often the case, in a great length of pipe, the air getting through, would render the pipe useless; whereas, should a leak happen above the bucket, it will occasion only the loss of some of the water. And by placing the valves under water, they will always be found more supple and pliant, and, consequently, be in a better condition for performing their offices.

There is another advantage of placing the pump-work

means of the pipe C, and the bucket playing in the barrel B C, the water will rise as if the well had been perpendicular to the pump ; because the water in the well being forced by the natural pressure of the atmosphere, will replenish the barrel B, through the pipe A C.

But when it happens that the barrel of the pump cannot go down directly to the well, as in the last case, the water may be led about any other way by means of a pipe E, and thus be conveyed to the pump D. And by making this pipe of conveyance E less in diameter than the barrel of the pump, it will sooner be exhausted of air, by moving the piston, and, consequently, the water will sooner follow.

But it will always be found more easy, in practice, to have the pipe of conveyance large, and of an equal bore throughout ; because the water will then have a velocity in it, and the friction will be less. This is the reason why the common pumps, made by the plumbers, do not work so easily as those which are bored out of trees ; for, by making the pipe so much less than the bucket, they, as it were, wire-draw the water. Therefore, in pumps that have a long pipe of conveyance, the diameter of the sucking-pipe should be nearly equal to half the diameter of the barrel. For, if the barrel be four inches in diameter, and the pipe of conveyance only one inch, the water will move 16 times as fast through the pipe as it will through the barrel ; which, consequently, requires more labour, and is attended with a greater wear and friction of the machine.

It is also a great fault to bore a pump conically upwards, because the water cannot with freedom run off so fast, as a vacuum may be made by the moving piston ; and the reflection of the water from the sides, will always be an hindrance in the operation.

branching or forcing pipe at E. These valves should also be air-tight, and so disposed as to let the water freely rise, but to prevent its returning back. The forcer C is leathered upwards, that it may withstand the pressure of the atmosphere from above, and that by sucking, when raised, it may bring up the water, to supply the barrel: and it is also leathered downwards, that, when repressed, it may resist the weight of the water to be forced up.

When the forcer C is moved upwards in the barrel, it lifts up the incumbent air; and the air between that and the water having room to dilate itself, will be rarefied, and the water will rise from the spring in the barrel A B, as in the sucking pump: and continuing the motion of the forcer, the water will at length rise up to the forcer, and fill the internal cavity of the pipes, between the two fixed valves D and E. And the water being prevented from descending again by the lower valve, will, by the forcer, be pressed, and make its way through the upper valve E. When the forcer rises, this pressure will be intermitted, and the valve at E will immediately close under the weight of the upper water, and thus prevent its return, while the forcer is rising with a fresh supply. The same is repeated at every stroke of the forcer.

M. de la Hire's Pump, which raises Water both by the Ascent and Descent of the Piston in the Pump-barrel.

B and C are two pipes (*fig. 6*), having their ends in the well of water A A. The pipe B has a valve *b* at the top, and is folded into the pump-barrel D. The pipe C also has a valve at the top, and is folded into the pipe S. The pipes E and F have each of them a valve *e* and *f* at the ends, and communicate with the pump-barrel D, and the hollow box G.

K is a solid plunger or forcer, exactly fitted to the bore
of

much water forced up the pipe N, to the spout-hole O, by the descent of the plunger, as by its ascent; and in each case as much water discharged at the spout-hole as fills that part of the pump-barrel through which the plunger moves.

P is a close air vessel, fixed on the top of the pipe N, which compresses the air, when the water rises up the pipe N, above the spout O: and this condensed air acting on the water, causes it to run off by the spout-hole nearly in an equal stream.

The pipe S at the top of the pipe C should never be above 32 feet above the surface of the water in the well; because if the pipe C be entirely exhausted of air, the pressure of the atmosphere, on the water in the well, would not force the water up the pipe to a greater height than 32 feet, at the most; but if S be within 24 feet of the water in the well, the pump will work so much the better.

The pipe N may be of any size required; but the pump barrel should be made in proportion to the height of the spout-hole above the surface of the water in the well; as follows:

For ten feet of perpendicular height of the spout-hole O, above the surface of the water in the well, the diameter of the bore of the barrel should be 6.9 inches: for 15 feet high, 5.6 inches: for 20 feet, 4.9 inches: for 25 feet, 4.4 inches: for 30 feet, 4.0 inches: for 35 feet, 3.7 inches: for 40 feet, 3.5 inches: for 45 feet, 3.3 inches: for 50 feet, 3.1 inches: for 55 feet, 2.9 inches: for 60 feet, 2.8 inches: for 65 feet, 2.7 inches: for 70 feet, 2.6 inches: for 75 feet, 2.5 inches: for 80 feet, 2.5 inches: for 85 feet, 2.4 inches: for 90 feet, 2.3 inches: for 95 feet, 2.2 inches: and for 100 feet, 2.1 inch; or at the most 2.2.

In pumps of this kind the pipes B and C should be made sufficiently large: for when they are too small, the velocity of the water through them being great, the water will have too much friction to suffer the pump to be worked with much advantage.

If this machine be constructed according to the above dimensions, it will raise more water than any person unskilled in hydraulics would imagine.

The force required to work a pump will always be as the height to which the water is raised; and as the square of the diameter of the pump-bore in that part where the piston works. Thus, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wider will raise four times as much water as the narrower one; and consequently will require four times the power to work it.

The piston rod of a pump is always raised by means of a lever, whose longer arm exceeds the shorter one, in length, generally five, or six times, and the power is applied at the end of the longer arm; by which means the rod is raised by a fifth, or sixth part of the power, which would be required to raise the rod without it.

The following table shews the quantity of water which a common sucking pump will discharge in one minute, the pump being of any given height above the surface of the well, from 10 to 100 feet inclusive; and the diameter of the bore of the barrel being from 6.93 inches to 8.19 inches inclusive.

3 C 2

Height

SECT. II.

OF STEAM ENGINES:—AND A DESCRIPTION OF
MR. WATT'S STEAM ENGINE.

OF all the uses to which steam has been applied, there are none where it has been used with greater success than in the application of it to raise water from any great depth, as in that machine called the steam engine, otherwise denominated the fire engine, on account of the fire employed in boiling the water to produce the steam.

The steam raised from hot water is an elastic fluid like air, and has its elasticity proportional to its density, when the heat is the same: or proportional to the heat when the density is the same. That steam raised from boiling water of an ordinary heat is near 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, and its elasticity is equal to that of common air. And it has been found by experiment, that water, when converted into very hot steam, will occupy 14000 times the space that it occupied when in water, and, consequently, it will become five times stronger than the atmosphere. And by accidents that have happened, it has been demonstrated that water, suddenly turned into steam by the immediate application of great heat, is vastly stronger than the atmosphere, or even gunpowder.

The steam engine is the most useful machine discovered in modern times; and were it not for this most important invention, we should never have been able to work the coal mines in England to the present advantage; as before the last century, for the want of this engine to draw the water, the attempts of our ancestors to procure coals were always ineffectual.

alternately turned by a man, in order to admit the steam, and to refill the vessel with cold water.

Captain Thomas Savery having read the Marquis's book, constructed an engine, which, after several experiments, he brought to some degree of perfection; upon which he bought up, and destroyed, all the books of the Marquis he could procure, claimed the honour of the invention to himself, and obtained a patent for the same. His engine, however, would not raise water to any great height, or in quantities sufficient to answer the purpose of draining a mine. The largest he ever erected was for the York Buildings Company, in London, for supplying the inhabitants of the Strand, and that neighbourhood, with water.

Several other gentlemen, both in England and France, attempted various improvements in the construction and manner of working these engines; but with little success, till the year 1705, when Mr. Newcomen, an ironmonger, and Mr. John Cowley, a glazier, both of Dartmouth, made a considerable improvement in these engines, by bringing the engine to work with a beam and piston (which had never been then introduced), and where the steam, even at the greatest depth of mines, is not required to be greater than the pressure of the atmosphere. These gentlemen obtained a patent for the sole use of this invention, for fourteen years; and the first engine they erected was in the year 1712, at a colliery at Griff, in Warwickshire; the cylinder of this engine being 22 inches in diameter. The next engine they erected was in the year 1718, at a colliery in the county of Durham, which was also improved by Mr. Henry Beighton, F. R. S. who introduced the manner of opening and shutting the cocks, by the hanging-beam, as at present used; and likewise made improvements in the pipes, valves, and some other parts of the machine.

When these engines came to be better understood, and their utility, particularly in draining mines, became more evident, from the great numbers of them every where erected,
they

pump-rods sink by their superior weight, and the piston, at the other end of the lever, rises; and when that steam is condensed, the piston descends, and the pump-rods rise with their quantity of water; and so on alternately, as long as the piston works.

As the piston does not descend with a force exceeding eight or nine pounds upon every square inch of its surface; and as, by reason of accidental frictions, and alterations in the density of the air, it is sometimes less than this, it will be safest in practice to calculate the weight at something less than eight pounds, viz.—at about seven pounds ten ounces for every square inch, or 7.64 pounds, which is six pounds upon every circular inch; and, as a gallon of water of 282 cubic inches weighs $10\frac{1}{2}$ pounds, we have the dimensions of the cylinder, pumps, &c. for any steam-engine, as follows:

c = the cylinder's diameter in inches.

p = the pump's diameter in inches.

f = the depth of the pit in fathoms.

g = gallons drawn by a stroke of six feet.

b = the hogheads drawn per hour.

s = the number of strokes per minute.

Then c^2 is the area of the cylinder in circular inches; therefore $6c^2$ is the power of the cylinder in pounds.

And $\frac{p^2 \times .7854 \times 72}{282}$, or $\frac{1}{4}p^2$ is $=g$, the gallons contained

in one fathom, or six feet of any pump; which, multiplied by f , gives $\frac{1}{4}p^2 f$ for the gallons contained in f fathoms of any pump, whose diameter is p .

Hence, $\frac{1}{4}p^2 f \times 10\frac{1}{2}$ pound gives $\frac{1}{2}p^2 f$, nearly, for the weight in pounds of the column of water, which is to be equal to the power of the cylinder, which was before found equal to $6c^2$. Thus, we have the second equation, viz.— $6c^2 = \frac{1}{2}p^2 f$, or $3c^2 = \frac{1}{4}p^2 f$; the first equation being $\frac{1}{4}p^2 = g$, or $p^2 = 4g$.

From which two equations any particulars may be determined.

The common Steam-Engine, (Fig. 9.)

A is the boiler.

B, the cylinder.

C, the injection cock.

D, the steam-cock, or regulator.

E, the shifting clack.

F, the eduction pipe, or sucking pipe.

G, the eduction valve.

H, the safety valve.

I, the piston.

K, the lever beam.

L, weights to counterpoise the piston, and press down the forcer in the pump-barrel **M**, to drive the water through the pipe **N**.

O, a cistern to hold the injection water.

P, an air vessel which prevents the pipe **N** from bursting, and serves to keep up a regular stream.

The boiler **A** is filled with water to the height of *da*, which being made to boil by a fire placed beneath it, will fill the upper part **AD** with a very elastic vapour or steam, which, when it is of sufficient strength, will force open the valve at **H**. This steam is let into the barrel or cylinder **B**, by turning the cock **D**; and by its elastic force raises the piston **I**, which drives the air above it through a proper clack, placed at the top; and the weights **L**, at the other end of the lever, cause it to descend, and drive the piston down the pump-barrel **M**. Then, in order to make the piston **I** descend, a little cold water is let into the cylinder, at the bottom, from the cistern **O**, by turning the cock **C**, which, rising in the form of a jet, condenses the hot steam in the cylinder into water, whereby it occupies about 13000 times less space than that it took up before; which creates a partial vacuum in the barrel, and thereby permits the piston to

212 degrees is necessary to produce steam; and the difference of heat at which water boils under different pressures increases in a less proportion than the pressures themselves; so that a double pressure requires less than a double increase of the heat.

There are two principal defects in the common steam-engine: first, as the vacuum in the cylinder is produced by throwing in cold water to condense the steam, the water thrown in becomes hot, and produces a steam from itself, which greatly resists the motion of the piston downwards, and thereby lessens the power of the engine. Secondly, upon attempting to fill a cold cylinder with hot steam, a great part of the steam will be destroyed; and the injection water that is let in to condense the steam, not only cools the cylinder, but remains there until it be extruded at the eduction pipe by the steam which is afterwards let into the cylinder, which steam will be condensed into water as fast as it enters, until all the matter it comes in contact with be nearly as hot as itself.

The great consumption of fuel also has been a material object in these engines; for it is well known, that a steam-engine of an ordinary size will consume near 3000 pounds worth of coals per annum, at any part near London.

Mr. Watt's Steam-Engine.

Mr. Watt has, in a great measure, if not wholly, remedied the foregoing inconveniences: he preserves an uniform heat in the cylinder of his engine, by suffering no cold water to touch it, and by protecting it from the air or other cold bodies, by a surrounding case filled with the steam, or with hot air, or water; and by coating it over with substances that transmit the heat very slowly. He makes his vacuum to approach nearly to that of the barometer, by condensing the steam in a separate vessel, called the condenser; which may be cooled at pleasure, without cooling the cylinder, either
by

L, the air-pump that exhausts the condenser both of air and the injection water that is let in at every stroke, and is fixed under water in the condensing back **M**, which is full of water.

N, the lever beam.

O, the great water pump for clearing the mine, or raising water for any other use through the pipes, &c.

This new engine differs from the common ones only in the foregoing particulars: having the cylinder, the great beam, the pumps, &c. in their usual positions.

The cylinder in this new engine is smaller than usual in proportion to the load, and is very accurately bored; and is surrounded at a small distance with another cylinder, furnished with a bottom and lid. The space between the cylinders communicates with the boiler by a large pipe **C**, open at both ends, so that it is always filled with steam, and thereby preserves the inner cylinder of the same degree of heat with the steam, and prevents the steam from condensing within it, which would be more prejudicial than an equal condensation in the outer cylinder.

The inner cylinder has a bottom and piston as usual; and as it does not reach up quite to the lid of the outer cylinder, the steam in the space between them has always a free access to the upper side of the piston. The lid of the outer cylinder has a hole in the middle, through which the piston-rod moves up and down; this hole is kept tight by a collar of oakum-screwed upon it.

There are two regulating valves at the bottom of the inner cylinder, one of which admits the steam to pass from the space between the two cylinders into the inner cylinder, below the piston, and shuts it out at pleasure; the other opens or shuts the end of a pipe that leads to the condenser. The condenser consists of one or more pumps furnished with clacks and buckets (nearly the same as in common pumps), which are wrought by chains fastened to the great working beam of the engine. To the bottom of these pumps

the piston, which is therefore forced down by the full power of the steam from the boiler, which is somewhat greater than the pressure of the atmosphere.

In the common engines, when they are loaded to about seven pounds upon each square inch of the piston, and are of a middle size, the quantity of steam which is condensed, in restoring to the cylinder the heat which it had been deprived of, by the former injection of cold water, is about one full of the cylinder, besides what is really required to fill that vessel; so that twice the full of the cylinder is employed to make it raise a column of water equal to seven pounds for each square inch: or more simply, a cubic foot of steam raises a cubic foot of water about eight feet higher, besides overcoming the friction of the engine, and the resistance of the water to motion.

But in the improved engine of Mr. Watt, about one full and a fourth of the cylinder is required to fill it, because the steam is one fourth more dense than in the common engine. This engine, therefore, raises a load equal to $12\frac{1}{4}$ pounds upon each square inch of the piston; and each cubic foot of steam, of the density of the atmosphere, raises one cubic foot of water 22 feet high.

These engines work more regular and steady than the common ones; and the savings amount at least to two thirds of the fuel; which is an important object where coals are dear. The new engines also will raise from 20000 to 24000 cubic feet of water, to the height of 24 feet, by only one hundred weight of good pit coals.

any false strokes of the charcoal. The black and red lead pencils are used to draw out the draught the second time, because the lines drawn with these will not be liable to be rubbed out with the hand, when the lines are again drawn with the pen. The pens made of crow-quills (though another good pen may answer the purpose) are to finish the work. Rulers are to draw the straight lines, triangles, squares, &c. which are to be done at first, till practice render them needless. The compasses should have steel points, which will take out, in order to use a black or red lead pencil; their use is to draw circles, ovals, arches, &c.; also to measure, by the help of a scale of equal parts, the proportions of the figures.

The Precepts of Drawing in general.

There are no arts that depend less upon theory than those of drawing and painting; in these it is principally practice and experience that can render any one a good artist. But here, as in every other art, a few rules may be of service to the inexperienced student; and in attending to the following rules, the young artist should be careful in following the outlines of the figure, which is the first process. He must also content himself with copying parts of objects, before he aims at any finished piece, and dwell upon each part; and never begin a second till he thoroughly understands the proportions of all the outlines of the first. He must also be very slow in his first operations; and he cannot too often contemplate the length, breadth, and every other proportion of each object of his original. For this purpose he should have it constantly in his eye, and cannot look at it too frequently.

1. The first part of drawing consists of plain geometrical figures; as lines, angles, triangles, quadrangles, polygons, cones, and the like; for these are the foundation of the outlines of all other figures. The circle assists in all orbicular forms; as the sun, moon, fruits, &c.: the oval, in giving a just proportion to the human face and mouth, the

more exactly drawn with the lead pencil, rubbing out any false or imperfect strokes of the charcoal; then having perused it well, mend it with the pencil, where there are any errors: this done, peruse it again, correcting by degrees all the errors of less magnitude, even to the least jot; then, compare it with the original copy, using neither the rule nor the compass, but giving every part its due place and proportion, according to judgment.

When the artist has arrived to some perfection in the art, he may begin to copy after life, for this is the most correct and complete method of drawing or painting; and here only he has the largest liberty of imitation. But there ought to be some perfection in the art before he aims at this.

Particular Directions for Drawing.

The print or painting which is to be copied, is to be placed so, that the gloss of the colours, or shades, may not fall upon the eye, and thereby prevent a perfect view of the piece: but it is to be so placed, that both the light and the eye may fall obliquely upon it. It must also be placed at such a distance, that the whole may be taken into the eye at once; for which purpose, the larger it is, the further distant it must be placed, and set a little reclining.

Make a small point upon the paper, to represent the centre. And observe, as a general rule, always to begin with the right side of the piece; for by that means, what is finished will not be hidden by the hand or pencil. Observe also the most perspicuous and uppermost figures in the piece (if there be more than one), which are to be touched upon the paper in their proper places, by the charcoal; thus running over the draught, there will be had the skeleton of the work, which is to be afterwards finished and filled up. But great care is to be taken in obtaining a true draught, and the more time there is bestowed upon it, the more it will improve the learner.

profile, which is equal to the length of the eye (*figs.* 18 and 16), and the nostril in height is nearly one third of the width of the nose. These proportions are to be attended to only during the learner's first practice: when he is arrived to a little proficiency, he is to follow his own judgment only, in the proportions, paying more regard to his original than to any verbal directions. When he is able to finish the outline of the profile and full face, he may proceed to the other different inclinations of the face, as seen in the plate.

LESSON III.

When the artist can easily imitate the different features and limbs, he may begin to attempt whole length figures; which is to be done in the following manner: sketch the whole over lightly with the charcoal (or, if the learner be able, he may use the black lead pencil at first), beginning with the head, next the shoulders, then the body; after which, the arms and hands, then the hips, legs, and feet; then examine the proportion of the different parts, rubbing out any strokes of the charcoal where necessary; and drawing the lines over again with the black lead pencil, to bring it as near as possible to the original. When this is done, proceed to finish the figure, by drawing it over again with the crow-quill pen, and Indian ink, departing from the black lead lines where it may be found necessary. Then rub out all the marks of the pencil with India rubber. The compasses are not to be used till after a very minute inspection with the eye: when, if there be any fault that cannot be easily discovered, by applying the compasses, first to the original, and then to the copy, the fault will be soon found. To ascertain the proportions of the several parts of the human body, a perpendicular line should be first drawn through that part intended for the middle of the figure: which should be divided into several equal parts, and from such mensuration
a scale

The learner should pay particular attention to these proportions, and retain them in his memory. It is also necessary that he have some knowledge of anatomy, as it will enable him to judge of the proportion and disproportion of the human figure.

LESSON IV.

The drapery, or clothing of the figure, is next to be considered. Having drawn the outline of the figure faintly with charcoal, correcting every part that appears faulty; proceed to draw the outline of the drapery lightly with the charcoal, with the several folds, not suffering the folds to cross each other. The quality of the drapery should also be considered; as stuffs and woollen cloth are more harsh than silk, which is always flowing and easy. The drapery should not stick too close to the body; but should appear to flow easily. If the drapery be supposed to be blown by a breeze of wind, it should all flow one way: and the parts next the body should be drawn first, before those which fly off. The garments should always bend with the figure; and the closer the drapery is to the body, the smaller must be the folds; and if it be quite close to the body, there should be no folds, but only a faint shadow, to represent that part of the body which it covers. The best rule in this case is, to remark the folds as they appear in the drapery of genteel persons, if the figure be to have a modern dress; but a few particular rules may, however, greatly assist the learner:

1. Carefully avoid a superfluity of drapery.—2. Let as much of the form of the body as possible appear under the drapery.—3. When the draperies are large, let them be thrown into large and graceful folds.—4. Drapery which is close to the body, should appear to be loosened by small folds, judiciously placed: for want of this caution, the figure will have a certain stiffness, and appear as if wrapped round with a bandage, instead of being clothed.—5. If there be much drapery, let the greater part, if possible, be thrown into shadow.—6. Those folds which fall in the light must have such

The shades should be rubbed with a piece of paper or glove-leather, rolled up hard, and cut almost to a point, like a pencil; which serves to blend the shades, and soften them into each other; and also to weaken them, where they appear too strong. India ink is prepared for different shades, by rubbing it, more or less, in water on a marble stone, cut in hollows for that purpose, reserving one of the hollows for the water. In finding a piece, as well as in drawing the outlines, the best rule is to follow nature, which will greatly improve the ideas of the learner in the disposal of light and shade.

To enlarge, or contract, a Piece.

Divide the original into any number of equal squares; then divide the paper, upon which the piece is to be drawn, into the same number of equal squares, either greater or less, than those in the original, according as the piece is to be enlarged or contracted, and draw the several parts of the piece as they fall in the squares of the original, in the corresponding squares of the paper: then outline it with India ink, and rub out the black lead marks of the squares; and shade the piece, &c. To prevent mistakes, the squares may be numbered, both in the original and copy (*fig. 14 and 15. plate 25*).

Though the human body be divided into eight or ten heads, as before observed, and which may answer for small figures, and where great exactness is not expected, yet it must be remembered, that these are not the exact proportions. Where the figures are large, the artist should be more correct; for which purpose, it may be necessary to attend to the proportions of the ancients, as they are given by M. Audran, from the proportions of the Apollo Pythius, in the garden of the Vatican, at Rome, and the Venus Aphrodite, belonging to the family of the Medicis: both which figures are supposed to stand upright, duly poised on both legs. The whole height of the former is divided into $31\frac{1}{2}$ parts, being 7 heads 2 parts and 6 minutes; and the latter into 31 parts, being 7 heads and 3 parts, as follows:

3 F 2

Length

Length of the Fore Arm, or Upper Extremities.

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
From the top of the shoulder to the elbow	1	2	3	1	2	3
The elbow to the hand	1	1	2	1	0	6
The joint of the hand to the root of the middle finger	0	1	8	0	1	6
The root to the tip of the middle finger	0	1	10	0	1	7
Length of the upper extremities	3	2	11	3	1	10
Breadth between the outward angles of the eyes	0	1	6	0	1	7
Of the face at the temples	0	2	2	0	2	2
Of the upper part of the neck	0	2	0	0	1	11
Over the shoulders	2	0	0	1	3	8
Of the body below the arm-pits	1	2	5	1	1	8
Between the nipples	1	0	7	0	3	8
From the bottom of the chin to the line crossing the nipples	1	0	7	1	0	1
Of the body, at the small of the waist	1	1	0	1	0	8
Over the loins	1	1	3	1	1	6
Over the haunches, or tops of the thigh bones	1	1	5	1	2	3
Of the thigh, at the top	0	3	0	0	3	1
Of the thigh, below the middle	0	2	8 $\frac{1}{2}$	0	2	7
Of the thigh, above the knee	0	1	8	0	2	0
Of the leg, below the knee	0	1	6	0	1	10 $\frac{1}{2}$
Of the calf of the leg	0	2	4	0	2	3
Below the calf	0	1	7	0	1	11 $\frac{1}{2}$
Above the ankle	0	1	2	0	1	2
Of the ankle	0	1	4	0	1	3
Below the ankle	0	1	1 $\frac{1}{2}$	0	1	1

Middle

	Apollo.			Venus.		
	Hd.	Pts.	Min.	Hd.	Pts.	Min.
From the fore to the back part of the skull	0	3	6	0	3	4
The wing of the nose to the tip of the ear	0	1	8½	0	1	6
The upper part of the neck	0	2	0	0	1	11
The breast to the back, over the nipple	1	0	6	1	0	6
The belly to the small of the back	0	3	6	0	3	7
The belly, above the navel, to the back of the loins	0	3	9	1	0	2
The bottom of the belly to the round of the hip	1	0	0	1	0	5
The fore part of the thigh to the bottom of the hip	0	3	2	0	3	7
The middle of the thigh	0	3	3	0	3	6½
The thigh, above the knee	0	2	1	0	2	3
The middle of the knee	0	2	1	0	2	2
The leg, above the knee	0	1	9	0	1	11
The leg, at the calf	0	1	8	0	1	9
The leg, at the ankle	0	1	5½	0	1	4
The foot, at the thickest part	—	—	—	0	1	3
Length of the foot	1	0	6	1	0	4½
The heel to the fore part of the bend of the foot	—	—	—	0	2	2
The arm, over the biceps	0	2	0	0	1	9
Over the elbow	0	1	6	0	1	6
Below the elbow	0	1	5	0	1	7
At the wrist	0	1	1	0	0	11
Below the joint of the wrist	0	1	0	0	0	10
The hand, at the roots of the fingers	0	0	5½	0	0	5
At the roots of the nails	0	0	3½	0	0	3

These

plates where there is a complexity of work, though they are generally finished by the graver.

The instruments proper for etching, are, needles, an oil-stone, brush pencils, a burnisher, scraper, compasses, ruler, tracer, and the graver; with the hard and soft varnish, prepared oil, and aqua-fortis.

The needles should be of a fine grain, and such as will break without bending, of which there should be several sizes. They are to be fixed in firm round sticks, about six inches in length, and the thickness of a large goose-quill; and may be fixed in such sticks, as have a pencil at the other end. They should stand at least a quarter of an inch out of the stick.

The oil-stone is to whet the needles upon: and, *note*, if the points are to be round, they are to be whetted short upon the stone, by turning them round; but if the points are to be sloped, they are first to be blunted upon the stone, and then whetted, sloping on one side only, till they come to a short oval.

The brush pencil is to cleanse the work, wipe off the dust, and strike the colours, even over the ground, when laid upon the plate.

The burnisher is a piece of tempered steel, somewhat round at the end, for smoothing and giving a lustre to the plate.

The scraper is used for clearing the plate of any scratches, or strokes, which the burnisher will not take out.

The compasses should have steel points, and are chiefly used in striking circles, measuring distances, &c.

The ruler is chiefly used to draw straight hatches, or lines, upon the plate.

The tracer is used for drawing through all the outermost lines, or circumference of the print or drawing, which is called etching after.

The manner in which etching is performed, is, by covering the surface of the plate with a proper varnish, or ground,

The manner of etching with the soft varnish is now more frequently intermixed with the use of the graver than formerly; which is generally attended with great advantages, even where the whole is intended to pass for the work of the graver; as it gives an opportunity of shewing the truth and spirit of the outline, and gives all the variety of shades which the different kinds of black can produce; while the exactness and regularity of the lines, which are requisite for finishing many kinds of designs, are supplied by the graver; and by a judicious application of both, that complete finishing and effect is produced, which either of them alone would be incapable of affording.

The Preparation of the soft Varnish, as directed by Mr. Lawrence, an eminent English Engraver at Paris.

“Take of virgin wax, and asphaltum, each two ounces; of black pitch, and Burgundy pitch, each half an ounce: melt the wax, and pitch, in a new earthenware glazed pot, and add to them by degrees the asphaltum, finely powdered; let the whole boil till such time, as that (taking a drop upon a plate) it will break, when it is cold, on bending it double two or three times betwixt the fingers. The varnish being then boiled enough, must be taken off the fire; and letting it cool a little, must be poured into warm water, that it may work the more easily with the hands, so as to be formed into balls; which must be rolled up, and put into a piece of taffety for use.”

In boiling the ingredients, it must be observed, first, that the fire be not too violent, lest they burn—a slight simmering will be sufficient: secondly, while the asphaltum is putting in, and even after it is mixed with them, the ingredients should be stirred continually with the spatula: and, thirdly, the water, into which this composition is thrown, should be nearly of the same heat with the composition, to prevent a kind of cracking, which will happen when the water is too

times two or four candles are used together, for dispatch; for the varnish must be blackened before it grows cold; for if it grows cold during the operation, the plate must be heated again, that the varnish be in a melted state when that operation is performed: great care must be taken, not to scorch it; which may be perceived, when it happens, by the varnish losing its gloss, and appearing burnt. Large plates are sometimes suspended from the ceiling by four cords, with an iron ring about four inches diameter at the end of each cord, to hold each corner of the plate. The plate being thus suspended with the varnish side downwards, may be blackened very conveniently.

In blackening the varnish, the candle or flambeau should be kept at a proper distance from the plate, that the wick may not touch the varnish. If, after the operation, it appears that the smoke has not penetrated the varnish, the plate must be again heated over the chafing-dish, and as the plate grows hot, the varnish will gradually melt and incorporate with the smoke, that lay above it, in such a manner that the whole will be equally pervaded by it.

The greatest caution is necessary in this operation, to keep a moderate fire all the time, to move frequently the plate, and change the place of every part of it, that the varnish may be equally melted every where, and kept from burning, and to keep the varnish entirely free from any filth, spark, or dust, till it be entirely cold.

The Method of applying the hard Varnish.

This is exactly the same as that of applying the soft varnish; being spread equally over the warm plate with the taffety ball, and smoked in the same manner; but after it is smoked, it must be baked, or else dried over a gentle charcoal fire, till the smoke of the varnish begins to decrease; observing not to heat the plate too much, which would burn, and soften the varnish.

The

The oval-pointed needle is most proper for making large and deep strokes. It should be held in the same manner as a pen, with the flat side next the thumb; though it may be used, with the face the other way: it must be held as upright and straight in the hand as possible, striking the strokes freely and firmly, which render them neat and clear.

The fine needles, with slender points, are proper for fine strokes, and for the faint strokes of those places at the greatest distance, in a landscape; and also for those places nearest the light. And it is requisite, when at work, to brush off all the loose dust which is worked up by the needles.

It is hardly necessary to observe, that the student should be so far master of the art of drawing, as to be able to copy any print exactly; before he attempts etching. It is also necessary that he be able to hatch with a pen or pencil exactly, from good copies; and then he will be able to draw from plaster, or from the life.

In shading his piece, he must be careful to observe how the original is shadowed, how close the hatches are joined, how they are laid, how they incline, and which way the light falls, which must always fall one way. If the light fall sideways in the print, that side, which is farthest from the light, must be hatched the darkest.

In landscapes the part next to the eye is to be hatched darkest; and the rest to decline in its shadow gradually, the farther it is off from view. The same is to be observed in etching a sky; for that which is nearest the eye, must have the deepest shades; but in general, as soft and faint as possible, gradually losing its shades as it comes nearer to the ground; and where they both meet, as it were, the sky must be entirely lost.

If any scratches, or false strokes, happen in the working, they are to be stopped up with a hair pencil, dipped in the Venetian varnish, mixed with lamp black, by which means these places will be defended from the aqua-fortis.

The

The management of the aqua-fortis is the principal matter in the whole art of etching, and on which the success of the work chiefly depends. For the exact strength of the aqua-fortis, the time it is to continue on the plate, &c. no certain rule can be given; but practice and experience alone can inform the artist.

Of etching Letters.

To etch letters, the copper-plate is to have a ground of virgin wax, which is to be spread very evenly with a feather, all over the plate, while it is warm; then the letters being wrote on paper with a black lead pencil, the written side of the paper is to be laid upon the ground of the plate, the paper being fastened at the four corners, as before directed. Then rub the back of the paper all over with a burnisher, taking care to rub every part of the paper; and taking the paper off the plate, the letters will all appear written on the wax, but reversed; they are then to be drawn through the wax on the plate with a tracer, cleaning the work from the loose wax with a linen rag, or pencil brush; then raising a border of wax, and pouring on the aqua-fortis, as before, the letters will be etched. The plate, being cleaned from the wax, is, in the next place, to be polished, as follows:—take a piece of good charcoal, and pulling off the rind, put fair water on the plate, and rub it with the charcoal: and by this means the plate will be cleared from all the varnish. But the charcoal should have no knots, or roughness. After this, wash the plate with a little aqua-fortis, added to twice its quantity of water. Lastly, the plate should be wiped dry, and rubbed with a little of the olive oil. Then, if any place require to be touched with the graver, it may be corrected; and the plate will be finished.

Of holding the Graver.

The graver should not exceed the length of five inches and a half, including the handle, except it be used for straight lines; and that part of the handle which is on the same line with the belly, or sharp edge of the graver, should be cut off flat, that it may be no obstruction in working.

The handle of the graver should be held in the hollow of the hand, with the fore-finger resting upon the back of the graver, in order that it may be moved parallel to the plate. Great care must be taken that the fingers do not interpose between the plate and the graver, which would prevent the graver from being carried level with the plate, and render the strokes not so clean.

Of laying the Design upon the Plate.

The plate being polished smooth, is to be heated, so as to melt virgin wax, with which it is to be rubbed thinly and equally all over, and suffered to cool. The design to be laid on must be drawn on paper, with a black lead pencil, and laid upon the plate, with the pencilled side upon the wax; it is then to be pressed close to the plate, and rubbed over every part with a burnisher. Then taking the paper off the plate, every line drawn with the black lead pencil will appear upon the wax, which are to be traced through the wax upon the plate with a sharp-pointed tool. The wax being taken off, the plate is to be engraved.

Of whetting and tempering the Graver.

Great care is required to whet the graver nicely; for which purpose, the two angles of the graver, which are to be held next the plate, are to be laid flat upon the stone, and rubbed steadily, till the belly rises gradually above the plate, so that, when the graver is laid upon it, the light may be seen under the point; if it be not whetted in this shape, it will dig into the copper, and it will be impossible to prevent the

cushion ; pressing more lightly, where they are to be fine, and leaning with greater force, where they are to be broad and deep. In making circular and curve lines, turn the plate upon the cushion against the graver. After some of the work is done, it is necessary to scrape off the roughness formed by the cutting of the graver, which is done with the scraper : or passing the graver over the plate in a level direction, taking care that it does not catch the copper. To render the work more visible, it may be rubbed over with a roll of felt dipped in oil. It is necessary to learn to carry the graver as level as possible with the surface of the plate ; for otherwise, if the fingers slip betwixt them, the line that is produced will become deeper and deeper in the progress of its formation, which will prevent making a stroke at one cut, that will be fine at the extremities, and larger in the middle ; and renders it necessary to retouch it. Therefore it is necessary to acquire the habit of making such strokes, both straight and curved, by lightening, or pressing, the hand, according to the occasion. And when the design is finished, if any scratches or false strokes appear in any part of the plate, they must be taken out by the burnisher.

In order to preserve a due equality in the work, the principal objects of the design should be sketched out, before any of them be finished. In working with the graver, the strokes should never be crossed too much in the lozenge manner (particularly in representing the flesh of the human body), except in the case of a cloud, waves of the sea, the skins of animals covered with hair, or the leaves of trees, where this method of crossing may be admitted. In the disposition of the strokes, the action of the figures, and the disposition of their parts, should be considered : and also the manner in which they advance towards, or depart from, the eye of the observer. The graver should be so guided, as to mark the rising or cavities of the muscles, making the strokes wider and fainter in the light, and closer and bolder in the shades. Thus, the hand should be lightened in such a manner,

by perpendicular strokes. When a gross stroke is made, it should be at right angles, and wider and thinner than the first stroke. In engraving mountains, as there are sharp and craggy objects, the strokes should be frequently broken; they should also be straight, in the lozenge manner, and accompanied with long points or dots. Rocks have some cross strokes in them more square and even. Distant objects towards the horizon are very slightly shaded, as in drawing. Calm, still waters should be represented by straight strokes, parallel to the horizon, and interlined with finer strokes; omitting those places where the light casts a shining reflection; and the form of objects reflected from the water at a small distance upon it, or on the banks of the water, are expressed by the same strokes, retouched more strongly or faintly, as it may be necessary: agitated waters, as the waves of the sea, have the first strokes in the figure of the waves, and are interlined; and the cross strokes should be very lozenge. The first strokes in cascades should follow the fall of the water, and be interlined. In clouds, that appear thick and agitated, the graver must be turned every way, according to their form and agitation. In dark clouds, where two strokes are necessary, they should be crossed more lozenge than the figures, and the second strokes should be rather wider than the first. The flat clouds, that are insensibly lost in a clear sky, should be formed by strokes parallel to the horizon, but a little waving; if second strokes be required, they should be more or less lozenge, and should be so lightened at the extremities, as to have no outline. The flat and clear sky is represented by strokes parallel and perfectly straight.

In all landscapes, in general, the trees, rocks, earth, and herbage should be etched as much as possible, leaving nothing to be done by the graver, but the perfecting, softening, and strengthening. And observe, once for all, that the dry needle produces a more delicate effect, and may be used to much greater advantage than the graver can, particularly
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be neatly imitated, if a plate be provided for every colour. And if it be well done, it will form such a good deception, that an able connoisseur cannot, from the first inspection, distinguish between the original drawing and the engraved imitation: therefore, this mode of engraving is very useful to multiply copies of drawings left by ancient artists who excelled in the use of chalks.



SECT. IV.

OF MEZZOTINTO AND AQUA-TINTA SCRAPING.

MEZZOTINTO prints have no hatches, or strokes of the graver; but the lights and shades are more blended together than in etchings and engravings, and appear like a drawing of India ink.

This art is of late invention, but is greatly used, and is admired for the amazing ease with which it is executed, particularly by persons who are deficient in drawing.

The principal tools used in this art, besides those used in etching and engraving, are the grounding tool and the scraper.

General Directions for laying the Mezzotinto Ground.

Leave a space upon the bottom of the plate for the writing, coat of arms, &c. then, laying the plate upon a piece of swan-

where there is not to be any shade, are to be softened or rubbed down with the burnisher, otherwise these parts will not appear clear, when the work is proved.

There is also another method, used by mezzotinto scrapers: which is, to etch the outlines of the original, with all the folds in the drapery, &c. marking the breadths of the shadows by dots; then having used the aqua-fortis, as in etching, and the ground being taken off the plate, the mezzotinto ground is to be laid, and the work finished by scraping, as above.

When the work is to be proved, it is necessary to have some French paper, which has been wetted down four or five days, as no other paper will do for this work, and it is necessary for it to lie wet that length of time. A proof is then to be taken of the plate; when the proof is dry, correct it, by touching it with white chalk, where it should be lighter; and with black chalk, where it should be darker. In retouching the plate, proceed as before, where it should be lighter, by using the scraper; and where it should be darker, use a small grounding tool, as much as is thought necessary to give it its proper shade. Then it is to be proved again, and again corrected and retouched; and thus proceed to prove, and retouch it, till it be finished.

It is to be observed, that the work should be proved the first time, before it is the least over-scraped in any part; as, by this caution, it will appear more elegant; for the small grounding tool, which is used to deepen any shades that are over-scraped, generally gives the work a coarse appearance.

Of Aqua-tinta Scraping.

Aqua-tinta is that method lately invented of etching, by which a soft and beautiful shade is given, resembling a drawing in water colours, or India ink.

The principal operation is as follows: the etching ground is to be laid on the plate, as in common etching, and the out-

as requisite, with the needle or point, by stippling with dots, and biting up those parts; or by a rolling wheel.

The foregoing method will only serve for prints of one single tint. When different colours are to be expressed, there must be as many different plates; each plate having only that part etched upon it, which is designed to be charged with its proper colour, unless (as sometimes happens) some of the colours are so distant from each other, as to allow the printer room to fill them in with his rubber, without blending them; in which case, two or more different colours may be printed upon the same plate at once. When different plates are requisite, there must be a separate one, having a pin in each corner, to serve as a sole or bottom to the aqua-tinta plates; and the aqua-tinta plates must be exactly fitted, having each a small hole in their corners for passing over the pins of the sole; the pins retain the plates in their due position, and also direct the printer in placing the paper exactly on each plate, so as not to shift; by which means each tint or colour will be exactly received on its proper place. This is the method practised by the Paris printers. Some subjects, however, such as landscapes, may be printed off at once in the different proper colours, by painting these upon the plate. Here the colours must be pretty thick in consistence, and the plate carefully wiped in the usual way, after laying in each tint, as well as to be wiped in general, when it is charged with all the tints.

In aqua-tinta plates, it must be observed, that the asphaltum and resin must be finely powdered, and well incorporated together, before they be fitted on the plate; for this purpose, it is necessary to sift each of them through a fine muslin sieve, sifting first a layer of one on a sheet of paper, and then a layer of the other; proceeding in this manner till the whole be finely sifted, and well incorporated together.

This art has been hitherto kept as secret as possible; but a strict attention to what has been delivered will enable the practitioner to finish his plate with success.

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flake white and white lead should be wholly avoided, as the slightest touch with either of these will always turn black, for these whites will stand only in oils. Therefore, when white is required, I would recommend the student to make use of common whiting, prepared in the following manner:—

Put some whiting in a large vessel of water, mixing them well together; when this has stood about half a minute, pour off the water into another vessel, and throw the gritty sediment away; after this water has rested about a minute, pour it off as before, which will purify the whiting from all dirt and grittiness. This being done, let the whiting settle, and pour the remainder of the water from it; after which lay it on the chalk to dry, and when dry, it will be fit for use; either for making white crayons, or for preparing tints with other colours. And, *Note*, if the student makes the crayons of the whiting immediately after it is washed, it is not necessary to dry it on the chalk; for it may be mixed with any other colour instantly, whereby much trouble will be saved. All colours of a heavy or gritty nature, particularly blue verditer, must be washed in this manner.

The student must be provided with a large flexible pallet-knife, a large stone, and muller, to levigate the colours; two or three large pieces of chalk, having large smooth surfaces, to absorb the moisture from the colours, after they are levigated; and a piece of flat glass, to prevent the moisture from being absorbed too much, till the colours are rolled into form. These implements being provided, the student may proceed to form his crayons from the following colours:

Reds are formed either from carmine, lake, or vermillion, or a composition of two or more of them; though it must be observed, that it is difficult to procure either good carmine or lake; good carmine is inclined to the vermillion tint, and should be an impalpable powder: a good lake should incline to the carmine tint.

The carmine crayons are prepared by mixing a sufficient quantity of good carmine with spirits of wine with the levigating

Lake crayons are somewhat difficult to form, on account of the harshness of the lake, therefore the student should observe the following rules in the formation of these crayons:— Take about half the quantity of the lake intended for the crayons, and grind it very fine in spirits of wine; when dry, pulverize it; and take the other half, and grind it with spirits of wine; after which, mix it with the pulverized lake, and lay it out directly in crayons on the chalk: this colour will not bear rolling. The simple colour being thus prepared, proceed with the compound crayons, as before directed in the carmine crayons, and in the same degree of gradation.

Vermilion crayons are formed by mixing the vermilion on the stone with the spirits of wine, or even soft water; after which it may be rolled into crayons. The different tints are produced by mixing the simple colour with whiting, according to the proportions given in the carmine. And, *Note*, that these crayons will sometimes be so soft, that they cannot be held in the fingers, but will break, and return to powder: which may be remedied by mixing the colour with some thin water gruel, well strained, which will give it sufficient cohesion.

Blues are formed of Prussian blue, and blue verditer.

Prussian blue crayons are formed in the same manner as the lake crayons: but as the Prussian blue is very apt to bind, it is somewhat more difficult to be softened than either lake or carmine. It is necessary to grind a large quantity of this colour, as it is chiefly used in draperies. The different tints may be made according to the fancy of the painter.

Blue verditer crayons are somewhat more difficult to form, on account of the coarse gritty nature of the verditer, which requires some binding matter to unite it, otherwise it will never adhere together. Therefore, to a quantity of blue verditer, sufficient to form two or three crayons, must be added a little sifted plaster of Paris, about the size of a pea: these are to be mixed well together, and the crayons formed

particularly in the light green ones, which will turn black on the pictures, if the least damp come to them: though the dark colours will remain perfect.

In order to discover whether there be any flake white in the crayons, the following experiment may be made: having bruised the crayon to a powder, mix it with an equal quantity of charcoal dust; put the whole into a crucible, which must be placed in a fierce fire till the charcoal dust be consumed; and if the crayon have any flake white in it, the lead will return to its original metallic state.

Browns are originally produced from Cullen's earth, or umber.

Cullen's earth crayons are of a fine dark brown, and several rich tints may be produced from a mixture of this colour with carmine, in various degrees: also, this colour mixed with black and carmine, makes useful tints for painting the hair. Several gradations may be made from each of these, by a mixture with whiting. Roman and brown ochre also form an excellent colour, either mixed together, or compounded with carmine. Whiting, tinged in several degrees with either of these, will prove very serviceable. Common sea-coal, ground to a fine powder, and mixed with carmine, forms a very fine brown.

Umbre crayons are formed in the same manner as the above; but it is necessary to levigate the umber with spirits of wine.

Purples are formed by a mixture of blue and red. Good purple crayons may be formed with Prussian blue, ground with spirits, and mixed with pulverized lake. Also Prussian blue and carmine produce a deep purple of an excellent hue. From either of these compounds various tints may be made, by a mixture with whiting.

Black crayons are formed of lamp black, as no other full black can be used with safety, all others being subject to mildew. But as lamp black is liable to great adulteration,

too wet, it must be laid upon the chalk again, to absorb more of the moisture. They should be rolled as quick as possible; and when finished, must be laid upon the chalk again to absorb the remaining moisture. When all the crayons of one colour are formed, the chalk and grinding stone should be well scraped, and washed with water, before they are used for another colour.

When a set of crayons is completed, they should be ranged in some thin drawer, divided into a number of partitions, and disposed according to the several gradations of light. The bottom of the partitions should be covered with bran, to preserve the crayons clean, and prevent them from breaking.

The box in which the crayons are placed for use, and which should be held in the lap, when the student paints, should be about a foot square, having nine partitions. In the upper corner, on the left hand, the black and gray crayons are usually placed, as they are the most seldom used; in the second partition are placed the blues; in the third the greens and browns: in the first partition, on the left hand of the second row, the carmines, lakes, vermilions, and all deep reds are deposited; the yellows and orange are in the middle partition; and in the next are placed the pearly tints, which being of a delicate nature, must be kept very clean, that the different gradations of colour may be easily distinguished: in the last row, the first partition contains a piece of linen rag to wipe the crayons with, while they are using; the second partition holds the pure lake and vermilion tints; and the last partition contains all those compounded tints, which cannot be classed with any colour.

Directions for the Artist.

To arrive at excellence in this art, the student should be as particular in the outline of the work, as in the disposal
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when the subject to be imitated, is in oils ; but if it be a crayon picture, the following method must be used, on account of the glass.

The picture being placed upon the esel, draw all the outlines upon the glass with a small camel's hair pencil, dipped in lake, ground very fine in oils ; then take a sheet of paper, and place it on the glass, stroking over all the lines with the hand, by which means the colours will adhere to the paper, which is then to be pierced with pin-holes pretty close to each other in all the outlines. The paper intended for the drawing is then to be laid upon the table, and the pierced paper to be laid upon it ; then, with some fine powdered charcoal, tied up in a piece of lawn, rub over all the pierced outlines, which will give an exact outline of the piece, upon the paper under it. This is not to be brushed off till the whole is drawn over with sketching chalk : which is a composition made of whiting and tobacco-pipe clay, rolled like a crayon.

But when the student paints immediately from life, it is best to make a correct drawing of the outlines on another paper, which he may trace by the first method : for if there be any false strokes of the sketching chalk, they will prevent the crayons from adhering to the paper.

The sitting posture is the most proper for painting with crayons, having the box of crayons in the lap. That part of the picture which the student is at work upon, should be below his face ; for when it is placed too high, it will fatigue the arm. The windows of the room, in which the artist works, should be darkened to the height of six feet from the ground, and the subject to be painted should be situated in such a manner, that the light may fall on the face to the greatest advantage, avoiding too much shadow, which seldom has a good effect in this kind of painting, particularly if the face have much delicacy.

In painting, as well as drawing, the student cannot be too attentive to the subject ; he must also learn to appropriate

Whatever colour the iris of the eyes is, the eyes must be first drawn with a crayon, inclined to the carmine tint; the colour must be laid in brilliant at first, and executed lightly, not meddling with the pupil yet. The light of the eye should incline very much to the blue cast; for if a staring white appearance is once introduced, it can seldom be altered: a broad shadow should also be thrown on the upper part by the eyelash. The eyebrows should be executed at first like a broad glowing shadow, on which is to be painted, in the finishing, the hair of the eyebrows, by which the former tints will shew themselves through, and produce a pleasing effect; but a black heavy tint is always to be avoided in first forming the eyebrows.

The lips should be begun with pure carmine and lake, shading them with carmine and black, and laying on the strong vermilion tints afterwards. Great caution is necessary to avoid stiff, harsh lines: each colour is to be gently intermixed with the neighbouring colour; the shadow beneath should be broad, and enriched with brilliant crayons. The corner of the mouth is formed with carmine, brown ochre, and greens, variously intermixed. If the hair be dark, it is necessary to use a good quantity of the lake, and deep carmine tints therein, which may be easily overpowered by the warmer hair tints, and which, as in the eyebrows, will produce a richer effect when the piece is finished, than if the lake and carmine be neglected.

When the student has dead-coloured the head, he is to sweeten the whole together, by rubbing it over with his finger, beginning at the strongest light upon the forehead, and passing his finger very lightly to the next tint, to unite them together; which he must continue to do, till the work is sweetened together, frequently wiping his finger on a towel, to prevent sullying the colours. In this process the student must be careful not to sweeten his picture too often; as that would produce a thin and scanty effect, and the piece would have more of the appearance of a drawing than a solid

the beauty of the face. This is requisite even in a simple back ground, where there is but one object in the piece; but more attention is required in the back ground of a picture which has several objects.

A great variety of colours are used for back grounds; but they should always be suited to the complexion of the figure. A strong coloured head generally should have a weak and tender tinted ground; and, on the contrary, a delicate complexion requires strong and powerful tints in the ground, by which proper contrast between the figure and the back ground, the picture receives great force.

But when several objects are introduced in one piece, as hills, trees, buildings, &c. the general rule to be observed is, that each grand object be disposed so, as to contrast each other, not merely in their forms, but in their colour, light, shade, &c. For example: suppose a figure in the piece, receiving the strongest light, and behind this figure, and near at hand, suppose there be stems of some large trees: these stems must have shade thrown over them, either from a driving cloud, or some other interposing object; behind these stems or trees, and at a distance, suppose there are seen trees on a rising ground; these again should receive the light, whereby they will serve as a contrast to the former; and the same may be observed in all other cases. The same rule holds good in an architectural back ground; as, suppose a building at a moderate distance, and behind this building, the figure which receives the light; a column, or some other object in the shade, intervenes to preserve proper decorum in the piece; or, what will have the same effect, a shadow may be thrown over the lower part of the building. In a word, it must be remembered, that the light must be always placed against the dark, and the weak against the strong; and *vice versa*, in order to produce force and effect.

In finishing the complexion, the student should be particularly attentive to Nature herself: for whoever carefully examines a clear and transparent skin, will discover a pleasing variety of colours on the surface, and discernible, through it, which will be greatly increased by the effect of light and shade; one part will appear to incline to the vermilion, another to the carmine or lake, one to the blue, another to the green, and another to the yellow, &c. Now, in order to produce these effects, a good artist will apply those colours corresponding to the tints, using, as often as he can, the compounded colours, instead of the simple colours; as, blue and yellow, instead of green, blue and carmine, instead of purple, and red and yellow for orange. In all other circumstances the compounded crayons already mixed should be used; but in this case no absolute rule can be given; the success of the piece depending upon the experience and discretion of the artist. And, observe, that it is impossible to give any set of rules for forming the complexion, that will hold in every case, the circumstances that require different treatments are so many and various; but great advantage will be derived, in the commencement of this art, by an able master, to direct the student, and point out the deformities and beauties of a piece, as they occur in practice; which, to a good capacity, will soon become clear and intelligible.

In finishing the cheeks, use the pure lake tint, which will clear them from any dust they may have contracted from the other crayons, mixing with the lake some bright vermilion; and lastly, (if the subject require it,) give a few touches of the orange crayons, but with great caution. This being done, sweeten the part with the finger as lightly as possible, lest it produce a heavy disagreeable effect on the cheeks; for the only method of imitating a beautiful complexion, consists in one colour shewing itself through, or rather between, other colours.

The eye is next to be executed. This is generally found the most difficult feature in the face, as every part must be expressed

completely correct; and in the finishing, have a little of the strong vermilion; but with great caution, as this colour is very predominant. This colour, if properly used, will give the complexion appearance equal, if not superior, to those executed in oils, notwithstanding the great advantage the latter have, by glazing, of which the former is destitute.

In drawing the neck, the student should carefully avoid giving too much expression to the muscles, in the stem; and also be careful that the bones appear not too prominent on the chest, as either of them has an unpleasant effect, and denotes a violent agitation of the body, which is seldom necessary in portrait painting. The most necessary part to be expressed, is a strong marking just above the place where the collar bones unite. This should always be expressed even in the most delicate subjects; and if the head be thrown much over the shoulders, the muscle that rises from behind the ear, and is inserted into the pit, between the collar bones, should be faintly marked. But, in general, all inferior muscles should be quite avoided, and not noticed. Many artists, in the portraits of thin persons, mark the muscles of the neck too evidently. The neck should, in general, have a small addition to the length, as few necks are too long; and nothing is more ungraceful than a neck too short; the stem of the neck should have a warm hue; and the light should not appear too strong upon the chest. The breast also (if any part appears) should be expressed by pearly tints, but blended with beautiful vermilion in the upper part thereof.

Of Drapery.

The drapery, by many young artists, is thought to require very little attention; but this is an egregious mistake. An eminent painter being asked, what part of the picture he thought the most difficult to execute? he answered, 'The drapery'—and the best judges of the art have universally allowed

A P P E N D I X.

NECESSARY RECEIPTS FOR THOSE WHO PAINT IN WATER COLOURS.

To make Gum Water.

DISSOLVE one ounce of pure gum-arabic, and half an ounce of double-refined sugar, in a quart of spring water: strain it through a fine sieve, or piece of fine muslin, and bottle it up for use, to keep it from the dust.

Or, secondly, take some of the whitest sort of gum-arabic, bruise it, and tie it up in a piece of woollen cloth; and steep it in spring water till it be dissolved. If it be too stiff, add more water; and if it be too thin, more gum.

With this water, most of the colours are to be mixed; and in such a proportion, that the colour may not rub off, when dry. If the colour shine, it is a sign there is too much gum in the water.

water. This will prevent the colours from sinking, also give them an additional beauty and lustre; and likewise preserve them from fading. If the paper is not good, it should be washed three or four times with the water, drying it every time.

To make Size for painting Scenes, or other Candle-light Pieces.

When the colours, mixed with gum water, are laid upon any surface, they are apt to produce a glare by candle-light: to prevent which, the colours should be mixed with the following size, while it is warm:—Steep a quarter of a pound of the cuttings of white leather for some time in water; or for the space of two or three days: then take them out, and boil them in three quarts of water, till it be consumed to one pint, and strain it through a cloth. If it feel firm under your hand when it is cold, it is a sign it is of a sufficient strength.

To lay Mezzotinto Prints upon Glass.

Having a clear plate of glass, as straight as possible, and a little larger than the print to be laid upon it, soak the print in warm water for about an hour, and with a thin, flexible pallet knife, spread some Venice turpentine, or good varnish, very thinly and evenly over one side of the glass; observing to keep the glass warm, that it spread the better, and taking care that there be not the least speck in the glass uncovered with the turpentine; then take the print out of the water, and spread it between two cloths, or several folds of soft paper, in order to absorb the superfluous water. Next lay the print on the glass by degrees, beginning at one end, and stroking outward that part which is fastened

To get the Colours out of the Bladders.

Prick a small hole near the bottom, and press the bladder until enough run out for present use, for if they stand open they are apt to spoil.

With these colours any tints or shades whatever may be exactly imitated, by the different ways and methods of mixing them, according to judgment.

To use the Colours.

The lighter colours are to be first laid on the lighter parts of the print, and the darker colours are next laid over the shaded part, and in the regular order in which the shades deepen; for when the brighter colours are once laid on, it is not material if the darker colours be laid a little over them; as the colour first laid on will always hide those laid on afterwards. The colours are not to be laid on too thick: and if any of them be too thick in consistence, they should be thinned, before they are used, with a little oil of turpentine.

If any of the colours be too strong, or dark, they may be lightened to any degree, by mixing more or less white with them on the pallet; or if they be too light, they may be darkened to any degree, by mixing them with a deeper shade of the same colour.

Note. It is necessary to have a pencil for each colour; but that pencil which has been used for green should never be used for any other colour, without first washing it well with oil of turpentine, as green will always appear predominant when the colours are dry. And it is also necessary to wash all the pencils in oil of turpentine after using them.

To make the Mastic Varnish.

Put two ounces of the clearest gum mastic, finely powdered, into a bottle, with six ounces of oil of turpentine: stop the bottle close, and shake them well together, in order to incorporate them with each other. Then hang the bottle in a vessel of boiling water for half an hour, taking it out three or four times to shake it. If it be necessary to make the varnish stronger, it may hang a quarter of an hour longer in the boiling water.

To make Camp Paper, with which a Person may write or draw, without Pen, Ink, or Pencil.

Mix some hard soap with lamp-black and water, into the consistence of a jelly; with this mixture brush over one side of the paper, and let it dry. When you use the paper, put it between two sheets of clean paper, with its black side downwards: then with a pin, a stick, or any other substance with a sharp point, draw, or write upon the clean paper; and where the point has touched, there will be the impression upon the lowermost sheet of paper, as if it had been drawn or written with a pen.

This camp paper may be made of any other colour, by mixing the soap with different colours.

By this paper also any print or drawing may be exactly copied, by laying it under the same, and tracing the outlines, &c.

To prepare a Plaster Mould so as to take an Impression from it.

Having prepared a plaster mould, according to the foregoing receipt, and letting it be quite dry, dip it in the following mixture : half a pint of boiled linseed oil, and one ounce of spirits of turpentine ; these are to be mixed well together in a bottle, and when wanted, the surface of the mould is to be dipped into it, and then suffered to dry. When the mould has sucked up the oil on its surface, it is to be dipped again in the oil. This operation is to be repeated till the mould will imbibe no more oil, and the oil begins to stagnate upon it ; then, with a little cotton wool, rolled up hard, wipe all the loose oil off the mould, and put it in a dry place for a day or two, to dry, and the mould will acquire a very hard surface from the effect of the oil. When it is to be used, it must be oiled with oil of olives, in the same manner as before directed. By these two methods, any medal, seal, or impression, may be so exactly imitated, that the new medal can scarcely be distinguished from the original.

The Method of casting Brimstone, and of giving it a metallic Gloss.

Melt some stone brimstone over the fire, in an iron ladle, and let it flame for about five or six minutes, then take it off the fire, and extinguish the flame, by covering the mouth of the ladle with a piece of board ; when it is a little cool, so as not to feel gluey, or run ropy, it is then fit for use, and may be poured into the mould, in which it should stand five or six minutes, and then be taken off ; part it as before, and rub the surface of the impression over with some cotton, and the best black lead in powder, which will give it a very fine metallic gloss.

